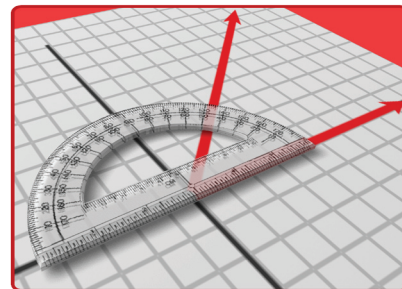


**PRE-ACTIVITY
PREPARATION****Points, Lines, Angles, and Parallel Lines**

Several new types of games illustrate and make use of the basic geometric concepts of points, lines, and planes. Whether the task is to find the location of hidden treasure or to collect as many points as possible while maneuvering through a maze of streets and alleys, you can apply the rules of geometry to many fun activities. Check out these web sites to learn more about how geometry fits into the world around us:

**Geocaching**

<http://www.geocaching.com>

Pac Manhattan

<http://www.pacmanhattan.com/rules.php>

Can You See Me Now?

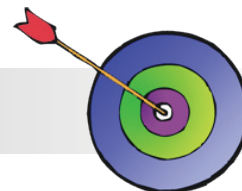
<http://www.canyouseemenow.co.uk/banff/en/intro.php>

Khet™: The Laser Game

<http://www.khet.com>

Play Billiards/Pool

<http://www.nabiscoworld.com/Games> (*sponsored by Nabisco®*)

LEARNING OBJECTIVES

- Start to build a working vocabulary of geometric terms
- Find complementary and supplementary angles
- Determine the measure of angles formed by intersecting lines

TERMINOLOGYPREVIOUSLY USEDNEW TERMS TO LEARN

acute angle

adjacent angle
angle

collinear

complementary angle

degrees

intersecting lines

line

line segment

obtuse angle

parallel

perpendicular

plane

point

ray

reflection

right angle

straight angle

supplementary angle

transversal

vertex

vertical angle

BUILDING MATHEMATICAL LANGUAGE



Geometric Terms

Geometric terms are used to describe figures in space. Listed below are terms that help us communicate ideas and build concepts linking algebra and geometry. Each term represents a basic concept that is a component of how we interact with and measure the world around us.

Point

Symbolized by a dot, a **point** has position, but not size.

A point on a line:



USES: Points describe intersections or locations. Global positioning uses intersecting latitude and longitude to locate a point on Earth.

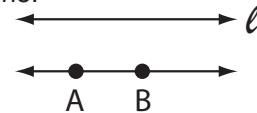
OBSERVATIONS: Points are like the “atoms” of geometry—everything else is made up of them.

Line

A **line** is a collection of points extending in both directions indefinitely. It has length, but no thickness.

A line can be named by a lower case letter or by two points on the line:

Line ℓ or line $\overline{AB}$



USES: Think of a line as a taut string, thread, or microfiber extending forever. Lines of latitude and longitude are imaginary lines on the Earth circling the globe or extending from pole to pole, respectively.

OBSERVATIONS: A line can be described or drawn between any two distinct points. Assume that “line” means a straight line. Points on the same line are **collinear**.

Plane

A **plane** is any flat surface containing points and lines. A plane has length and width, but no depth.

Figures that lie in a plane are called plane figures—they are two-dimensional. Examples are triangles, squares, circles, etc.

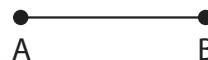


USES: Think of a plane as a wall, or the surface of a mirror. In its purest sense, however, a plane extends indefinitely in all directions.

OBSERVATIONS: Two airplanes flying at different altitudes are in different geometric planes.

Line Segment

A **line segment** is a section or part of a line.



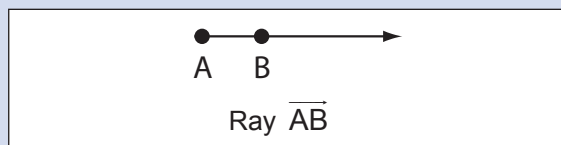
Name segments by their endpoints: $\overline{AB}$

USES: Think of a highway extending in a straight line in both directions. A segment can be between mile marker 102 and 130.

OBSERVATIONS: Unlike lines, line segments have an end and beginning—they can be measured. Typical measurements of length include feet, inches, meters, etc.

Ray

A **ray** is sometimes described as a half line—it has a beginning point but no ending point.



USES: A ray is like a beam of light shone into space—it has a source or beginning—but goes on forever.

OBSERVATIONS: In physics, a vector is represented as a ray.

Angle

An **angle** is formed when two rays with the same beginning point open in different directions. Measure how wide the rays are apart to find the size of the angle in **degrees** ($^\circ$).

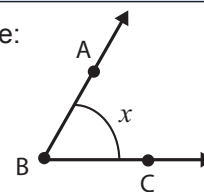
Ways to name a given angle:

Angle B: $\angle B$

Angle ABC: $\angle ABC$

Angle CBA: $\angle CBA$

Angle x: $\angle x$.



USES: Clock hands form angles. A complete revolution of the minute hand measures 360° .

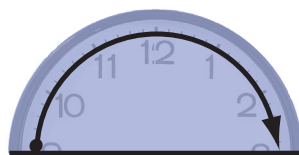
OBSERVATIONS: In the example, B is called the **vertex** of the angle. The angle is named by either its vertex or by three points on the angle with the vertex in the middle. Name angles so that there is no ambiguity and you know exactly which angle you are dealing with.



Angles



One complete revolution of a clock hand is 360° .

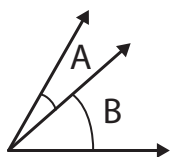


One-half of a revolution of a circle (such as a clock face) represents 180° ; we call this a **straight angle**.

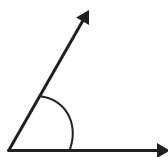


From 12:00 to 3:00, the hour hand makes a sweep of 90° . At 3:00, the hour hand is at a 90° angle with the minute hand.

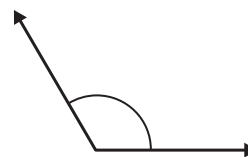
One-fourth of a revolution of a circle is 90° ; notice the corner shape. This size angle is a **right angle**.



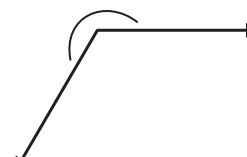
Two angles that are arranged side-by-side, sharing a common ray, are **adjacent angles**.



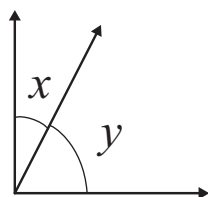
Acute angles are angles measuring less than 90° (from 0° to 90°).



Obtuse angles measure greater than 90° but less than 180° .

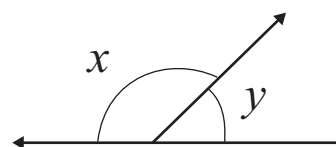


An oblique angle measures greater than 180° .



Two angles are **complementary** if the sum of their angle measures is equal to 90° . If the angles are adjacent they form a right angle (a corner).

$$\angle x = 25^\circ \text{ and } \angle y = 65^\circ \text{ so } \angle x + \angle y = 90^\circ$$



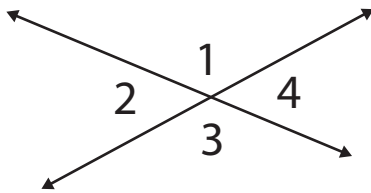
Two angles are **supplementary** if their angle measures add to 180° . If the angles are adjacent they form a straight angle.

$$\angle x = 135^\circ \text{ and } \angle y = 45^\circ \text{ so } \angle x + \angle y = 180^\circ$$



Lines

Given two lines in a plane, one of three situations can occur. The two lines may be:



Intersecting lines

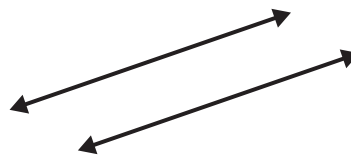
(crossing at one point)

Intersecting lines form four angles: two pairs of equal **vertical angles** ($\angle 2 = \angle 4$ and $\angle 1 = \angle 3$) and four pairs of supplementary angles.

$$(\angle 1 + \angle 2) = (\angle 2 + \angle 3) =$$

$$(\angle 3 + \angle 4) = (\angle 4 + \angle 1) = 180^\circ$$

OR



Parallel lines

Parallel lines do not intersect.

OR



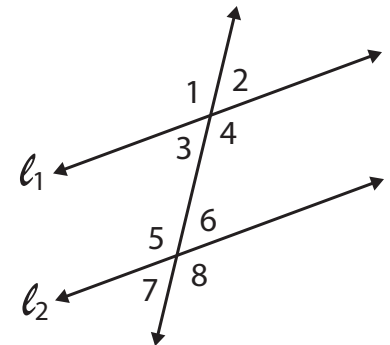
Coincident lines

Coincident lines lay directly on top of each other.



Parallel lines

If two parallel lines ($\ell_1 \parallel \ell_2$) are intersected by a third line, called a **transversal**, eight angles are formed. What can we say about the angles? Examine the figure on the right. The relationships among the eight angles will always be as follows:



Acute angles: $\angle 2 = \angle 3 = \angle 6 = \angle 7$.

Obtuse angles: $\angle 1 = \angle 4 = \angle 5 = \angle 8$.

Vertical angles: $\angle 1 = \angle 4$; $\angle 2 = \angle 3$; $\angle 5 = \angle 8$; $\angle 6 = \angle 7$.

Corresponding angles: $\angle 1 = \angle 5$; $\angle 3 = \angle 7$; $\angle 2 = \angle 6$; $\angle 4 = \angle 8$.

Alternate interior angles: $\angle 3 = \angle 6$ and $\angle 4 = \angle 5$.

Alternate exterior angles: $\angle 1 = \angle 8$ and $\angle 2 = \angle 7$.

Given the measure of *any one* angle, we can find the other seven angles by using the above relationships. For example, if $\angle 8 = 120^\circ$, then we also know that $\angle 5 = \angle 1 = \angle 4 = 120^\circ$.

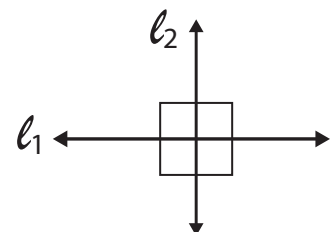
We also know that $\angle 8$ is supplementary to $\angle 7$ because they make a straight angle of 180° .

Therefore $\angle 7 = 60^\circ$ as do $\angle 2$, $\angle 3$, and $\angle 6$.



Perpendicular lines

Two lines are **perpendicular** ($\ell_1 \perp \ell_2$) if their intersection forms four right angles.



MODELS



Model 1

Segment AB is 12 units and BC is 1.5 times as long as AB. Find the length of segment AC.



The length of $\overline{AB} = 12$

The length of $\overline{BC} = 1.5 \times 12 = 18$

$\overline{AB} + \overline{BC} = \overline{AC}$

$12 + 18 = 30$

Answer: $\overline{AC}$ is 30 units long.

Model 2

Two parallel lines are cut by a transversal.

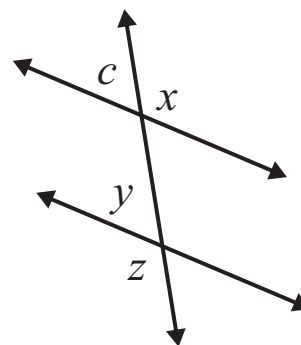
Find the measures of angles y and z if angle x is 125° .

Reasoning:

Angle x and its adjacent angle, $\angle c$, are supplementary; therefore, their sum is 180° . So $\angle c = 55^\circ$.

y is a corresponding angle to $\angle c$, so $y = 55^\circ$

z is supplementary to y , so $z = 125^\circ$



Model 3

Determine the measure of $\angle AOC$ if $\overline{OA} \perp \overline{OB}$ and $\angle BOC$ is $\frac{1}{3} \angle AOB$.

Reasoning:

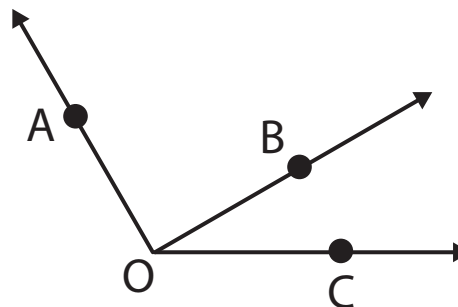
Because $\overline{OA} \perp \overline{OB}$,

$\angle AOB = 90^\circ$; $\angle BOC = \frac{1}{3}(90^\circ) = 30^\circ$

$\angle AOC = \angle AOB + \angle BOC$

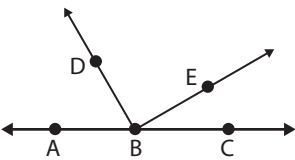
$= 90^\circ + 30^\circ$

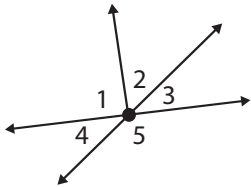
Answer: $\angle AOC = 120^\circ$





ADDRESSING COMMON ERRORS

Issue	Incorrect Process	Resolution	Correct Process	Validation
Mathematical language errors	If $\angle a = 37^\circ$ and $\angle a$ is supplementary to $\angle b$, what is the measure of angle b? $\angle a + \angle b = 90^\circ$ Answer: $\angle b = 53^\circ$. Validate: $37 + 53 = 90$.	Collect all terms in a learning journal with their definitions. Quiz yourself until the terms are solidly in your knowledge base. Vocabulary is critical to success. If you do not know the correct language, you cannot understand the directions.	The word <i>supplementary</i> means that two angles add up to 180°. $\angle a + \angle b = 180^\circ$ $\angle b = 143^\circ$	$37^\circ + 143^\circ = 180^\circ$
Misidentifying angles	 In the figure above, $\angle ABC$ is a straight angle. Which angle is supplementary to $\angle EBC$? Answer: Angle B is supplementary to angle EBC.	Use the three point naming pattern to precisely identify an angle. In the example, angle B could refer to any of the three adjacent angles in the diagram—it is not clear which angle is referenced.	Supplementary angles add to 180°. $\angle EBC$ is adjacent to and makes a straight angle (180°) with $\angle EBA$. Therefore, $\angle EBA$ is supplementary to $\angle EBC$.	
Reasoning errors	Find the complementary angle to an angle measuring 35°. Answer: Complementary angles are 90°, therefore, $35^\circ + 90^\circ = 125^\circ$.	While knowing the definition is required, it is often not enough; you must be able to apply the definition to each situation as needed to get the correct result.	Two angles are complementary if they add to 90°. $35 + \text{what number} = 90?$ $90 - 35 = 65$ 65° is therefore complementary to 35°.	$35^\circ + 65^\circ = 90^\circ$

Issue	Incorrect Process	Resolution	Correct Process	Validation
Making false assumptions	 In the figure above, $\angle 4$ and $\angle 5$ are supplementary, as are $\angle 3$ and $\angle 5$. If $\angle 5 = 147^\circ$, What can you determine about $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$? Answer: $\angle 4$ and $\angle 5$ are supplementary, so $\angle 4 = 33^\circ$. $\angle 4 = \angle 3$. $\angle 1$ is a right angle, so $\angle 1 = 90^\circ$; $\angle 2$ and $\angle 3$ are complements, so $\angle 2 = 57^\circ$	Do not assume that the visual representation is “given” information. In the example, it cannot be determined that $\angle 1 = 90^\circ$, even though it “looks like” it is a right angle.	From the given information, $\angle 4 + \angle 5 = 180^\circ$ $\angle 3 + \angle 5 = 180^\circ$ If $\angle 5 = 147^\circ$, then $\angle 4 = 33^\circ$ and $\angle 3 = 33^\circ$ We also know that $\angle 1 + \angle 2 = \angle 5 = 147^\circ$ (vertical angles) and that $\angle 1 + \angle 2 + \angle 3 = 180^\circ$, but we do not have enough information to determine the measures of $\angle 1$ and $\angle 2$.	$147^\circ + 33^\circ = 180^\circ$

PREPARATION INVENTORY

Before proceeding, you should be able to:

- ☐ Understand and accurately use the vocabulary of geometry
- ☐ Find complementary and supplementary angles
- ☐ Find angle measures made by a line crossing two parallel lines

Points, Lines, Angles, and Parallel Lines

PERFORMANCE CRITERIA



- Using vocabulary correctly
 - correct and appropriate application
 - correct spelling
- Determining angle measures from the given information for a geometric figure
 - demonstration of correct reasoning
 - demonstration of logical process
 - mathematical accuracy

TEAM ACTIVITY

1. Divide into groups.
2. Each group chooses one of the web sites given in the Pre-Activity Preparation section.
3. Investigate the web site and the game rules. Play the game if possible.
4. Record your responses to the following:
 - a. Describe the purpose of the game.
 - b. Describe the basic rules of the game.
 - c. How does the game incorporate basic geometric concepts like points, lines, and planes into the purpose of the game?
 - d. Make a list of the geometric terms and the corresponding game terms, pieces, or moves.
5. Report to the class. Describe the game, the outcome of the game, the geometric concepts used or described in the game, and the degree to which mathematics (geometry) would enhance an expert's chances of winning.

CRITICAL THINKING QUESTIONS

1. Given two angles that are complementary, what types of angles can they be: acute, obtuse, or oblique?
2. What time is it if the hands on a clock make a straight angle and the little hand is on five? (Tip: draw hands on the clock below to show the appropriate angle.)



3. What is the sum of the four angles made by two intersecting lines?
4. If two lines (ℓ_1 and ℓ_2) are both perpendicular to a transversal, what can you say about the alternate interior angles formed by these three lines?
5. Why does a football goal line represent a plane rather than a line?
6. Why do you think it is important to have a common language for any issue?



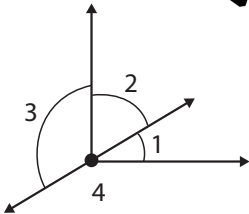
TIPS FOR SUCCESS

- If a diagram is not given, draw one to represent the given situation
- Make your diagram as accurate and to scale as space allows



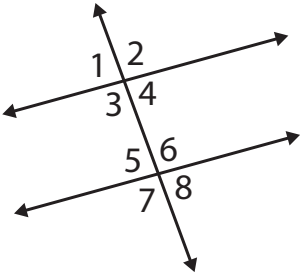
DEMONSTRATE YOUR UNDERSTANDING

1. In the figure at right, $\angle 1$ and $\angle 2$ are complementary angles; $\angle 2$ and $\angle 3$ are supplementary angles; $\angle 1$ and $\angle 4$ are supplementary angles. Given this information, find the requested angle measure(s) for problems a) through e) below.



Problem	Worked Solution	Validation
a) If $\angle 1 = 39^\circ$, what is the measure of $\angle 3$?		
b) If $\angle 4 = 123^\circ$, what is the measure of $\angle 2$?		
c) If $\angle 4 = 131^\circ$, what is the measure of $\angle 3$?		
d) If $\angle 2 = 57^\circ$, what are the measures of $\angle 1$, $\angle 3$, and $\angle 4$?		
e) If $\angle 1 = \angle 2$, what are the measures of $\angle 3$ and $\angle 4$?		

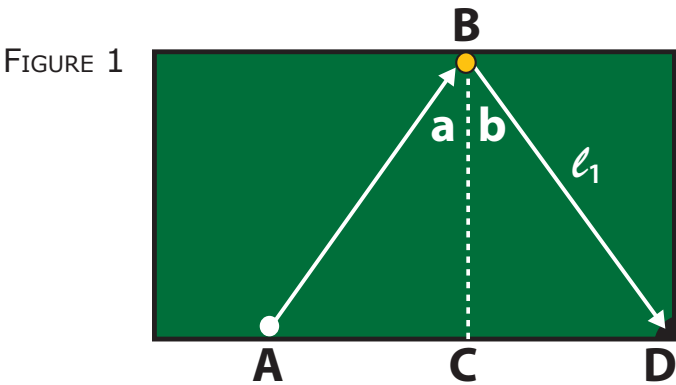
2. In the figure at right, two parallel lines are cut by a transversal to form eight angles. For problems a) through e) below, find the angle measures of the requested angles from the information given.



Problem	Worked Solution	Validation
a) If $\angle 1 = 79^\circ$, what is the measure of $\angle 3$?		
b) If $\angle 4 = 83^\circ$, what is the measure of $\angle 5$?		
c) If $\angle 4 = 131^\circ$, what is the measure of $\angle 3$?		
d) If $\angle 1 = \angle 2$, what are the measures of $\angle 3$ and $\angle 4$?		
e) If $\angle 2 = 67^\circ$, what are the measures of $\angle 5$, $\angle 6$, and $\angle 7$ and $\angle 8$?		

3. Answer the following.

Problem	Worked Solution	Validation
a) Three points A, B, and C, are collinear. What is the length of the segment AC if BC is twice as long as AB and BC measures 6 cm?		
b) The sum of the measures of two angles is 165° and both angles are acute. List three possible pairs of angle measures that would meet these conditions.	Pair 1: $\angle a = \underline{\hspace{1cm}}$ $\angle b = \underline{\hspace{1cm}}$ Pair 2: $\angle a = \underline{\hspace{1cm}}$ $\angle b = \underline{\hspace{1cm}}$ Pair 3: $\angle a = \underline{\hspace{1cm}}$ $\angle b = \underline{\hspace{1cm}}$	
c) In the game of pool, a bank shot is an application of parallel lines and angles. The angle of reflection is equal to the angle of incidence as defined by a line perpendicular to the cushion at impact (point B). If the pocket is collinear with the point of impact and the ball leaves the cushion at a 40° angle ($\angle b$), at what angle must you hit the ball (at point A)? <i>(See the Hints and Figure 1 below for help.)</i>		



Hints:

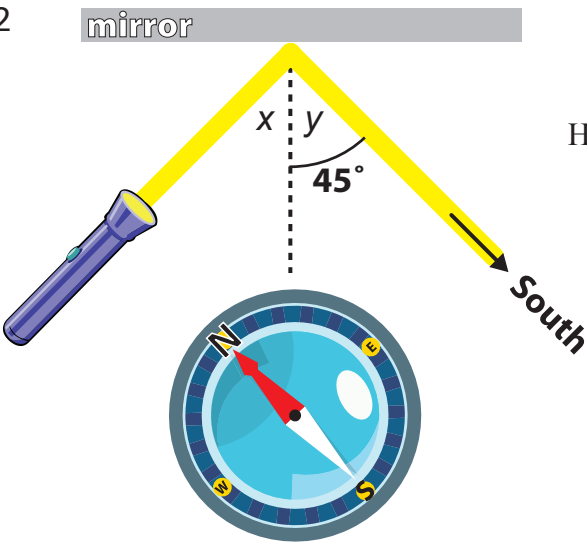
- B = point of impact
- $\angle a$ is the *angle of incidence*
- $\angle b$ is the *angle of reflection*
- $\overline{CB} \perp \overline{CD}$
- Line ℓ_1 is a straight line
(the pocket is collinear with the point of impact)

Problem	Worked Solution	Validation
d) What is the measure of the angle between the hands of a clock, if the time is 10 minutes after 6 o'clock? (Assume that the hour hand will be precisely on the 6.) <i>(Use the clock face below to help you work through the problem.)</i>		



Problem	Worked Solution	Validation
e) If a beam of light is reflected off a mirror at a 45° angle, what was the original direction if the beam is now directed south? <i>(Use Figure 2 below to help you work through the problem.)</i>		

FIGURE 2

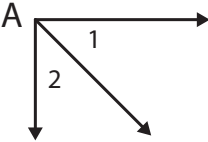
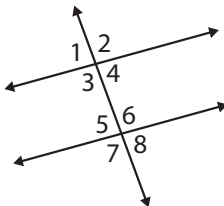


Hint: for a beam of light shined at a mirror, the *angle of incidence* ($\angle x$) is equal to the *angle of reflection* ($\angle y$).

IDENTIFY AND CORRECT THE ERRORS



In the second column, identify the error(s) in the worked solution or validate its answer. If the worked solution is incorrect, solve the problem correctly in the third column and validate your answer.

Worked Solution	Identify Errors or Validate	Correct Process	Validation
1) Find the angle supplementary to 115°. $115 - 90 = 25$ Answer: the angle is 25°			
2) Points A, B, and C are collinear and B is between A and C. Name the angles formed if another line passes through point B. Answer: The new angle is ABC.			
3) Find the measure of angles 1 and 2 if A is a right angle and 1 and 2 are equal.  Answer: $\angle A = 90^\circ$; therefore, $\angle 1 + \angle 2 = 90^\circ$ and $\angle 1$ and $\angle 2$ are both 45°.			
4) Find $\angle 7$ if $\angle 1 = 70^\circ$.  Answer: $\angle 7 = 70^\circ$ as well because every alternate angle is equal.			