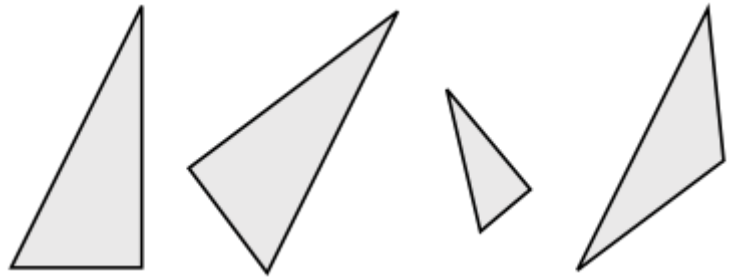


Congruence (geometry)

In [geometry](#), two figures or objects are **congruent** if they have the same [shape](#) and size, or if one has the same shape and size as the [mirror image](#) of the other.^[1]

More formally, two sets of [points](#) are called **congruent** if, and only if, one can be transformed into the other by an [isometry](#), i.e., a combination of **rigid motions**, namely a [translation](#), a [rotation](#), and a [reflection](#). This means that either object can be repositioned and reflected (but not resized) so as to coincide precisely with the other object. So two distinct plane figures on a piece of paper are congruent if we can cut them out and then match them up completely. Turning the paper over is permitted.



An example of congruence. The two triangles on the left are congruent, while the third is similar to them. The last triangle is neither similar nor congruent to any of the others. Congruence permits alteration of some properties, such as location and orientation, but leaves others unchanged, like distance and angles. The unchanged properties are called [invariants](#).

In elementary geometry the word *congruent* is often used as follows.^[2] The word *equal* is often used in place of *congruent* for these objects.

- Two [line segments](#) are congruent if they have the same length.
- Two [angles](#) are congruent if they have the same measure.
- Two [circles](#) are congruent if they have the same diameter

In this sense, *two plane figures are congruent* implies that their corresponding characteristics are "congruent" or "equal" including not just their corresponding sides and angles, but also their corresponding diagonals, perimeters and areas.

The related concept of [similarity](#) applies if the objects have the same shape but do not necessarily have the same size. (Most definitions consider congruence to be a form of similarity, although a minority require that the objects have different sizes in order to qualify as similar)

Contents

Determining congruence of polygons

Congruence of triangles

- Determining congruence
 - Side-side-angle
 - Angle-angle-angle

Definition of congruence in analytic geometry

Congruent conic sections

Congruent polyhedra

Congruent triangles on a sphere

Notation

See also

References

External links

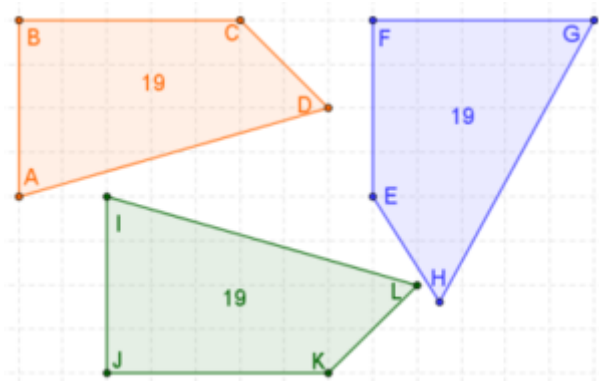
Determining congruence of polygons

For two polygons to be congruent, they must have an equal number of sides (and hence an equal number—the same number—of vertices). Two polygons with n sides are congruent if and only if they each have numerically identical sequences (even if clockwise for one polygon and counterclockwise for the other) side-angle-side-angle... for n sides and n angles.

Congruence of polygons can be established graphically as follows:

- First, match and label the corresponding vertices of the two figures.
- Second, draw a vector from one of the vertices of the one of the figures to the corresponding vertex of the other figure. *Translate* the first figure by this vector so that these two vertices match.
- Third, *rotate* the translated figure about the matched vertex until one pair of corresponding sides matches.
- Fourth, *reflect* the rotated figure about this matched side until the figures match.

If at any time the step cannot be completed, the polygons are not congruent.



The orange and green quadrilaterals are congruent; the blue is not congruent to them. All three have the same perimeter and area. (The ordering of the sides of the blue quadrilateral is "mixed" which results in two of the interior angles and one of the diagonals not being congruent.)

Congruence of triangles

Two triangles are congruent if their corresponding sides are equal in length, in which case their corresponding angles are equal in measure.

If triangle ABC is congruent to triangle DEF, the relationship can be written mathematically as:

$$\triangle ABC \cong \triangle DEF$$

In many cases it is sufficient to establish the equality of three corresponding parts and use one of the following results to deduce the congruence of the two triangles.

Determining congruence

Sufficient evidence for congruence between two triangles in Euclidean space can be shown through the following comparisons:

- **SAS** (Side-Angle-Side): If two pairs of sides of two triangles are equal in length, and the included angles are equal in measurement, then the triangles are congruent.
- **SSS** (Side-Side-Side): If three pairs of sides of two triangles are equal in length, then the triangles are congruent.
- **ASA** (Angle-Side-Angle): If two pairs of angles of two triangles are equal in measurement, and the included sides are equal in length, then the triangles are congruent.
The ASA Postulate was contributed by Thales of Miletus (Greek). In most systems of axioms, the three criteria – SAS, SSS and ASA – are established as theorems. In the School Mathematics Study Group system **SAS** is taken as one (#15) of 22 postulates.
- **AAS** (Angle-Angle-Side): If two pairs of angles of two triangles are equal in measurement, and a pair of corresponding non-included sides are equal in length, then the triangles are congruent. AAS is equivalent to an ASA condition, by the fact that if any two angles are given, so is the third angle, since their sum should be 180° . ASA and AAS are sometimes combined into a single condition **AACorrs** – any two angles and a corresponding side.^[3]
- **RHS** (Right-angle-Hypotenuse-Side), also known as **HL** (Hypotenuse-Leg): If two right-angled triangles have their hypotenuses equal in length, and a pair of shorter sides are equal in length, then the triangles are congruent.

Side-side-angle

The SSA condition (Side-Side-Angle) which specifies two sides and a non-included angle (also known as ASS, or Angle-Side-Side) does not by itself prove congruence. In order to show congruence, additional information is required such as the measure of the corresponding angles and in some cases the lengths of the two pairs of corresponding sides. There are a few possible cases:

If two triangles satisfy the SSA condition and the length of the side opposite the angle is greater than or equal to the length of the adjacent side (SsA, or long side-short side-angle), then the two triangles are congruent. The opposite side is sometimes longer when the corresponding angles are acute, but it is *always* longer when the corresponding angles are right or obtuse. Where the angle is a right angle, also known as the Hypotenuse-Leg (HL) postulate or the Right-angle-Hypotenuse-Side (RHS) condition, the third side can be calculated using the Pythagorean Theorem thus allowing the SSS postulate to be applied.

If two triangles satisfy the SSA condition and the corresponding angles are acute and the length of the side opposite the angle is equal to the length of the adjacent side multiplied by the sine of the angle, then the two triangles are congruent.

If two triangles satisfy the SSA condition and the corresponding angles are acute and the length of the side opposite the angle is greater than the length of the adjacent side multiplied by the sine of the angle (but less than the length of the adjacent side), then the two triangles cannot be shown to be congruent. This is the ambiguous case and two different triangles can be formed from the given information, but further information distinguishing them can lead to a proof of congruence.

Angle-angle-angle

In Euclidean geometry, AAA (Angle-Angle-Angle) (or just AA, since in Euclidean geometry the angles of a triangle add up to 180°) does not provide information regarding the size of the two triangles and hence proves only similarity and not congruence in Euclidean space.

However, in spherical geometry and hyperbolic geometry (where the sum of the angles of a triangle varies with size) AAA is sufficient for congruence on a given curvature of surface.^[4]

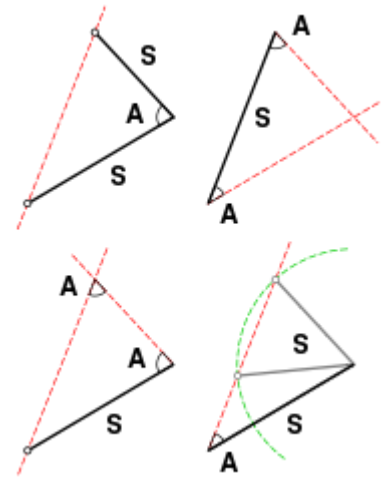
Definition of congruence in analytic geometry

In a Euclidean system, congruence is fundamental; it is the counterpart of equality for numbers. In analytic geometry, congruence may be defined intuitively thus: two mappings of figures onto one Cartesian coordinate system are congruent if and only if, for any two points in the first mapping, the Euclidean distance between them is equal to the Euclidean distance between the corresponding points in the second mapping.

A more formal definition states that two subsets A and B of Euclidean space $\mathbf{R}^n$ are called congruent if there exists an isometry $f: \mathbf{R}^n \rightarrow \mathbf{R}^n$ (an element of the Euclidean group $E(n)$) with $f(A) = B$. Congruence is an equivalence relation

Congruent conic sections

Two conic sections are congruent if their eccentricities and one other distinct parameter characterizing them are equal. Their eccentricities establish their shapes, equality of which is sufficient to establish similarity, and the second parameter then establishes size. Since two circles, parabolas, or rectangular hyperbolas always have the same eccentricity (specifically 0 in the case of circles, 1 in the case of parabolas, and $\sqrt{2}$ in the case of rectangular hyperbolas), two circles, parabolas, or rectangular hyperbolas need to have only one other common parameter value, establishing their size, for them to be congruent.



The shape of a triangle is determined up to congruence by specifying two sides and the angle between them (SAS), two angles and the side between them (ASA) or two angles and a corresponding adjacent side (AAS). Specifying two sides and an adjacent angle (SSA), however, can yield two distinct possible triangles.

Congruent polyhedra

For two polyhedra with the same number E of edges, the same number of faces, and the same number of sides on corresponding faces, there exists a set of at most E measurements that can establish whether or not the polyhedra are congruent.^{[5][6]} For cubes, which have 12 edges, only 9 measurements are necessary

Congruent triangles on a sphere

As with plane triangles, on a sphere two triangles sharing the same sequence of angle-side-angle (ASA) are necessarily congruent (that is, they have three identical sides and three identical angles).^[7] This can be seen as follows: One can situate one of the vertices with a given angle at the south pole and run the side with given length up the prime meridian. Knowing both angles at either end of the segment of fixed length ensures that the other two sides emanate with a uniquely determined trajectory, and thus will meet each other at a uniquely determined point; thus ASA is valid.

The congruence theorems side-angle-side (SAS) and side-side-side (SSS) also hold on a sphere; in addition, if two spherical triangles have an identical angle-angle-angle (AAA) sequence, they are congruent (unlike for plane triangles^[7]).

The plane-triangle congruence theorem angle-angle-side (AAS) does not hold for spherical triangles.^[8] As in plane geometry, side-side-angle (SSA) does not imply congruence.

Notation

A symbol commonly used for congruence is an equals symbol with a tilde above it, $\cong$, corresponding to the Unicode character 'approximately equal to' (U+2245). In the UK, the three-bar equal sign $\equiv$ (U+2261) is sometimes used.

See also

- CPCTC (Corresponding parts of congruent triangles are congruent)
- Euclidean plane isometry
- Isometry

References

- Clapham, C.; Nicholson, J. (2009). "Oxford Concise Dictionary of Mathematics, Congruent Figures" (<https://web.archive.org/web/20131029203826/http://web.cortland.edu/matresearch/OxfordDictionaryMathematics.pdf>) (PDF). Addison-Wesley. p. 167. Archived from the original on 29 October 2013Retrieved 2 June 2017.
- "Congruence" (<http://mathopenref.com/congruent.html>) Math Open Reference. 2009 Retrieved 2 June 2017.
- Parr, H. E. (1970). *Revision Course in School mathematics* Mathematics Textbooks Second Edition. G Bell and Sons Ltd. ISBN 0-7135-1717-4
- Cornel, Antonio (2002). *Geometry for Secondary Schools* Mathematics Textbooks Second Edition. Bookmark Inc. ISBN 971-569-441-1
- Alexander Boriso, Mark Dickinson, and Stuart Hastings, "A congruence problem for polyhedra" *American Mathematical Monthly* 117, March 2010, pp. 232-249.^[1] (<http://www.maa.org/programs/maa-awards/writing-awards/a-congruence-problem-for-polyhedra>)
- Alexa Creech, "A congruence problem" http://146.163.152.131/teaching/projects/creech_final.pdf Archived (https://web.archive.org/web/20131111162553/http://146.163.152.131/teaching/projects/creech_final.pdf) November 11, 2013, at the Wayback Machine
- Michael Bolin, "Exploration of Spherical Geometry", September 9, 2003, pp.6-7. http://math.iit.edu/~mccomic/420/notes/Bolin_spherical.pdf
- Hollyer, L., <http://www.uh.edu/~hollyer/Module6/m6ppt/slides089.htm>

External links

- [The SSS at Cut-the-Knot](#)
 - [The SSA at Cut-the-Knot](#)
 - Interactive animations demonstrating [Congruent polygons](#) [Congruent angles](#) [Congruent line segments](#) [Congruent triangles](#) at Math Open Reference
-

Retrieved from '[https://en.wikipedia.org/w/index.php?title=Congruence_\(geometry\)&oldid=826949271](https://en.wikipedia.org/w/index.php?title=Congruence_(geometry)&oldid=826949271)

This page was last edited on 21 February 2018, at 22:20.

Text is available under the [Creative Commons Attribution-ShareAlike License](#); additional terms may apply. By using this site, you agree to the [Terms of Use](#) and [Privacy Policy](#). Wikipedia® is a registered trademark of the [Wikimedia Foundation, Inc.](#), a non-profit organization.