

5-4 The Triangle Inequality

What You'll Learn

- Apply the Triangle Inequality Theorem.
- Determine the shortest distance between a point and a line.

How can you use the Triangle Inequality Theorem when traveling?

Chuck Noland travels between Chicago, Indianapolis, and Columbus as part of his job. Mr. Noland lives in Chicago and needs to get to Columbus as quickly as possible. Should he take a flight that goes from Chicago to Columbus, or a flight that goes from Chicago to Indianapolis, then to Columbus?



THE TRIANGLE INEQUALITY In the example above, if you chose to fly directly from Chicago to Columbus, you probably reasoned that a straight route is shorter. This is an example of the Triangle Inequality Theorem.

Theorem 5.11

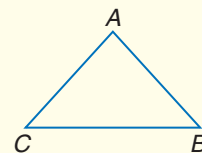
Triangle Inequality Theorem The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

Examples:

$$AB + BC > AC$$

$$BC + AC > AB$$

$$AC + AB > BC$$



You will prove Theorem 5.11 in Exercise 40.

The Triangle Inequality Theorem can be used to determine whether three segments can form a triangle.

Study Tip

Inequality

If the sum of the smallest number and the middle number is greater than the largest number, then each combination of inequalities are true.

Example 1 Identify Sides of a Triangle

Determine whether the given measures can be the lengths of the sides of a triangle.

a. 2, 4, 5

Check each inequality.

$$2 + 4 \stackrel{?}{>} 5$$

$$6 > 5 \checkmark$$

$$2 + 5 \stackrel{?}{>} 4$$

$$7 > 4 \checkmark$$

$$4 + 5 \stackrel{?}{>} 2$$

$$9 > 2 \checkmark$$

All of the inequalities are true, so 2, 4, and 5 can be the lengths of the sides of a triangle.

b. 6, 8, 14

$$6 + 8 \stackrel{?}{>} 14$$

$$14 \not> 14$$

Because the sum of two measures equals the measure of the third side, the sides cannot form a triangle.

When you know the lengths of two sides of a triangle, you can determine the range of possible lengths for the third side.

Standardized Test Practice

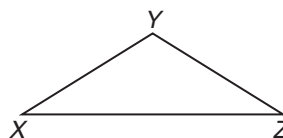
A B C D

Example 2 Determine Possible Side Length

Multiple-Choice Test Item

In $\triangle XYZ$, $XY = 8$, and $XZ = 14$. Which measure cannot be YZ ?

- (A) 6 (B) 10
(C) 14 (D) 18



Read the Test Item

You need to determine which value is not valid.

Solve the Test Item

Solve each inequality to determine the range of values for YZ .

Let $YZ = n$.

$$XY + XZ > YZ$$

$$8 + 14 > n$$

$$22 > n \text{ or } n < 22$$

$$XY + YZ > XZ$$

$$8 + n > 14$$

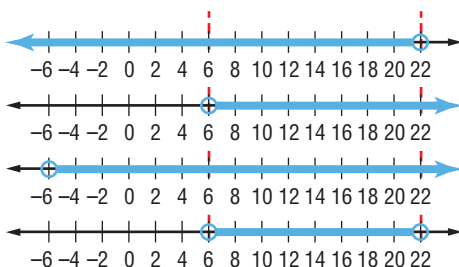
$$n > 6$$

$$YZ + XZ > XY$$

$$n + 14 > 8$$

$$n > -6$$

Graph the inequalities on the same number line.



Graph $n < 22$.

Graph $n > 6$.

Graph $n > -6$.

Find the intersection.

The range of values that fit all three inequalities is $6 < n < 22$.

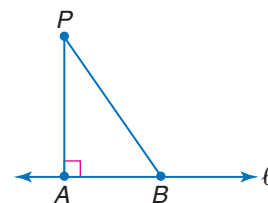
Examine the answer choices. The only value that does not satisfy the compound inequality is 6 since $6 = 6$. Thus, the answer is choice A.

Test-Taking Tip

Testing Choices If you are short on time, you can test each choice to find the correct answer and eliminate any remaining choices.

DISTANCE BETWEEN A POINT AND A LINE

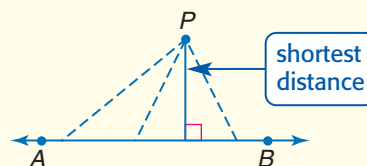
Recall that the distance between point P and line ℓ is measured along a perpendicular segment from the point to the line. It was accepted without proof that $\overline{PA}$ was the shortest segment from P to ℓ . The theorems involving the relationships between the angles and sides of a triangle can now be used to prove that a perpendicular segment is the shortest distance between a point and a line.



Theorem 5.12

The perpendicular segment from a point to a line is the shortest segment from the point to the line.

Example: $\overline{PQ}$ is the shortest segment from P to $\overleftrightarrow{AB}$.

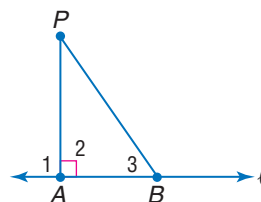


Example 3 Prove Theorem 5.12

Given: $\overline{PA} \perp \ell$
 $\overline{PB}$ is any nonperpendicular segment from P to ℓ .

Prove: $PB > PA$

Proof:



Study Tip

Shortest Distance to a Line

If a line is horizontal, the shortest distance from a point to that line will be along a vertical line. Likewise, the shortest distance from a point to a vertical line lies along a horizontal line.

Statements

1. $\overline{PA} \perp \ell$
2. $\angle 1$ and $\angle 2$ are right angles.
3. $\angle 1 \cong \angle 2$
4. $m\angle 1 = m\angle 2$
5. $m\angle 1 > m\angle 3$
6. $m\angle 2 > m\angle 3$
7. $PB > PA$

Reasons

1. Given
2. $\perp$ lines form right angles.
3. All right angles are congruent.
4. Definition of congruent angles
5. Exterior Angle Inequality Theorem
6. Substitution Property
7. If an angle of a triangle is greater than a second angle, then the side opposite the greater angle is longer than the side opposite the lesser angle.

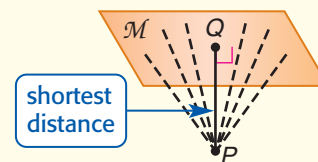
Corollary 5.1 follows directly from Theorem 5.12.

Corollary 5.1

The perpendicular segment from a point to a plane is the shortest segment from the point to the plane.

Example:

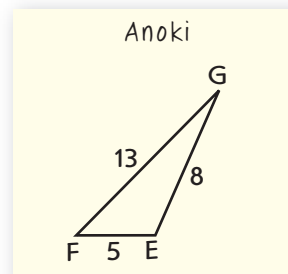
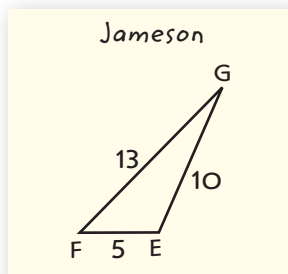
$\overline{QP}$ is the shortest segment from P to Plane M .



You will prove Corollary 5.1 in Exercise 12.

Check for Understanding

- Concept Check**
1. **Explain** why the distance between two nonhorizontal parallel lines on a coordinate plane cannot be found using the distance between their y -intercepts.
 2. **FIND THE ERROR** Jameson and Anoki drew $\triangle EFG$ with $FG = 13$ and $EF = 5$. They each chose a possible measure for GE .



Who is correct? Explain.

3. **OPEN ENDED** Find three numbers that can be the lengths of the sides of a triangle and three numbers that cannot be the lengths of the sides of a triangle. Justify your reasoning with a drawing.



Guided Practice

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

4. 5, 4, 3

5. 5, 15, 10

6. 30.1, 0.8, 31

7. 5.6, 10.1, 5.2

Find the range for the measure of the third side of a triangle given the measures of two sides.

8. 7 and 12

9. 14 and 23

10. 22 and 34

11. 15 and 18

12. **PROOF** Write a proof for Corollary 5.1.

Given: $\overline{PQ} \perp \text{plane } \mathcal{M}$

Prove: $\overline{PQ}$ is the shortest segment from P to plane $\mathcal{M}$.

13. An isosceles triangle has a base 10 units long. If the congruent sides have whole number measures, what is the least possible length of the sides?

(A) 5

(B) 6

(C) 17

(D) 21

Standardized Test Practice

Practice and Apply

Homework Help

For Exercises

14–25
26–37
38–40

See Examples

1
2
3

Extra Practice
See page 764.

Determine whether the given measures can be the lengths of the sides of a triangle. Write *yes* or *no*. Explain.

14. 1, 2, 3

15. 2, 6, 11

16. 8, 8, 15

17. 13, 16, 29

18. 18, 32, 21

19. 9, 21, 20

20. 5, 17, 9

21. 17, 30, 30

22. 8.4, 7.2, 3.5

23. 4, 0.9, 4.1

24. 14.3, 12, 2.2

25. 0.18, 0.21, 0.52

Find the range for the measure of the third side of a triangle given the measures of two sides.

26. 5 and 11

27. 7 and 9

28. 10 and 15

29. 12 and 18

30. 21 and 47

31. 32 and 61

32. 30 and 30

33. 64 and 88

34. 57 and 55

35. 75 and 75

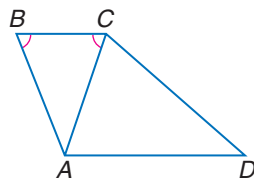
36. 78 and 5

37. 99 and 2

PROOF Write a two-column proof.

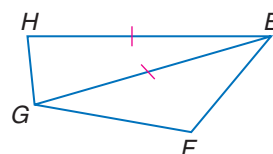
38. **Given:** $\angle B \cong \angle ACB$

Prove: $AD + AB > CD$



39. **Given:** $\overline{HE} \cong \overline{EG}$

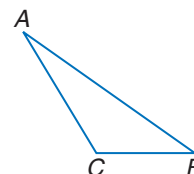
Prove: $HE + FG > EF$



40. **Given:** $\angle ABC$

Prove: $AC + BC > AB$ (Triangle Inequality Theorem)

(Hint: Draw auxiliary segment $\overline{CD}$, so that C is between B and D and $\overline{CD} \cong \overline{AC}$.)



ALGEBRA Determine whether the given coordinates are the vertices of a triangle. Explain.

41. $A(5, 8), B(2, -4), C(-3, -1)$

42. $L(-24, -19), M(-22, 20), N(-5, -7)$

43. $X(0, -8), Y(16, -12), Z(28, -15)$

44. $R(1, -4), S(-3, -20), T(5, 12)$

CRAFTS For Exercises 45 and 46, use the following information.

Carlota has several strips of trim she wishes to use as a triangular border for a section of a decorative quilt she is going to make. The strips measure 3 centimeters, 4 centimeters, 5 centimeters, 6 centimeters, and 12 centimeters.

45. How many different triangles could Carlota make with the strips?

46. How many different triangles could Carlota make that have a perimeter that is divisible by 3?

- 47. **HISTORY** The early Egyptians used to make triangles by using a rope with knots tied at equal intervals. Each vertex of the triangle had to occur at a knot. How many different triangles can be formed using the rope below?



PROBABILITY For Exercises 48 and 49, use the following information.

One side of a triangle is 2 feet long. Let m represent the measure of the second side and n represent the measure of the third side. Suppose m and n are whole numbers and that $14 < m < 17$ and $13 < n < 17$.

48. List the measures of the sides of the triangles that are possible.

49. What is the probability that a randomly chosen triangle that satisfies the given conditions will be isosceles?

50. **CRITICAL THINKING** State and prove a theorem that compares the measures of each side of a triangle with the differences of the measures of the other two sides.

51. **WRITING IN MATH** Answer the question that was posed at the beginning of the lesson.

How can you use the Triangle Inequality when traveling?

Include the following in your answer:

- an example of a situation in which you might want to use the greater measures, and
- an explanation as to why it is not always possible to apply the Triangle Inequality when traveling.

**Standardized
Test Practice**

A B C D

52. If two sides of a triangle measure 12 and 7, which of the following cannot be the perimeter of the triangle?

(A) 29

(B) 34

(C) 37

(D) 38



53. **ALGEBRA** How many points of intersection exist if the equations $(x - 5)^2 + (y - 5)^2 = 4$ and $y = -x$ are graphed on the same coordinate plane?

(A) none

(B) one

(C) two

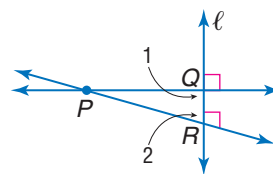
(D) three



Maintain Your Skills

Mixed Review 54. **PROOF** Write an indirect proof. (Lesson 5-3)

Given: P is a point not on line ℓ .
Prove: $\overline{PQ}$ is the only line through P perpendicular to ℓ .



ALGEBRA List the sides of $\triangle PQR$ in order from longest to shortest if the angles of $\triangle PQR$ have the given measures. (Lesson 5-2)

55. $m\angle P = 7x + 8$, $m\angle Q = 8x - 10$, $m\angle R = 7x + 6$

56. $m\angle P = 3x + 44$, $m\angle Q = 68 - 3x$, $m\angle R = x + 61$

Determine whether $\triangle JKL \cong \triangle PQR$ given the coordinates of the vertices. Explain. (Lesson 4-4)

57. $J(0, 5)$, $K(0, 0)$, $L(-2, 0)$, $P(4, 8)$, $Q(4, 3)$, $R(6, 3)$

58. $J(6, 4)$, $K(1, -6)$, $L(-9, 5)$, $P(0, 7)$, $Q(5, -3)$, $R(15, 8)$

59. $J(-6, -3)$, $K(1, 5)$, $L(2, -2)$, $P(2, -11)$, $Q(5, -4)$, $R(10, -10)$

Getting Ready for the Next Lesson **PREREQUISITE SKILL** Solve each inequality. (To review solving inequalities, see pages 739 and 740.)

60. $3x + 54 < 90$

61. $8x - 14 < 3x + 19$

62. $4x + 7 < 180$

Practice Quiz 2

Lessons 5-3 and 5-4

Write the assumption you would make to start an indirect proof of each statement. (Lesson 5-3)

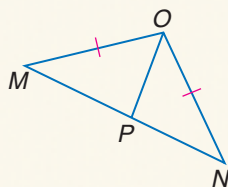
- The number 117 is divisible by 13.
- $m\angle C < m\angle D$

Write an indirect proof. (Lesson 5-3)

- If $7x > 56$, then $x > 8$.

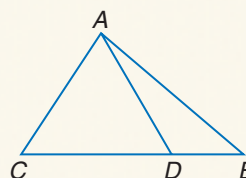
4. **Given:** $\overline{MO} \cong \overline{ON}$, $\overline{MP} \not\cong \overline{NP}$

Prove: $\angle MOP \not\cong \angle NOP$



5. **Given:** $m\angle ADC \neq m\angle ADB$

Prove: $\overline{AD}$ is not an altitude of $\triangle ABC$.



Determine whether the given measures can be the lengths of the sides of a triangle.

Write *yes* or *no*. Explain. (Lesson 5-4)

6. 7, 24, 25

7. 25, 35, 60

8. 20, 3, 18

9. 5, 10, 6

- If the measures of two sides of a triangle are 57 and 32, what is the range of possible measures of the third side? (Lesson 5-4)