

10

Mid-point Theorem

INTRODUCTION

We have learnt many properties of a triangle. Now, let us learn another result which is related to the mid-points of sides of a triangle.

Perform the following activity:

Draw any triangle ABC. Mark mid-points E and F of sides AB and AC respectively. Join points E and F. Measure EF and BC. Also measure $\angle AEF$ and $\angle ABC$.

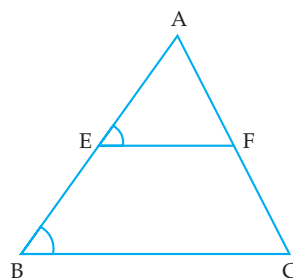
We observe that $EF = \frac{1}{2} BC$ and $\angle AEF = \angle ABC$.

But these angles are a pair of corresponding angles formed by lines EF, BC and transversal AB, therefore, $EF \parallel BC$.

Repeat this activity with more triangles. Every time we get the same result.

The result is known as mid-point theorem.

In this chapter, we shall learn mid-point theorem and its converse and theorem on intercepts. We shall also learn their applications.



10.1 MID-POINT THEOREM

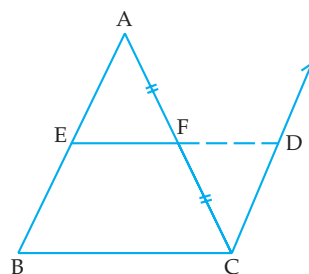
The line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is equal to half of it.

Given. A triangle ABC, E and F are the mid-points of sides AB and AC respectively.

To prove. $EF \parallel BC$ and $EF = \frac{1}{2} BC$.

Construction. Through C, draw a line parallel to BA to meet EF produced at D.

Proof.



Statements	Reasons
In $\triangle AEF$ and $\triangle CDF$	
1. $AF = CF$	1. F is mid-point of AC (given).
2. $\angle AFE = \angle CFD$	2. Vert. opp. $\angle$ s

3. $\angle EAF = \angle DCF$	3. Alt. $\angle$ s, $BA \parallel CD$ (by construction) and AC is a transversal.
4. $\triangle AEF \cong \triangle CDF$	4. ASA rule of congruency.
5. $EF = FD$ and $AE = CD$	5. c.p.c.t.
6. $AE = BE$	6. E is mid-point of AB (given).
7. $BE = CD$	7. From 5 and 6
8. $EBCD$ is a parallelogram.	8. $BA \parallel CD$ (const.) and $BE = CD$ (from 7)
9. $EF \parallel BC$ and $ED = BC$	9. Since $EBCD$ is a parallelogram.
10. $EF = \frac{1}{2} ED$	10. Since $EF = FD$, from 5
11. $EF = \frac{1}{2} BC$	11. Since $ED = BC$, from 9
Hence, $EF \parallel BC$ and $EF = \frac{1}{2} BC$.	

The converse of the above theorem is also true. In fact, we have

Converse of mid-point theorem

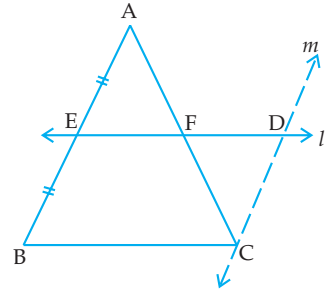
The line drawn through the mid-point of one side of a triangle parallel to another side bisects the third side.

Given. A triangle ABC , E is mid-point of AB . Line l drawn through E and parallel to BC meeting AC at F .

To prove. $AF = FC$.

Construction. Through C , draw a line m parallel BA to meet line l at D .

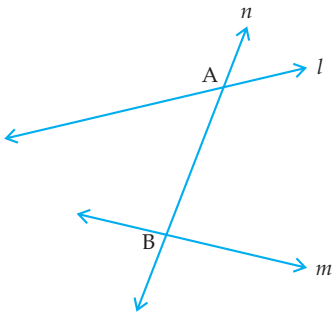
Proof.



Statements	Reasons
1. $EBCD$ is a parallelogram	1. $EF \parallel BC$ (given), $BA \parallel CD$ (const.).
2. $BE = CD$	2. Opp. sides of a parallelogram are equal.
3. $EA = BE$	3. E is mid-point of AB (given).
4. $EA = CD$	4. From 2 and 3.
In $\triangle AEF$ and $\triangle CDF$	
5. $\angle EAF = \angle DCF$	5. Alt. $\angle$ s, $CD \parallel BA$ and AC is a transversal.
6. $\angle EFA = \angle DFC$	6. Vert. opp. $\angle$ s
7. $EA = CD$	7. From 4
8. $\triangle AEF \cong \triangle CDF$	8. AAS rule of congruency
9. $AF = FC$	9. c.p.c.t.
Hence, F is mid-point of AC .	

□ Intercepts

If a line n intersects two lines l and m (drawn in a plane) at points A and B respectively, then the line segment AB is called the *intercept* made on line n by the lines l and m .



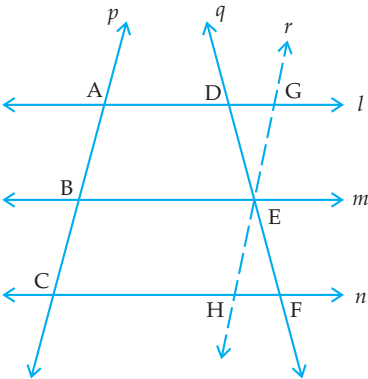
□ Theorem on intercepts

If a transversal makes equal intercepts on three (or more) parallel lines, then any other line cutting them also makes equal intercepts.

Given. Three parallel lines l, m and n . A transversal p cutting them at points A, B and C respectively such that $AB = BC$. Any other line q cuts them at points D, E and F respectively.

To prove. $DE = EF$.

Construction. Through E, draw a line r parallel to line p to meet line l at G and line n at H.



Proof.

Statements	Reasons
1. ABEG is a parallelogram.	1. $AG \parallel BE$ (given), $AB \parallel GE$ (const.).
2. $AB = GE$	2. Opp. sides of a parallelogram are equal.
3. BCHE is a parallelogram.	3. $BE \parallel CH$ (given), $BC \parallel EH$ (const.).
4. $BC = EH$	4. Opp. sides of a parallelogram are equal.
5. $GE = EH$	5. From 2 and 4; also $AB = BC$ (given)
In $\triangle DEG$ and $\triangle FEH$	
6. $\angle EGD = \angle FHE$	6. Alt. $\angle$ s, $AG \parallel CH$ (given) and GH is a transversal.
7. $\angle DEG = \angle FEH$	7. Vert. opp. $\angle$ s are equal.
8. $GE = EH$	8. From 5 (proved above)
9. $\triangle DEG \cong \triangle FEH$	9. ASA rule of congruency.
10. $DE = EF$	10. c.p.c.t.

Illustrative Examples

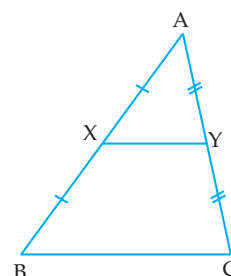
Example 1. In the adjoining figure, X and Y are mid-points of sides AB and AC respectively of $\triangle ABC$. If $BC = 6$ cm, $AB = 7.4$ cm and $AC = 6.4$ cm, then find the perimeter of trapezium $XBCY$.

Solution. Since X is mid-point of AB and Y is mid-point of AC , therefore, XY is parallel to BC and $XY = \frac{1}{2} BC = (\frac{1}{2} \times 6)$ cm
 $= 3$ cm.

Also
$$XB = \frac{1}{2} AB = (\frac{1}{2} \times 7.4) \text{ cm} = 3.7 \text{ cm and}$$

$$YC = \frac{1}{2} AC = (\frac{1}{2} \times 6.4) \text{ cm} = 3.2 \text{ cm}$$

$$\therefore \text{ Perimeter of trapezium } XBCY = (3.7 + 6 + 3.2 + 3) \text{ cm} = 15.9 \text{ cm.}$$



Example 2. D , E and F are mid-points of the sides BC , CA and AB respectively of an equilateral triangle ABC . Show that $\triangle DEF$ is also an equilateral triangle.

Solution. As F and E are mid-points of AB and CA respectively of $\triangle ABC$.

$$FE = \frac{1}{2} BC \quad (\text{by Mid-point theorem})$$

Similarly,
$$FD = \frac{1}{2} AC \text{ and } DE = \frac{1}{2} AB$$

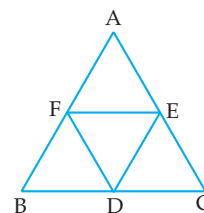
Given, $\triangle ABC$ is equilateral triangle.

$$\therefore AB = BC = CA$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} BC = \frac{1}{2} CA$$

$$\Rightarrow DE = FE = FD.$$

Hence, $\triangle DEF$ is also an equilateral triangle.



Example 3. Show that the line segments joining the mid-points of the opposite sides of a quadrilateral bisect each other.

Solution. Let $ABCD$ be a quadrilateral and P , Q , R , S be mid-points of sides AB , BC , CD , AD respectively. We need to show that the line segments PR and SQ bisect each other.

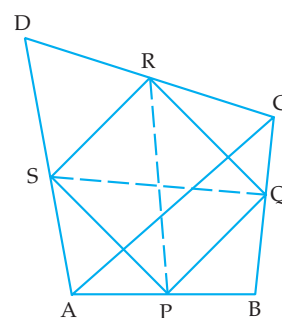
In $\triangle ABC$, P and Q are mid-points of sides AB and BC respectively, therefore,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC.$$

Similarly,
$$SR \parallel AC \text{ and } SR = \frac{1}{2} AC$$

$$\Rightarrow PQ \parallel SR \text{ and } PQ = SR \Rightarrow PQRS \text{ is a parallelogram.}$$

As the diagonals of a parallelogram bisect each other, therefore, the line segments PR and SQ bisect each other.



Example 4. $ABCD$ is a rhombus and P , Q , R and S are the mid-points of the sides AB , BC , CD and DA respectively. Show that the quadrilateral $PQRS$ is a rectangle.

Solution. Given $ABCD$ is a rhombus and P , Q , R and S are mid-points of the sides AB , BC , CD and DA respectively. We need to show that $PQRS$ is a rectangle.

Let the diagonals AC and BD of the rhombus ABCD intersect at O.

In $\triangle ABC$, P and Q are the mid-points of the sides AB and BC respectively, therefore, by mid-point theorem,

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC.$$

Similarly, $SR \parallel AC \text{ and } SR = \frac{1}{2} AC$

$$\Rightarrow PQ \parallel SR \text{ and } PQ = SR$$

Thus, in quadrilateral PQRS, $PQ \parallel SR$ and $PQ = SR$, therefore, PQRS is a parallelogram.

In $\triangle ABD$, P and S are mid-points of AB and AD respectively, therefore,

$$PS \parallel BD$$

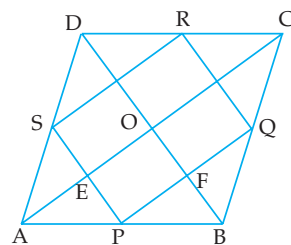
Thus, $EP \parallel OF$ and $PF \parallel EO$, therefore, OEFP is a parallelogram

But $\angle EOF = 90^\circ$ ($\because$ diagonals of a rhombus intersect at right angles)

Also, in a parallelogram opposite angles are equal

$$\therefore \angle EPF = \angle EOF = 90^\circ.$$

Thus, PQRS is a parallelogram in which one angle i.e. $\angle P = 90^\circ$, therefore, PQRS is a rectangle.



Example 5. ABCD is a rectangle and P, Q, R and S are the mid-points of the sides AB, BC, CD and DA respectively. Show that the quadrilateral PQRS is a rhombus.

Solution. Given ABCD is a rectangle and P, Q, R and S are mid-points of the sides AB, BC, CD and DA respectively. We need to prove that PQRS is a rhombus.

In $\triangle ABC$, P and Q are mid-points of the sides AB and BC respectively, therefore, by mid-point theorem

$$PQ \parallel AC \text{ and } PQ = \frac{1}{2} AC.$$

Similarly, $SR \parallel AC \text{ and } SR = \frac{1}{2} AC$

$$\Rightarrow PQ \parallel SR \text{ and } PQ = SR$$

Thus, in quadrilateral PQRS, $PQ \parallel SR$ and $PQ = SR$, therefore, PQRS is a parallelogram.

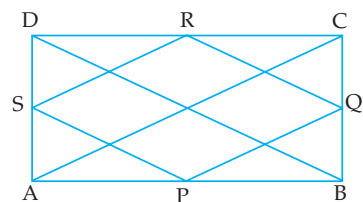
In $\triangle ABD$, P and S are mid-points of the sides AB and AD respectively, therefore,

$$PS \parallel BD \text{ and } PS = \frac{1}{2} BD.$$

But $AC = BD$ ($\because$ diagonals of a rectangle are equal)

$$\therefore PQ = PS.$$

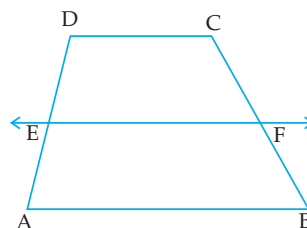
Thus, PQRS is a parallelogram in which two adjacent sides are equal, therefore, PQRS is a rhombus.



Example 6. In the adjoining figure, ABCD is a trapezium in which $AB \parallel DC$ and E is mid-point of AD. A line is drawn through E parallel to AB intersecting BC at F. Show that F is mid-point of BC.

Solution. Join BD and let BD intersect EF at G.

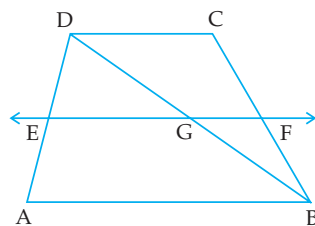
In $\triangle DAB$, E is mid-point of AD and $EF \parallel AB$ i.e. $EG \parallel AB$, therefore, by the converse of mid-point theorem, G is mid-point of BD.



Now, $DC \parallel AB$ (given) and $EF \parallel AB$

$\Rightarrow DC \parallel EF \Rightarrow DC \parallel GF$.

In $\triangle BCD$, G is mid-point of BD and $GF \parallel DC$, therefore, by the converse of mid-point theorem, F is mid-point of BC .



Example 7. ABC is a triangle right angled at C . A line through the mid-point M of hypotenuse AB and parallel to BC intersects AC at D . Show that

(i) D is mid-point of AC (ii) $MD \perp AC$ (iii) $CM = MA = \frac{1}{2} AB$.

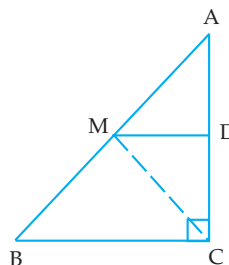
Solution. (i) In $\triangle ABC$, M is mid-point of AB and MD is drawn parallel to BC , therefore, by the converse of mid-point theorem, D is mid-point of AC .

(ii) As $MD \parallel BC$ and AC is a transversal,

$$\angle BCD + \angle CDM = 180^\circ \quad (\text{sum of co-int. } \angle\text{s})$$

$$\Rightarrow 90^\circ + \angle CDM = 180^\circ \quad (\because \angle BCA = 90^\circ, \text{ given})$$

$$\Rightarrow \angle CDM = 90^\circ \Rightarrow MD \perp AC.$$



(iii) In $\triangle AMD$ and $\triangle CMD$,

$$AD = DC$$

($\because D$ is mid-point of AC from part (i))

$$\angle ADM = \angle CDM$$

(each = 90° , $MD \perp AC$)

$$MD = MD$$

(common)

$$\therefore \triangle ADM \cong \triangle CMD$$

(by SAS rule of congruency)

$$\therefore AM = CM$$

(c.p.c.t.)

Also, as M is mid-point of AB , $AM = \frac{1}{2} AB$.

$$\therefore CM = AM = \frac{1}{2} AB.$$

Example 8. In the adjoining figure, $ABCD$ is a parallelogram and E, F are the mid-points of the sides AB, CD respectively. Show that the line segments AF and EC trisect the diagonal BD .

Solution. As $ABCD$ is a parallelogram, $AB \parallel DC$

$$\Rightarrow AE \parallel FC.$$

$$\text{Also } AB = DC$$

$$\Rightarrow \frac{1}{2} AB = \frac{1}{2} DC$$

$$\Rightarrow AE = FC \quad (\because E \text{ is mid-point of } AB \text{ and } F \text{ is mid-point of } CD)$$

Thus, in the quadrilateral $AECF$, $AE = FC$ and $AE \parallel FC$

$\Rightarrow AECF$ is a parallelogram $\Rightarrow AF \parallel EC$.

$$\text{In } \triangle DQC, \quad PF \parallel QC$$

($\because AF \parallel EC$ proved above)

and F is mid-point of CD

$\therefore P$ is mid-point of DQ

(using converse of mid-point theorem)

$$\Rightarrow DP = PQ$$

...(i)

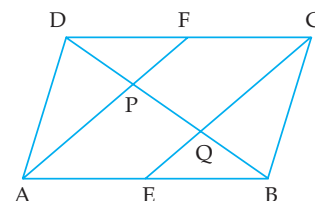
$$\text{In } \triangle ABP, \quad EQ \parallel AP$$

($\because EC \parallel AF$, from above)

and E is mid-point of AB

$\therefore Q$ is mid-point of BP

(using converse of mid-point theorem)



(opp. sides of a $\parallel$ gm are equal)

$$\Rightarrow \quad BQ = PQ \quad \dots(ii)$$

From (i) and (ii), we get $DP = PQ = BQ$

$\Rightarrow$ the line segments AF and EC trisect the diagonal BD .

Example 9. P is the mid-point of the side CD of a parallelogram. A line through C parallel to PA intersects AB at Q and DA produced at R . Prove that $DA = AR$ and $CQ = QR$.

Solution. Draw the figure according to given conditions.

In $\triangle DCR$, P is mid-point of side CD and $PA \parallel CR$, therefore, by converse of mid-point theorem, A is mid-point of DR and $AP = \frac{1}{2} CR$.

As A is mid-point of DR , so $DA = AR$.

Now $AB \parallel DC$ ($\because$ $ABCD$ is a $\parallel$ gm)

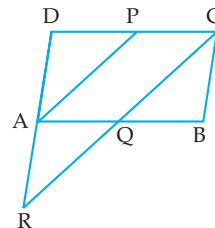
$\Rightarrow AQ \parallel PC$.

Also, $PA \parallel CR$ i.e. $PA \parallel CQ$, therefore, $AQCP$ is a parallelogram

$\Rightarrow AP = CQ \Rightarrow CQ = \frac{1}{2} CR$ ($\because AP = \frac{1}{2} CR$)

$\Rightarrow 2CQ = CR \Rightarrow 2CQ = CQ + QR$

$\Rightarrow CQ = QR$.



Example 10. E is mid-point of the median AD of $\triangle ABC$ and BE is produced to meet AC at F . Show that $AF = \frac{1}{3} AC$.

Solution. Through D , draw a line parallel to BF to meet AC at G .

In $\triangle ADG$, E is mid-point of side AD and $DG \parallel BF$ i.e. $DG \parallel EF$, therefore, by converse of mid-point theorem, F is mid-point of AG i.e. $AF = FG$ $\dots(i)$

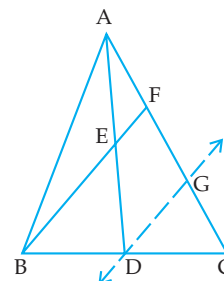
In $\triangle BCF$, D is mid-point of side BC and $DG \parallel BF$, therefore, by converse of mid-point theorem, G is mid-point of CF i.e. $FG = GC$ $\dots(ii)$

From (i) and (ii), we get

$$AF = FG = GC \quad \dots(iii)$$

Now $AC = AF + FG + GC = AF + AF + AF$ (using (iii))

$\Rightarrow AC = 3AF \Rightarrow AF = \frac{1}{3} AC$.



Example 11. In $\triangle ABC$, the medians BE and CF are produced to points P and Q respectively such that $EP = BE$ and $FQ = CF$. Prove that :

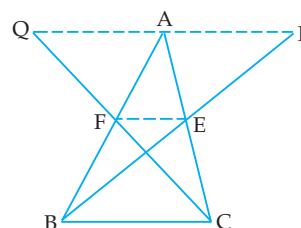
- (i) Q, A and P are collinear (ii) A is mid-point of QP .

Solution. Join AP , AQ and FE .

Given. $EP = BE$ and $FQ = CF$, therefore, E and F are mid-points of BP and CQ respectively.

In $\triangle ABP$, F is mid-point of AB and E is mid-point of BP , therefore, $FE \parallel AP$ and $FE = \frac{1}{2} AP$.

In $\triangle ACQ$, E is mid-point of AC and F is mid-point of CQ , therefore, $FE \parallel QA$ and $FE = \frac{1}{2} QA$.



- (i) Thus $FE \parallel AP$ and $FE \parallel QA$, therefore, QA and AP lie along the same straight line (because both QA and AP pass through the same point A and are parallel to the straight line EF).

It follows that Q, A and P are collinear.

- (ii) As $FE = \frac{1}{2} AP$ and $FE = \frac{1}{2} QA$

$$\Rightarrow \frac{1}{2} AP = \frac{1}{2} QA \Rightarrow AP = QA$$

$$\Rightarrow A \text{ is mid-point of } QP.$$

Example 12. E and F are respectively the mid-points of non-parallel sides AD and BC of a trapezium $ABCD$. Prove that $EF \parallel AB$ and $EF = \frac{1}{2} (AB + CD)$.

Solution. $ABCD$ is a trapezium in which $AB \parallel DC$ and E, F are mid-points of AD, BC respectively.

Join CE and produce it to meet BA produced at G .

In $\triangle EDC$ and $\triangle EAG$,

$$ED = EA \quad (\because E \text{ is mid-point of } AD)$$

$$\angle CED = \angle GEA \quad (\text{Vert. opp. } \angle\text{s})$$

$$\angle ECD = \angle EGA \quad (\text{Alt. } \angle\text{s, } DC \parallel AB \text{ i.e. } DC \parallel GB \text{ and } CG \text{ is transversal})$$

$$\therefore \triangle EDC \cong \triangle EAG$$

$$\Rightarrow CD = GA \text{ and } EC = EG \quad (\text{c.p.c.t.})$$

In $\triangle CGB$,

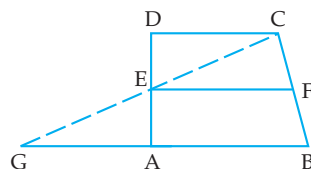
E is mid-point of CG ($\because EC = EG$ proved above)

F is mid-point of BC (given)

Therefore, by mid-point theorem, $EF \parallel AB$ and $EF = \frac{1}{2} GB$.

But $GB = GA + AB = CD + AB$. ($\because GA = CD$ proved above)

Hence, $EF \parallel AB$ and $EF = \frac{1}{2} (AB + CD)$.



Example 13. Prove that the line segment joining the mid-points of the diagonals of a trapezium is parallel to the parallel sides of trapezium and is equal to half the difference of these sides.

Solution. Let $ABCD$ be a trapezium in which $AB \parallel CD$. E and F are mid-points of its diagonals AC and BD respectively.

We want to prove that $EF \parallel AB$ and $EF = \frac{1}{2} (AB - DC)$.

Join DE and produce it to meet AB at G .

In $\triangle ECD$ and $\triangle EAG$,

$$EC = EA \quad (\because E \text{ is mid-point of } AC)$$

$$\angle ECD = \angle EAG \quad (\text{Alt. } \angle\text{s, } AB \parallel DC \text{ i.e. } AG \parallel DC \text{ and } AC \text{ is transversal})$$

$$\angle CED = \angle AEG \quad (\text{Vert. opp. } \angle\text{s})$$

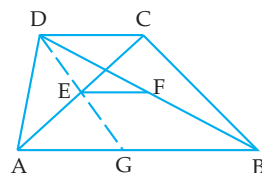
$$\therefore \triangle ECD \cong \triangle EAG \quad (\text{by ASA rule of congruency})$$

$$\Rightarrow AG = DC \text{ and } DE = EG.$$

In $\triangle DGB$,

E is mid-point of DG ($\because DE = EG$, proved above)

F is mid-point of BD (given)



Therefore, by mid-point theorem,

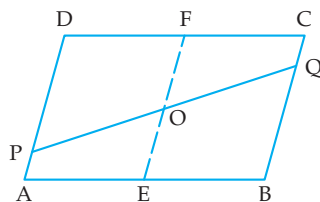
$$EF \parallel GB \text{ i.e. } EF \parallel AB \text{ and } EF = \frac{1}{2} GB.$$

$$\text{But } GB = AB - AG = AB - DC$$

$$(\because AG = DC, \text{ proved above})$$

$$\text{Hence, } EF \parallel AB \text{ and } EF = \frac{1}{2} (AB - DC).$$

Example 14. In the adjoining figure, ABCD is a parallelogram. E and F are mid-points of sides AB and CD respectively. PQ is any line that meets AD, EF and BC in points P, O and Q respectively. Prove that PO = OQ.



Solution. $AE = \frac{1}{2} AB$ ($\because$ E is mid-point of AB)

and $DF = \frac{1}{2} DC$ ($\because$ F is mid-point of CD)

$$\Rightarrow AE = DF \quad (\because AB \parallel DC, \text{ opp. sides of } \parallel \text{ gm})$$

Also $AE \parallel DF$ ($AB \parallel DC$, ABCD is a $\parallel$ gm)

$$\Rightarrow \text{AEFD is a } \parallel \text{ gm} \quad (\because \text{ opp. sides of quad. AEFD are equal and parallel})$$

$$\Rightarrow AD \parallel EF$$

Also $AD \parallel BC$ ($\because$ ABCD is a $\parallel$ gm)

$$\Rightarrow AD, EF \text{ and } BC \text{ are parallel lines.}$$

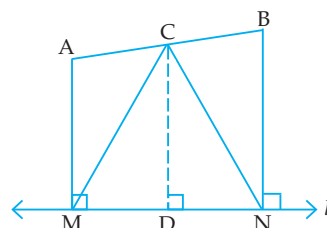
Now, as the transversal AB makes equal intercepts $AE = EB$ on the three parallel lines AD, EF and BC, therefore, the transversal PQ also makes equal intercepts on these parallel lines (theorem on intercepts)

$$\Rightarrow PO = OQ, \text{ as required.}$$

Example 15. Points A and B are on the same side of a line l . AM and BN are perpendiculars to the line l . If C is the mid-points of AB, prove that $CM = CN$.

Solution. From C, draw CD perpendicular to the line l .

Since AM, CD and BN are perpendiculars to the same line l , AM, CD and BN are parallel lines. Now, as the transversal AB makes equal intercepts $AC = CB$ (given C is mid-point of AB) on the three parallel lines AM, CD and BN, therefore, the transversal l also makes equal intercepts on these parallel lines



$$\Rightarrow MD = DN.$$

In $\triangle CMD$ and $\triangle CND$,

$$MD = DN \quad (\text{proved above})$$

$$\angle CDM = \angle CDN \quad (\text{each angle} = 90^\circ, \text{ CD is perpendicular to } l)$$

and CD is common.

$$\therefore \triangle CMD \cong \triangle CND \quad (\text{by SAS rule of congruency})$$

$$\Rightarrow CM = CN \quad (\text{c.p.c.t.})$$

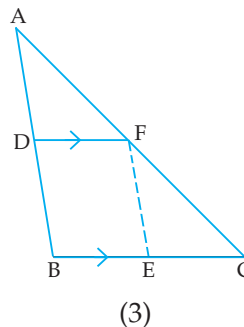
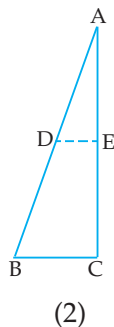
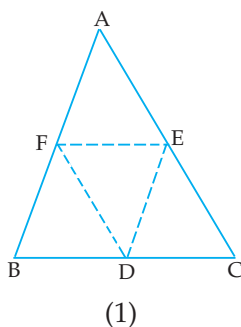
Exercise 10

- (a) In the figure (1) given below, D, E and F are mid-points of the sides BC, CA and AB respectively of $\triangle ABC$. If $AB = 6$ cm, $BC = 4.8$ cm and $CA = 5.6$ cm, find the perimeter of
 - the trapezium FBCE
 - the triangle DEF.

(b) In the figure (2) given below, D and E are mid-points of the sides AB and AC respectively. If $BC = 5.6$ cm and $\angle B = 72^\circ$, compute

- (i) DE (ii) $\angle ADE$.

(c) In the figure (3) given below, D and E are mid-points of AB, BC respectively and $DF \parallel BC$. Prove that DBEF is a parallelogram. Calculate AC if $AF = 2.6$ cm.



- Prove that the four triangles formed by joining in pairs the mid-points of the sides of a triangle are congruent to each other.
- If D, E and F are mid-points of the sides AB, BC and CA respectively of an isosceles triangle ABC, prove that $\triangle DEF$ is also isosceles.
- The diagonals AC and BD of a parallelogram ABCD intersect at O. If P is the mid-point of AD, prove that

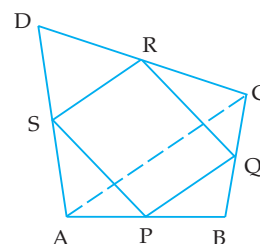
- (i) $PO \parallel AB$ (ii) $PO = \frac{1}{2} CD$.

- In the adjoining figure, ABCD is a quadrilateral in which P, Q, R and S are mid-points of AB, BC, CD and DA respectively. AC is its diagonal. Show that

- (i) $SR \parallel AC$ and $SR = \frac{1}{2} AC$

- (ii) $PQ = SR$

- (iii) PQRS is a parallelogram.



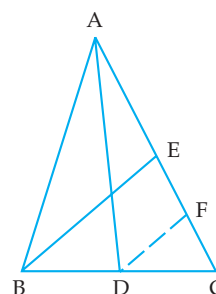
- Show that the quadrilateral formed by joining the mid-points of the adjacent sides of a square, is also a square.

- In the adjoining figure, AD and BE are medians of $\triangle ABC$.

If $DF \parallel BE$, prove that $CF = \frac{1}{4} AC$.

Hint: In $\triangle BCE$, D is mid-point of BC and $DF \parallel BE$, therefore, F is mid-point of CE (converse of mid-point theorem)

$$\Rightarrow CF = \frac{1}{2} CE = \frac{1}{2} \left(\frac{1}{2} AC \right).$$



- (a) In the figure (1) given below, ABCD is a parallelogram. E and F are mid-points of the sides AB and CD respectively. The straight lines AF and BF meet the straight lines ED and EC in points G and H respectively. Prove that
 - $\triangle HEB \cong \triangle HCF$
 - GEHF is a parallelogram.