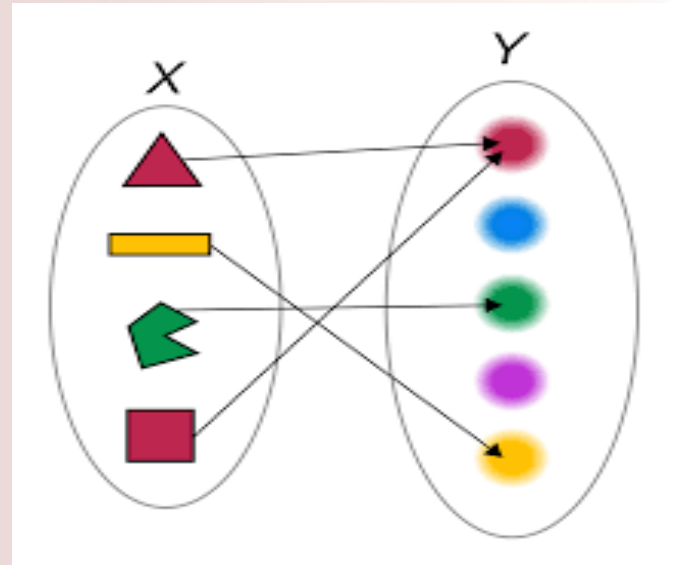


# Relations and Functions



# Look at these Sets

$$A = \{ \text{apple}, \text{grapes} \} \quad B = \{ \text{tennis ball}, \text{cricket ball}, \text{golf ball} \}$$



How many pairs can you make from these two sets?

$$\begin{aligned} & ( \text{apple}, \text{tennis ball} ), ( \text{apple}, \text{cricket ball} ), ( \text{apple}, \text{golf ball} ), ( \text{grapes}, \text{tennis ball} ), ( \text{grapes}, \text{cricket ball} ) \\ & ( \text{grapes}, \text{golf ball} ) \end{aligned}$$

In orderly manner we get 6 pairs from these sets.

Mathematically Which are called ORDERED PAIRS.

Let A,B,C are three non empty Sets.

### Ordered Pair

$(a,b)$  is ordered pair where  $a \in A, b \in B$

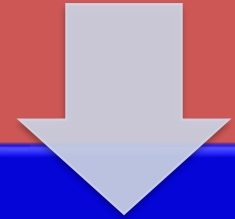
### Ordered Triplet

$(a,b,c)$  is ordered triplet where  $a \in A, b \in B, c \in C$



### Cartesian Product

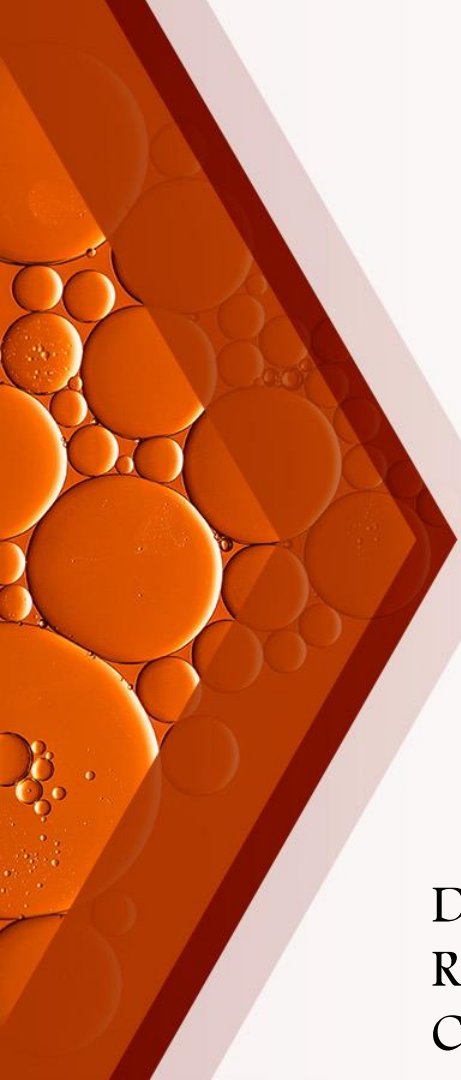
Set of all ordered pairs,  $A \times B = \{ (a,b) \text{ s.t. } a \in A, b \in B \}$



### Relation

Any subset of  $A \times B$  forms a relation.

$(a,b) \in R$  can be read as  $a$  is related to  $b$ .



## Representation Of Relation

### Roster Form;

Set of all ordered pairs belongs to relation.

### Set-Builder Form:

$R = \{ (a, b) : a \in A, b \in B \}$

### Arrow Diagram:

Draw arrows from First element to second element of all ordered pairs in a relation.

Domain of Relation =  $\{ a : (a, b) \in R \}$

Range of Relation =  $\{ b : (a, b) \in R \}$

Co-domain of Relation = Set B

# Function

Let A and B are two non empty sets then a function  $f$  from set A to set B is a **rule** or **method** or **correspondence** which associates elements of set A to elements of set B s.t.

- (i) All elements of set A are associated to elements of set B.
- (ii) An element of set A is associated to a **UNIQUE** element of set B.

Written as,  $f : A \rightarrow B$

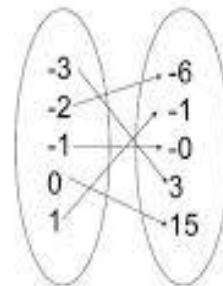
Read as, function from A to B,  $f$  maps from A to B

Domain ( $D_f$ ) = Set A

Range ( $R_f$ ) = Set of images

Co-domain = Set B

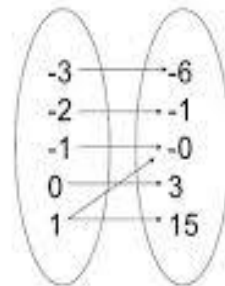
Function



A

B

Not a Function



A

B



**Real Valued Function:** A Function which has either  $\mathbb{R}$  or one of its subsets as its Range is called Real Valued Function.

**Domain of Real Valued Function** : Either  $\mathbb{R}$  or subset of  $\mathbb{R}$

**Range of Real Valued Function** : Either  $\mathbb{R}$  or subset of  $\mathbb{R}$

**Equality of Two Sets:** Let  $f$  and  $g$  are two sets and are said to be equal if

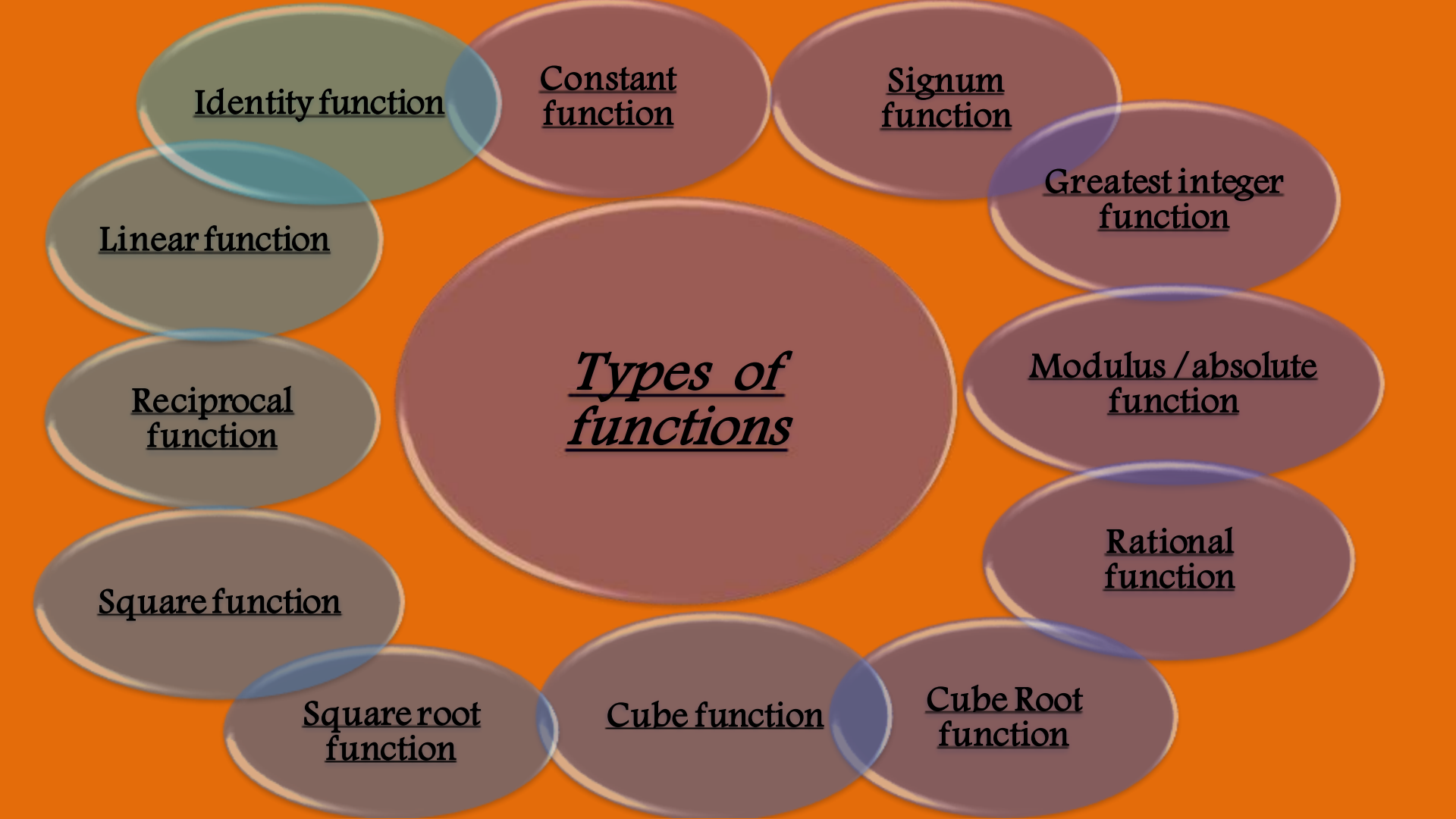
- (i) Domain of  $f$  = Domain of  $g$
- (ii) Co-domain of  $f$  = co-domain of  $g$

## Algebra of Real Functions:

let  $f : A \rightarrow \mathbb{R}$  ,  $g : B \rightarrow \mathbb{R}$  be two real functions with domain  $A$  and  $B$  respectively , then

- (i)  $(f + g)(x) = f(x) + g(x) \quad \forall x \in A \cap B$
- (ii)  $(f - g)(x) = f(x) - g(x) \quad \forall x \in A \cap B$
- (iii)  $(f \cdot g)(x) = f(x) \cdot g(x) \quad \forall x \in A \cap B$
- (iv)  $(f/g)(x) = f(x)/g(x) \quad \forall x \in A \cap B - \{x; g(x) \neq 0\}$
- (v)  $(c f)(x) = c f(x) \quad \forall x \in A$  ,  $c$  is a scalar.

# *Types of functions*



Identity function

Constant function

Signum function

Greatest integer function

Modulus / absolute function

Rational function

Cube Root function

Cube function

Square root function

Square function

Reciprocal function

Linear function

## Constant Function:

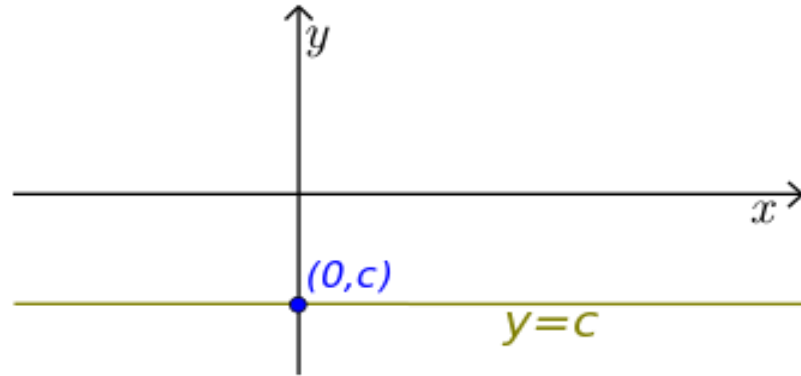
If  $C$  is a fixed real no. then  
function  $f(x)$  given by

$$f(x) = C, \forall x \in \mathbb{R}$$

$$D_f = \mathbb{R}$$

$$R_f = \{C\}$$

Graph is parallel  
to x-axis.



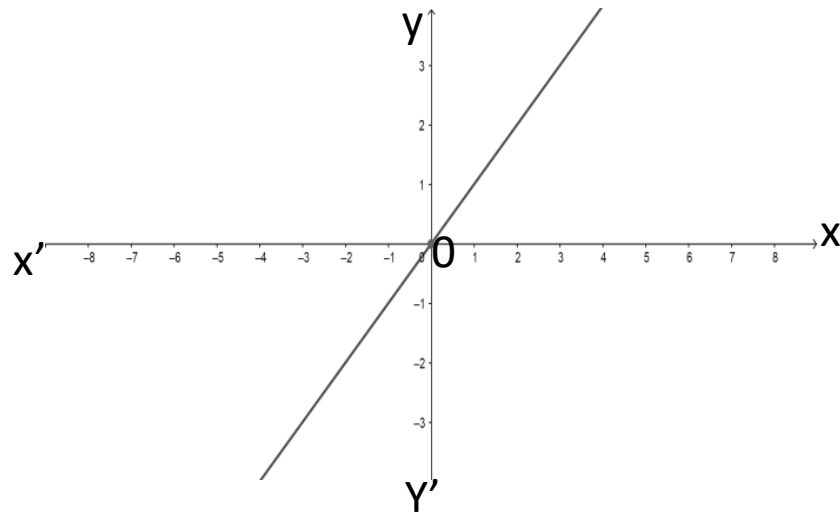
## Identity Function:

A function which associate to each real no. to itself .

It is denoted by  $I$ ,  $I: \mathbb{R} \rightarrow \mathbb{R}$  s.t.

$$I(x) = f(x) = x$$

$$D_f = \mathbb{R} \quad R_f = \mathbb{R}$$



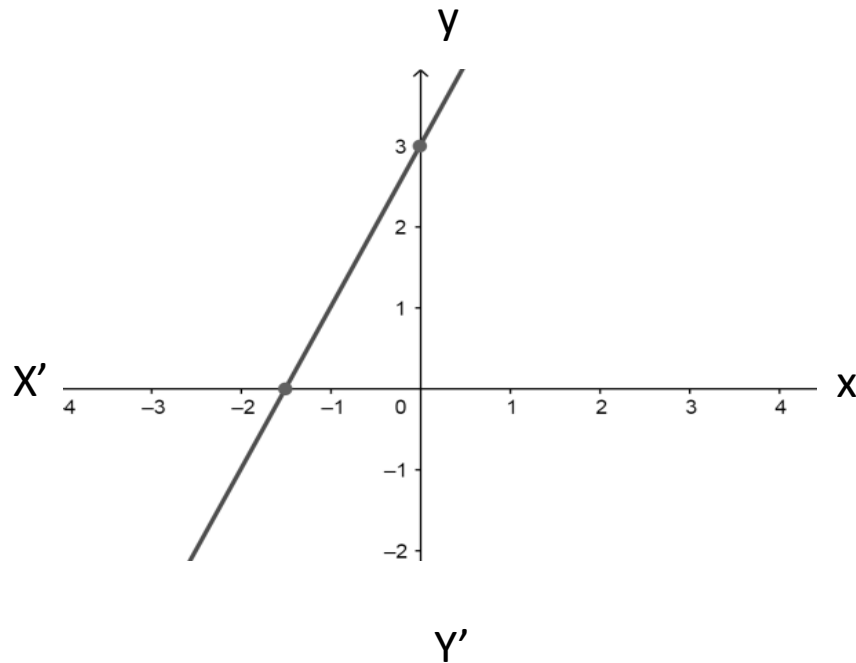
## Linear Function:

A Function is of the form  
 $f(x) = ax + by$ ,  $a \neq 0$ ,  $a, b$  are  
constants.

$$D_f = \mathbb{R} = (-\infty, \infty)$$

$$R_f = \mathbb{R} = (-\infty, \infty)$$

Graph is a straight Line.

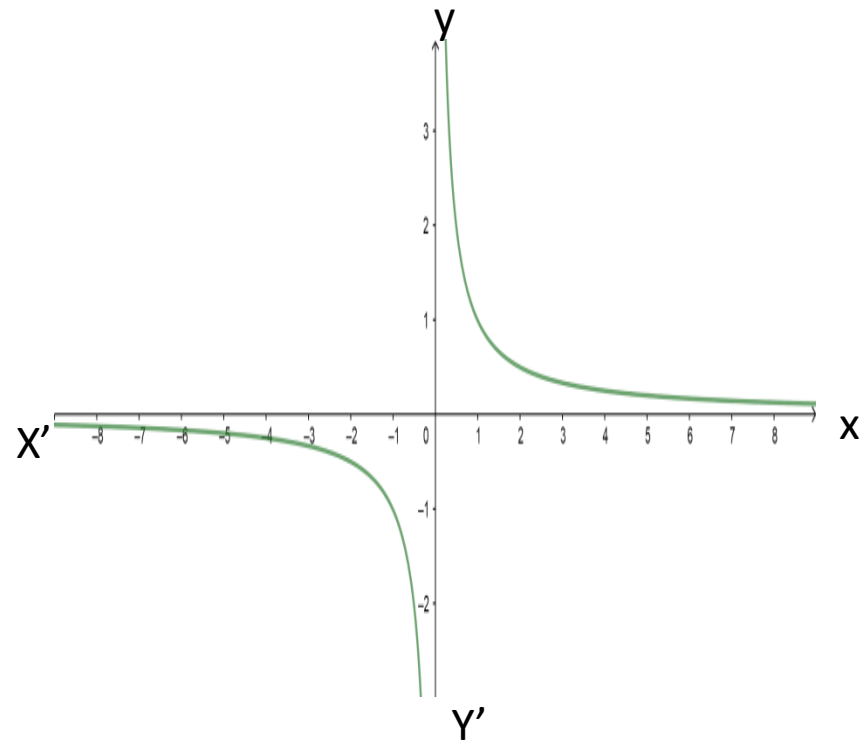


# Reciprocal Function:

A Function that associates  $x$  to its reciprocal  $f : \mathbb{R} \setminus \{0\} \rightarrow \mathbb{R} \setminus \{0\}$  defined by  $f(x) = \frac{1}{x}$ ,  $x \neq 0$

$$D_f = \mathbb{R} \setminus \{0\}$$

$$R_f = \mathbb{R} \setminus \{0\}$$

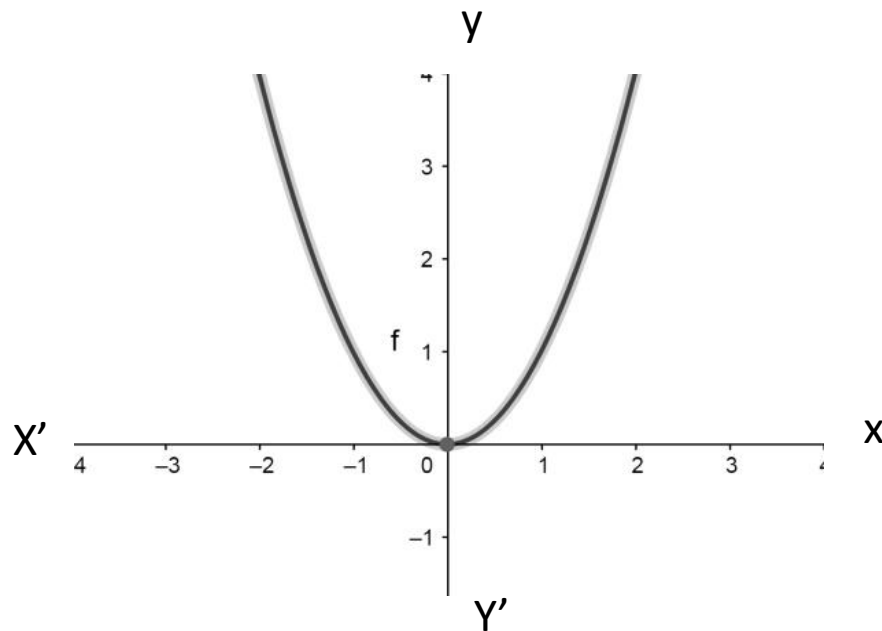


# Square Function:

A Function is defined as  $f(x) = x^2$

$$D_f = \mathbb{R}$$

$$R_f = [0, \infty)$$

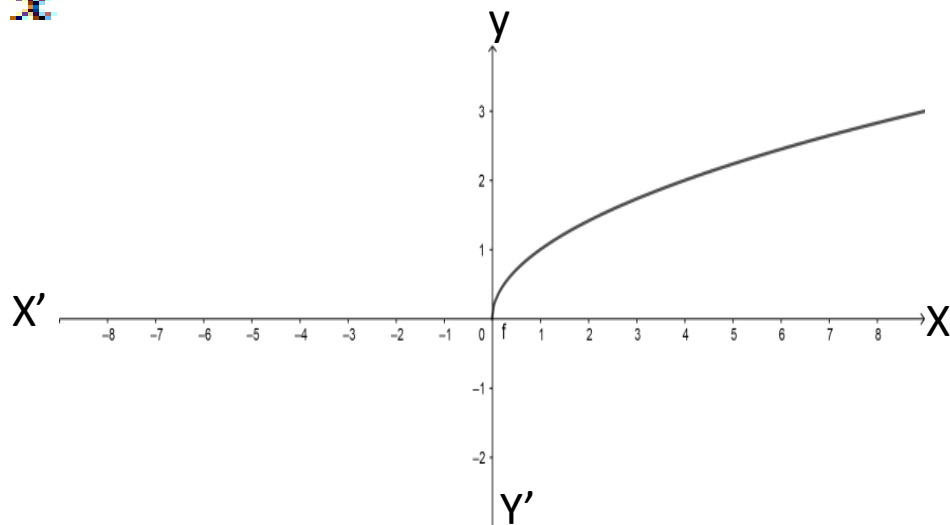


# Square Root Function:

A Function is defined as  $f(x) = \sqrt{x}$

$$D_f = [0, \infty)$$

$$R_f = [0, \infty)$$



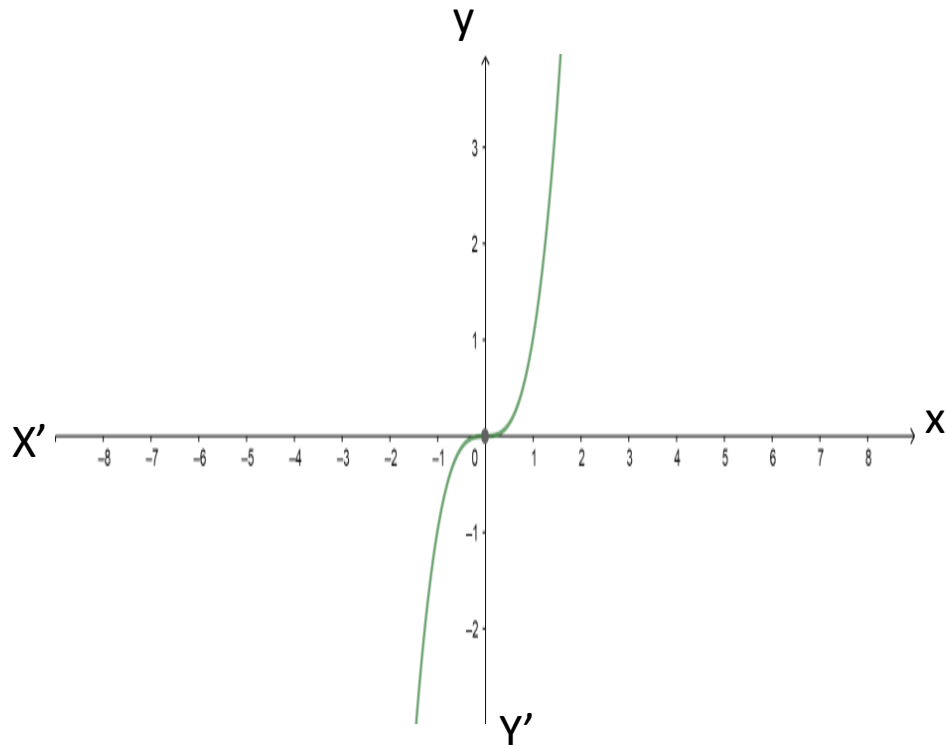
# Cube Function:

A Function is defined as

$$f(x) = x^3$$

$$D_f = \mathbb{R}$$

$$R_f = \mathbb{R}$$



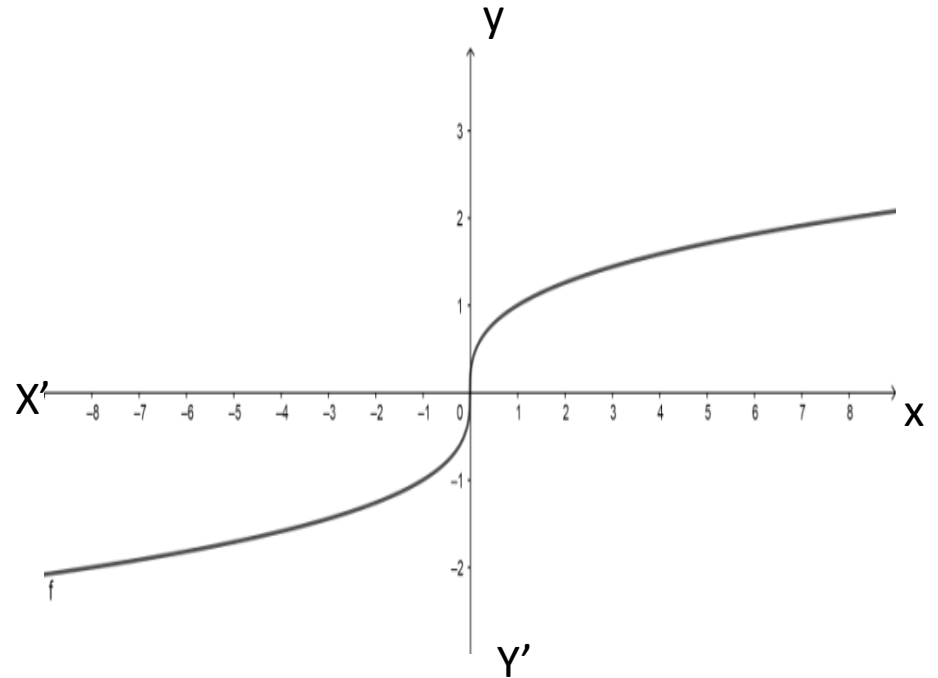
# Cube Root Function:

A Function is defined as

$$f(x) = \sqrt[3]{x}$$

$$D_f = \mathbb{R}$$

$$R_f = \mathbb{R}$$



# Modulus Function/Absolute Function:

A Function is defined by  $f(x) = |x| = \sqrt{x^2}$   $x'$

$$|x| = \begin{cases} x & x \geq 0 \\ -x & x \leq 0 \end{cases}$$

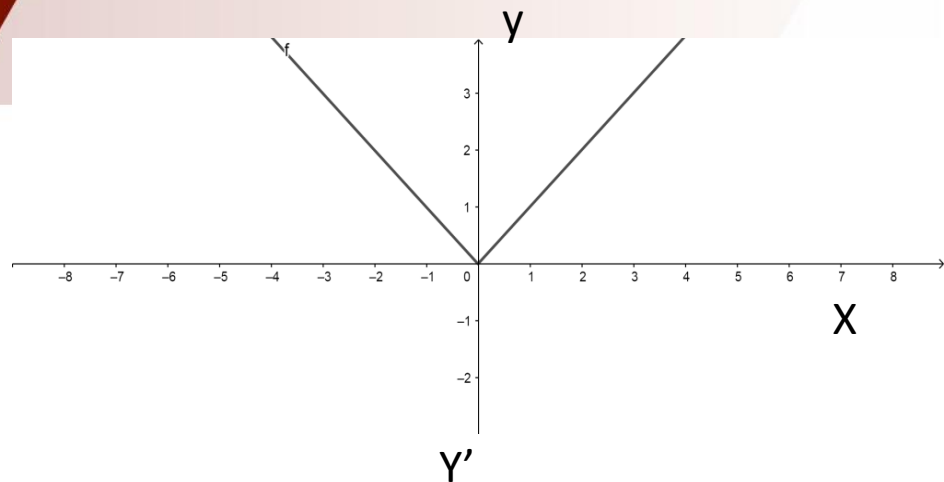
$$D_f = R = (-\infty, \infty)$$

$$R_f = R^+ = [0, \infty)$$

Identity:

$$x^2 \leq a^2 \Leftrightarrow |x| \leq |a| \rightarrow -a \leq x \leq a$$

$$x^2 \geq a^2 \Leftrightarrow |x| \geq |a| \rightarrow x \leq -a \text{ or } x \geq a$$



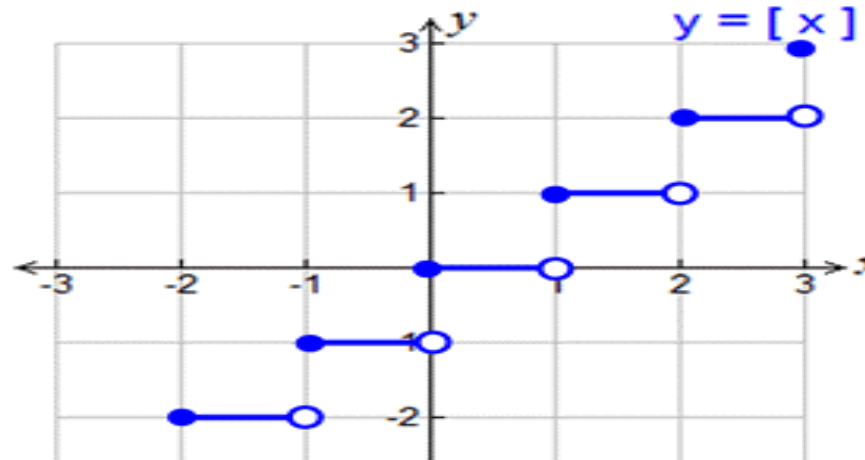
| X | ..... | -2 | -1 | 0 | 1 | 2 | ..... |
|---|-------|----|----|---|---|---|-------|
| Y | ..... | 2  | 1  | 0 | 1 | 2 | ..... |

## Greatest Integer Function :

A function is defined by  
 $f(x) = [x], \forall x \in \mathbb{R}$

$$D_f = \mathbb{R} \quad R_f = \mathbb{Z}$$

| X | .... | $-2 \leq x < -1$ | $-1 \leq x < 0$ | $0 \leq x < 1$ | $1 \leq x < 2$ | .... |
|---|------|------------------|-----------------|----------------|----------------|------|
| Y | .... | -2               | -1              | 0              | 1              | .... |



## Signum Function:

A function is defined as

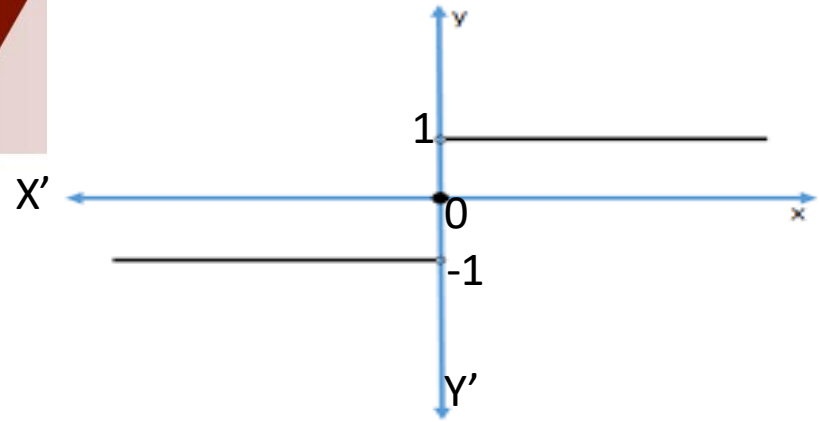
$$\text{Sgn}(x) = \begin{cases} +1 & \text{if } x > 0 \\ 0 & \text{if } x = 0 \\ -1 & \text{if } x < 0 \end{cases}$$

Also a more devious way to define it would be:

$$\text{Sgn}(x) = \begin{cases} \frac{x}{|x|} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

$$D_f = \mathbb{R}$$

$$R_f = \{-1, 0, 1\}$$



| X | ..... | -2 | -1 | 0 | 1 | 2 | ..... |
|---|-------|----|----|---|---|---|-------|
| Y | ..... | -1 | -1 | 0 | 1 | 1 |       |

## Rational Function:

A Function is defined by  $f(x) = p(x)/q(x)$ ,  $q(x) \neq 0$  where  $p(x), q(x)$  are polynomials.