

## CHAPTER-3- CURRENT ELECTRICITY

### Electric Current

The amount of charge flowing through a given cross-section of a conductor per unit time constitutes electric current.

$$I = \frac{q}{t} \text{ where, } I \text{ is the current, and } q \text{ is the charge that flows across that area in time } t.$$

The S.I. unit of current is Ampere (A).

1 ampere =  $\frac{1 \text{ coulomb}}{1 \text{ second}}$  One ampere of current is said to flow through a wire if at any cross section, one coulomb of charge flows in one second.

In solid conductors, the current carriers are the free electrons; in electrolytes, the anions and cations; and in gases, the electrons and anions.

### Resistance and Resistivity

It is found experimentally that the current  $I$  flowing through a conductor is directly proportional to the potential difference  $V$  across its ends, provided the physical conditions (temperature, mechanical strain, etc.,) remain constant. That is

$$V \propto I$$

$$\text{or } V = IR$$

where,  $R$  is the resistance of the conductor.

This equation is called **Ohm's law**,

The resistance of a resistor depends on the nature of the material, its geometrical features (length and cross-sectional area) and on the temperature and pressure.

$$R = \rho \frac{l}{A}$$

Where  $\rho$ , the constant of proportionality, is called resistivity. It depends only on the nature of the material and its physical conditions. The unit of  $\rho$  is ohm m ( $\Omega\text{m}$ ).

The reciprocal of resistance of a conductor is called its conductance ( $G$ ). Therefore,  $G = 1/R$ . The S.I. unit of conductance is mho or siemen (S).

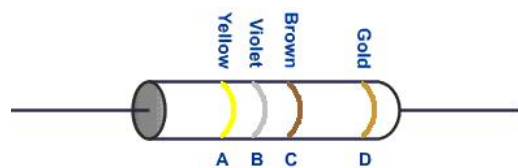
The inverse of resistivity of a conductor is called its conductivity ( $\sigma$ );

$$\sigma = 1/\rho.$$

$$G = \frac{1}{R} = \frac{A}{\rho l} = \sigma \frac{A}{l}$$

The S.I. unit of conductivity is siemen metre<sup>-1</sup> (sm<sup>-1</sup>) or mho m<sup>-1</sup> or ( $\Omega\text{m}$ )<sup>-1</sup>.

### Colour Code for Resistors



| Equation Colour | Number | In order to remember the colour code, the following sentence may be helpful.         |
|-----------------|--------|--------------------------------------------------------------------------------------|
| (Black) B       | 0      | <u>BB</u> <u>Roy</u> <u>Great</u> <u>Britain</u> <u>Very</u> <u>Good</u> <u>Wife</u> |
| (Brown) B       | 1      |                                                                                      |
| (Red) R         | 2      |                                                                                      |

|            |   |  |
|------------|---|--|
| (Orange) O | 3 |  |
| (Yellow) Y | 4 |  |
| (Green) G  | 5 |  |
| (Blue) B   | 6 |  |
| (Violet) V | 7 |  |
| (Grey) G   | 8 |  |
| (White) W  | 9 |  |

The first two colour bands (i.e., A and B) from the end indicate the first two significant figures of the resistance in ohms. The third colour band i.e., C indicates the decimal multiplier and the last band D stands for the tolerance in percent about the indicated value. If this last band is gold, the tolerance is 5%, if silver it is 10%. If it is absent, the tolerance is 20%.

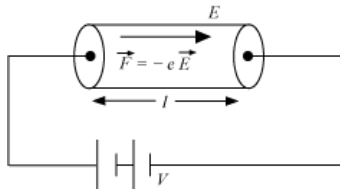
Example: If ABCD represent colours yellow, violet, brown and gold respectively, the value of the resistance will be

$$47 \times 10^1 (\pm) 5\% \text{ Or } 470 \Omega (\pm) 5\% .$$

### **Relation between current and drift velocity**

Free electrons are in continuous random motion. They undergo change in direction at each collision and the thermal velocities are randomly distributed in all directions.

$$\therefore \text{Average thermal velocity, } u = \frac{u_1 + u_2 + \dots + u_n}{n} \text{ is zero ... (1)}$$



The electric field  $E$  exerts an electrostatic force ' $-Ee$ '.

$$\text{Acceleration of each electron is, } \vec{a} = \frac{-e\vec{E}}{m} \quad \dots(2)$$

Where,

$m \rightarrow$  Mass of an electron

$e \rightarrow$  Charge on an electron

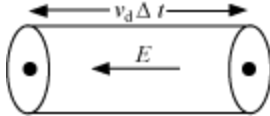
- Drift velocity – It is the average velocity with which free electrons get drifted towards the positive terminal under the effect of the applied electric field.

$$\therefore v_d = a \tau$$

where  $\tau = \frac{\tau_1 + \tau_2 + \tau_3 + \dots + \tau_n}{n}$ , is the average time elapsed

Substituting for  $a$  from equation (2),

$$\vec{v}_d = \frac{-e\vec{E}}{m} \tau \quad \dots(4)$$



Electron drift to a small distance in a time  $\Delta t = V_d \Delta t$

Amount of charge passing through the area  $A$  in time  $\Delta t$ ,  $q = I \Delta t$

$$I \Delta t = neAv_d \Delta t$$

Where,

$n \rightarrow$  Number of free electrons per unit volume

From equation (4),

$$I \Delta t = \frac{e^2 A}{m} \tau n \Delta t |E|$$

$$\text{Current density (J)} = \frac{I}{A}$$

$$\therefore |J| = \frac{ne^2}{m} \tau |E|$$

We know,

$$J = \sigma E$$

$$\therefore \sigma = \frac{ne^2}{m} \tau$$

- resistivity  $\rho = m/ne^2 \tau$

The resistivity of a metallic conductor increases with increase in temperature.

The resistivity of an alloy has a weak dependence on temperature.

The resistivity of a semiconductor decreases rapidly with increasing temperature.

### **Mobility ( $\mu$ )**

It is defined as the magnitude of the drift velocity per unit electric field.

$$\mu = \frac{V_d}{E}$$

Unit of mobility is  $\text{m}^2/\text{Vs}$ .

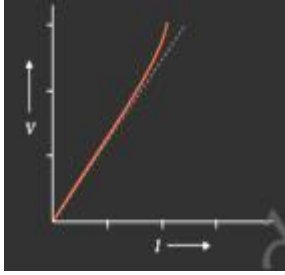
$$\therefore V_d = \frac{e\tau E}{m} \quad \therefore \mu = \frac{e\tau}{m}$$

### **Limitations of Ohm's Law**

There are several materials and devices for which the proportionality of  $V$  and  $I$  are as follows:

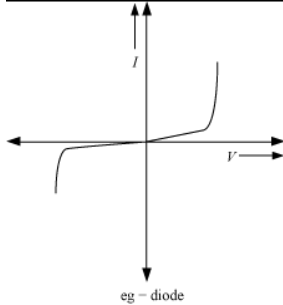
$V$  ceases to be proportional to  $I$ .

### **$V$ versus current $I$ for a good conductor**



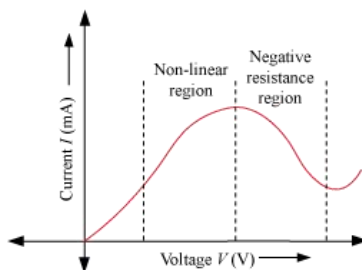
The dashed line represents the linear Ohm's law.  
The solid line is the voltage

### Characteristic curve of a diode:



There is more than one value of  $V$  for the same current.

### Variation of current versus voltage for GaAs



Ohm's law is not a fundamental law of nature. There are a number of commonly used circuit elements which do not obey this law. They have one or more of the following properties:

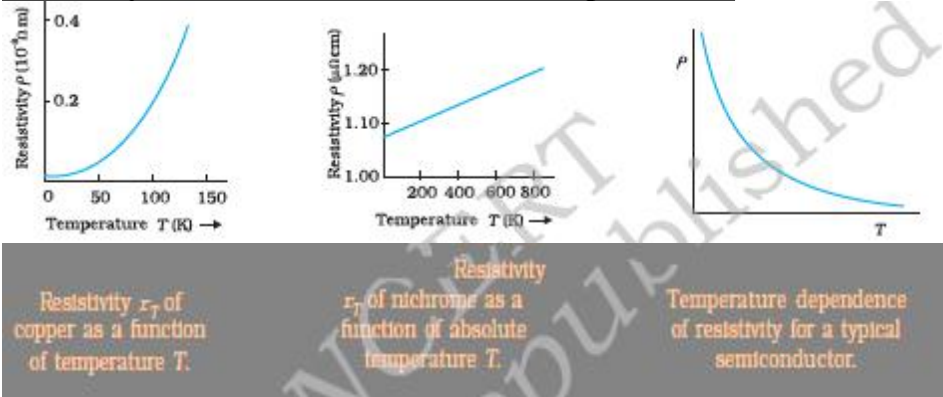
1.  $V$  depends on  $I$  non-linearly.
2. The relationship between  $V$  and  $I$  depends on the sign of  $V$  for the same absolute value of  $V$ .
3. The relation between  $V$  and  $I$  is non-unique, that is, for the same current  $I$ , there is more than one value of voltage  $V$ .

### TEMPERATURE DEPENDENCE OF RESISTIVITY

Over a small temperature range, the resistivity of a metal can be represented by a linear relation

$$\rho(T) = \rho_0 [1 + \alpha(T - T_0)]$$

### Resistivity of material as a function of temperature T



### Power:

In an actual conductor, an amount of energy dissipated as heat in the conductor during the time interval  $\Delta t$  is,

$$\Delta W = I V \Delta t$$

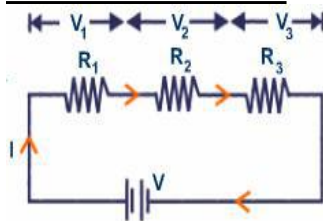
The energy dissipated per unit time is the power dissipated  $P = \Delta W / \Delta t$  and we have,

$$P = I V$$

Using Ohm's law  $V = IR$ , we get

$$P = I^2 R = V^2 / R$$

### Resistances in Series



If the resistances are connected end-to-end, the same current flows through each resistance, as there is no alternative path. Then

$$I = \frac{V}{R_S}$$

where  $I$  is the current,  $V$  is the potential difference of the battery and  $R_S$  is the equivalent resistance of the combination.

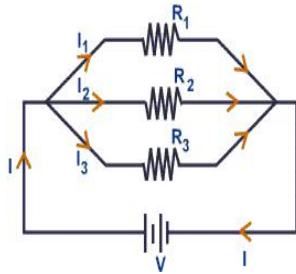
Now,

$$V = V_1 + V_2 + V_3$$

$$I R_S = I R_1 + I R_2 + I R_3 \text{ Therefore,}$$

$$R_S = R_1 + R_2 + R_3$$

## Resistance in Parallel



If the resistance are in parallel, the potential difference across each is the same, but the current is not. Then,

$$I = I_1 + I_2 + I_3$$

$$\text{But } I_1 = \frac{V}{R_1}, I_2 = \frac{V}{R_2}, I_3 = \frac{V}{R_3}$$

Since charge is not accumulated at a point,

$$\therefore \frac{V}{R_p} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3}$$

where  $R_p$  is the equivalent resistance of the combination. Therefore,

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

For the special case of only two resistors in parallel, the expression for the equivalent resistance takes on a particularly simple form, i.e.,

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2}$$

$$\therefore R_p = \frac{R_1 R_2}{R_1 + R_2}$$

## Internal Resistance, Potential Difference and E.M.F. of a cell:

Resistance offered by the electrolyte of the cell when the electric current flows through it.

**The internal resistance of a cell depends on the distance between the plates, the nature of the electrolytes, the concentration of electrolytes, the nature of the electrodes and the area of the plates.**

Let  $\epsilon$ ,  $r$  be the emf and internal resistance of a cell and  $R$  - the external resistance. A high resistance voltmeter  $V$  is connected.

Emf- When  $K$  is opened (i.e., open circuit) emf, the voltmeter reads the emf ( $\epsilon$ ) of the cell as no current flows through the circuit.

Potential difference-When  $K$  is closed (i.e., closed circuit), a current ' $I$ ' flows in the circuit.

Hence, we have

$$I = \frac{\epsilon}{R + r}$$

$$\epsilon = IR + Ir$$

' $Ir$ ' is the potential difference across the internal resistance  $r$ .

But,  $V = IR$

$$\therefore \mathcal{E} = V + Ir$$

$$\text{or } V = \mathcal{E} - Ir$$

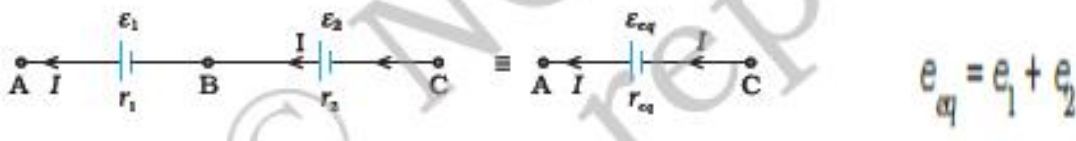
Therefore, the external voltage  $V$  is less than the emf of the cell,  $\mathcal{E}$ . It is as though an internal resistance  $r$  is in series with the external resistance  $R$ , and this determines the current in the circuit for a given source of emf.

Also,

$$r = \frac{\mathcal{E} - V}{I}$$

$$r = \left[ \frac{\mathcal{E} - V}{V} \right] R$$

### CELLS IN SERIES AND IN PARALLEL



we had connected the negative electrode of the first to the positive electrode of the second. If instead we connect the two negatives,

$$e_{eq} = e_1 - e_2 \quad (e_1 > e_2)$$

### Cells in Series



$E_1, E_2$  – emf of two cells

$r_1, r_2$  – Internal resistance of two cells

$I$  – Current in the circuit

Terminal potential difference across the first cell,  $V_1 = E_1 - Ir_1$

Terminal potential difference across the second cell,  $V_2 = E_2 - Ir_2$

Potential difference between the points A and B,

$$V = V_1 + V_2 = (E_1 - Ir_1) + (E_2 - Ir_2)$$

$$= (E_1 + E_2) - I(r_1 + r_2)$$

Let

$E$  – Effective emf

$r$  – Effective internal resistance

$$V = E - Ir$$

$$\therefore E = E_1 + E_2$$

$$r = r_1 + r_2$$

Current in the circuit,

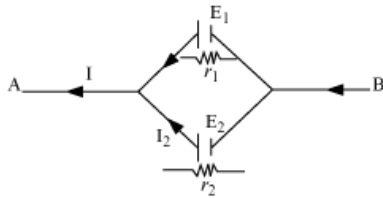
$$I = \frac{E_1 + E_2}{R + (r_1 + r_2)}$$

If the two cells are connected in opposite direction, then

$$E = E_1 - E_2$$

$$\therefore I = \frac{E_1 - E_2}{R + (r_1 + r_2)}$$

### Cell in Parallel



$E_1, E_2$  – *emf* of two cells

$r_1, r_2$  – Internal resistances of cell

$I_1, I_2$  – Current due to the two cells

Terminal potential difference across the first cell,

$$V = E_1 - I_1 r_1$$

$$I_1 = \frac{E_1 - V}{r_1}$$

For the second cell,

$$I_2 = \frac{E_2 - V}{r_2}$$

$$\therefore I = \frac{E_1 - V}{r_1} + \frac{E_2 - V}{r_2}$$

$$I = \left( \frac{E_1}{r_1} + \frac{E_2}{r_2} \right) - \left( \frac{1}{r_1} + \frac{1}{r_2} \right) \times V$$

$$V = \left( \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2} \right) - I \left( \frac{r_1 r_2}{r_1 + r_2} \right)$$

Let  $E$  be effective *emf* and  $r$  is effective internal resistance.

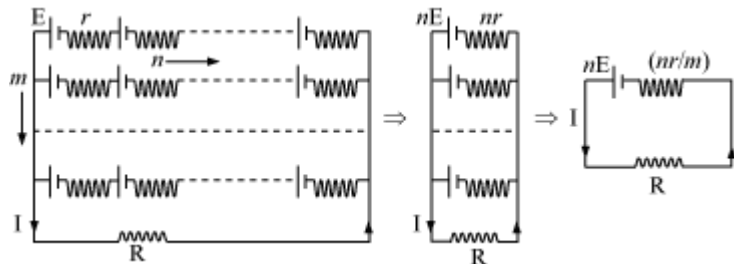
$$V = E - Ir$$

$$\therefore E = \frac{E_1 r_2 + E_2 r_1}{r_1 + r_2}$$

$$\text{And, } r = \frac{r_1 r_2}{r_1 + r_2}$$

### Mixed Grouping of Cells

Suppose 'n' cells of emf 'E' and internal resistance 'r' are connected in series in each branch; and there are 'm' such branches. 'R' is the external resistance connected to the circuit.



In each branch, the net emf is  $nE$  and the net internal resistance is  $nr$ . The equivalent resistance of all the internal resistances in parallel is,

$$\frac{1}{r_{eq}} = \frac{1}{nr} + \frac{1}{nr} + \frac{1}{nr} \dots \text{upto } m$$

$$\Rightarrow \frac{1}{r_{eq}} = \frac{m}{nr}$$

$$\Rightarrow r_{eq} = \frac{nr}{m}$$

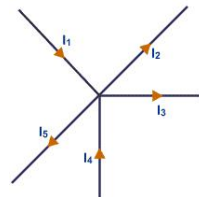
$$\frac{nr}{m}$$

We finally have a circuit with a cell of emf  $nE$ , internal resistance  $\frac{nr}{m}$  connected to a resistance  $R$ . The current through the circuit is,

$$I = \frac{nE}{R + \frac{nr}{m}}$$

### Kirchhoff's Junction Rule:

The algebraic sum of the currents at a junction in a closed circuit is zero.



Therefore,  $I_1 + I_4 = I_2 + I_3 + I_5$

Hence,  $I_1 + I_4 - I_2 - I_3 - I_5 = 0$

or  $\Sigma I = 0$

(Sum of currents entering a junction = Sum of currents leaving the junction)

This rule is based on the fact that charge cannot be accumulated at any point in a conductor in a steady situation.

### Kirchhoff's Loop Rule:

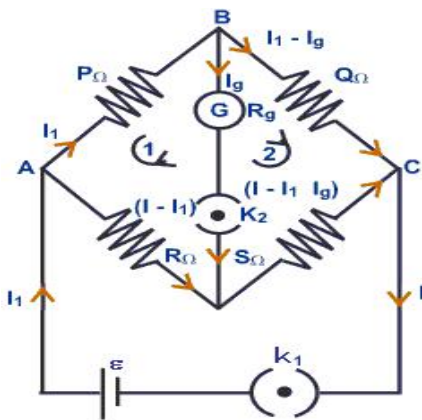
The algebraic sum of all the potential drops and emfs along any closed path in a network is zero.

$\Sigma V = 0$  (Valid for any closed loop)

This rule is based on energy conservation, i.e., the net change in the energy of a charge after completing the closed path is zero. Otherwise, one can continuously gain energy by circulating charge in a particular direction.

### Wheatstone Bridge:

This is used to measure an unknown resistance accurately. It consists of 4 resistors (2 fixed known resistances P and Q, a known variable resistance R and the unknown resistance X) connected as shown in the figure.



Using Kirchhoff's rules.

Applying the Kirchhoff's law to loop 1, we have

$$-I_1 P - I_g R_g + (I - I_1) R = 0 \dots(1)$$

Similarly for loop 2, we have

$$-(I_1 - I_g) Q + (I - I_1 + I_g) X + I_g R_g = 0 \dots(2)$$

(where  $R_g$  is the resistance of the galvanometer)

In the balanced condition, putting  $I_g = 0$ , we have

$$-I_1 P + (I - I_1) R = 0 \dots(1)$$

$$\text{and } -(I_1) Q + (I - I_1) X = 0 \dots(2)$$

Simplifying the two equations, we get

$$I_1 P = (I - I_1) R \dots(1)$$

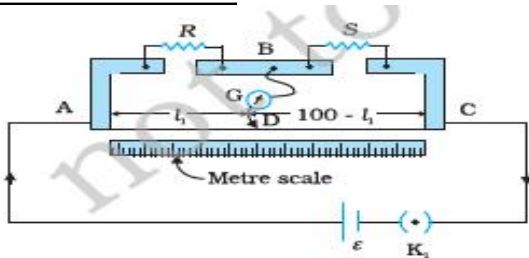
$$I_1 Q = (I - I_1) X \dots(2)$$

Dividing the above two equations, we get

$$\frac{P}{Q} = \frac{R}{X}$$
$$\therefore X = \frac{Q}{P} \cdot R$$

Resistor Q is called the standard arm of the bridge, and resistor P and R are called the ratio arms.

## METER BRIDGE



$R$  – Unknown resistance

$S$  – Standard resistance

$l_1$  – Distance from A

$R_{cm}$  – Resistance of the wire per unit centimetre

$R_{cm}l_1$  – Resistance of length AD

$R_{cm}(100 - l_1)$  – Resistance of length DC

From the figure, the balance condition gives

$$\frac{R}{S} = \frac{R_{cm}l_1}{R_{cm}(100 - l_1)}$$

$$\therefore R = \frac{Sl_1}{100 - l_1}$$

## POTENTIOMETER

**Potentiometer:**

**Principle:**

$$V = IR$$

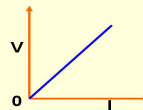
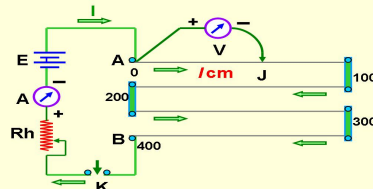
$$= I \rho l/A$$

If the constant current flows through the potentiometer wire of uniform cross sectional area (A) and uniform composition of material ( $\rho$ ), then

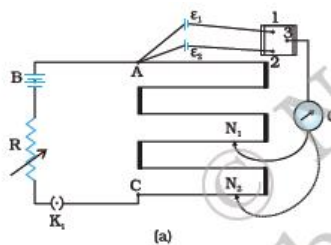
$$V = K I (\text{LENGTH}) \quad \text{or} \quad V \propto l$$

$V/I$  is a constant.

The potential difference across any length of a wire of uniform cross-section and uniform composition is proportional to its length when a constant current flows through it.



## COMPARISON OF EMF OF TWO CELLS



When 1 and 3 are connected,  $E_1$  is connected to the galvanometer (G).

Jockey is moved to  $N_1$ , which is at a distance  $l_1$  from A, to find the balancing length.

Applying loop rule to  $AN_1G31A$ ,

$$\Phi l_1 + 0 - E_1 = 0 \quad (1)$$

Where,  $\Phi$  is the potential drop per unit length

Similarly, for  $E_2$  balanced against  $l_2$  ( $AN_2$ ),

$$\Phi l_2 + 0 - E_2 = 0 \quad (2)$$

From equations (1) and (2),

$$\frac{E_1}{E_2} = \frac{l_1}{l_2}$$

### Internal Resistance of a cell using a potentiometer

We can also use a potentiometer to measure internal resistance of a cell. For this the cell (emf  $e$ ) whose internal resistance ( $r$ ) is to be determined is connected across a resistance box through a key  $K_2$ , as shown in the figure. With key  $K_2$  open, balance is obtained at length  $l_1$  ( $AN_1$ ). Then,

$$e = \phi l_1 \quad [3.93(a)]$$

When key  $K_2$  is closed, the cell sends a current ( $I$ ) through the resistance box ( $R$ ). If  $V$  is the terminal potential difference of the cell and balance is obtained at length  $l_2$  ( $AN_2$ ),

$$V = \phi l_2 \quad [3.93(b)]$$

$$\text{So, we have } e/V = l_1/l_2$$

$$[3.94(a)]$$

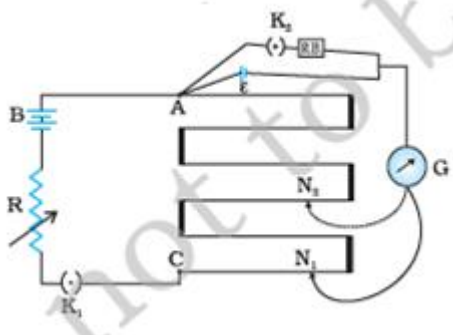
But,  $e = I(r + R)$  and  $V = IR$ . This gives

$$e/V = (r+R)/R$$

$$[3.94(b)]$$

From Eq. [3.94(a)] and [3.94(b)] we have

$$(R+r)/R = l_1/l_2$$



### RESOURCES:

NCERT TEXT BOOK- CLASS XII, TUTOR VISTA

PPT- MR.C. MANI, RETD DEPUTY COMMISSIONER, KVS RO, CHENNAI