

ELECTRIC POTENTIAL DUE TO *A DIPOLE*

ELECTRIC POTENTIAL DUE TO A DIPOLE, SPECIAL CASES

In the previous module we have seen about the potential due to a point charge as

$$V = \frac{1}{4\pi\epsilon_0} \frac{q}{r}$$

Here let us see for a dipole.

AB is a dipole. P is a point which is at a distance r_1 and r_2 from $+q$ and $-q$ respectively.

It is at a distance r from the centre of the axis.

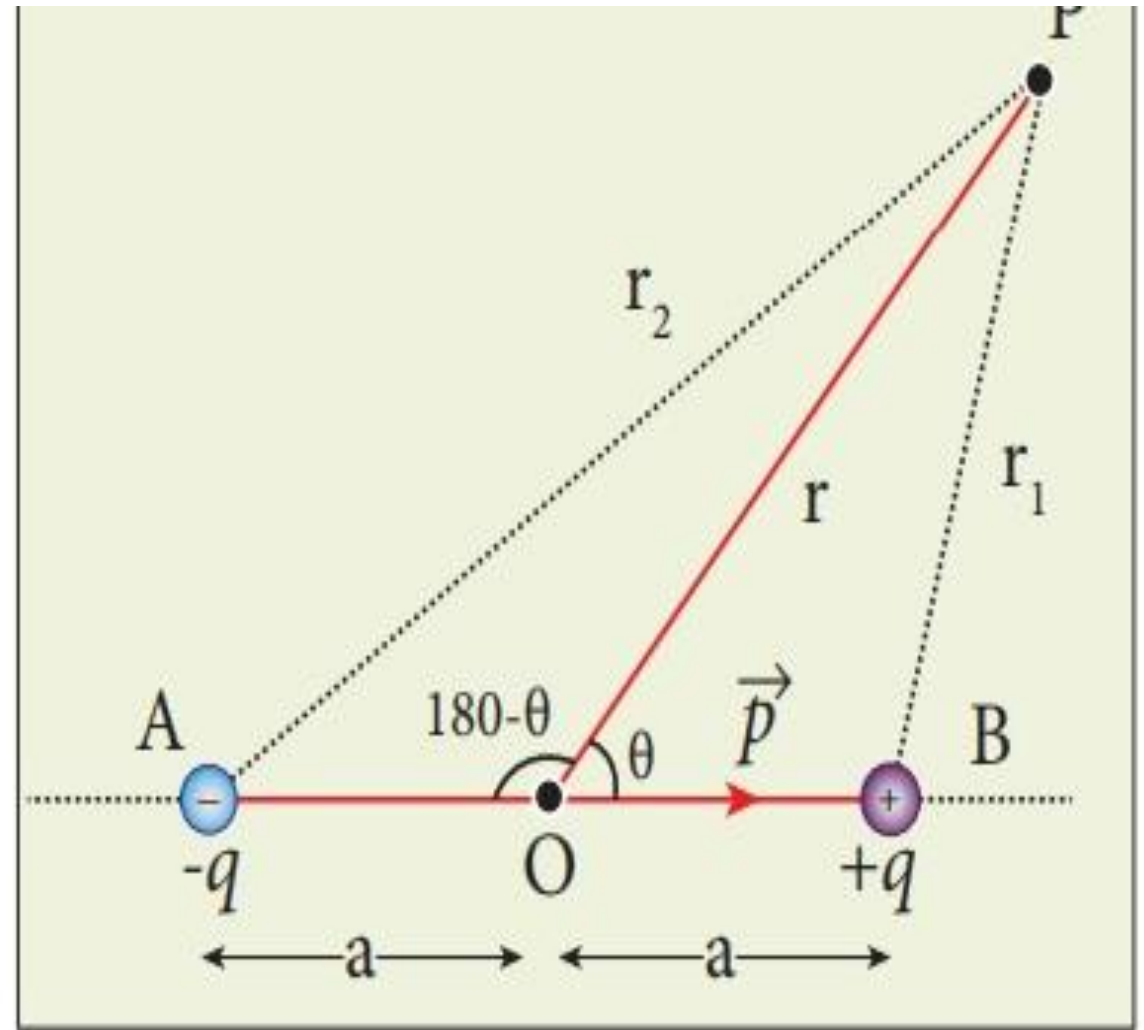


Figure 1.26 Potential due to electric dipole

potential at P due to +q $V_1 = \frac{1}{4\pi\epsilon_0} \frac{q}{r_1}$

Potential at P due to -q $V_2 = -\frac{1}{4\pi\epsilon_0} \frac{q}{r_2}$

Total potential at P is $V = V_1 + V_2 = \frac{1}{4\pi\epsilon_0} q \left(\frac{1}{r_1} - \frac{1}{r_2} \right)$

From the diagram, consider the $\triangle OBP$, and

Apply cosine law,

$$r_1^2 = r^2 + a^2 - 2ra \cos \theta$$

$$r_1^2 = r^2 + r^2 \frac{a^2}{r^2} - 2r^2 \frac{a}{r} \cos \theta$$

$$r_1^2 = r^2 \left(1 + \frac{a^2}{r^2} - 2 \frac{a}{r} \cos \theta \right)$$

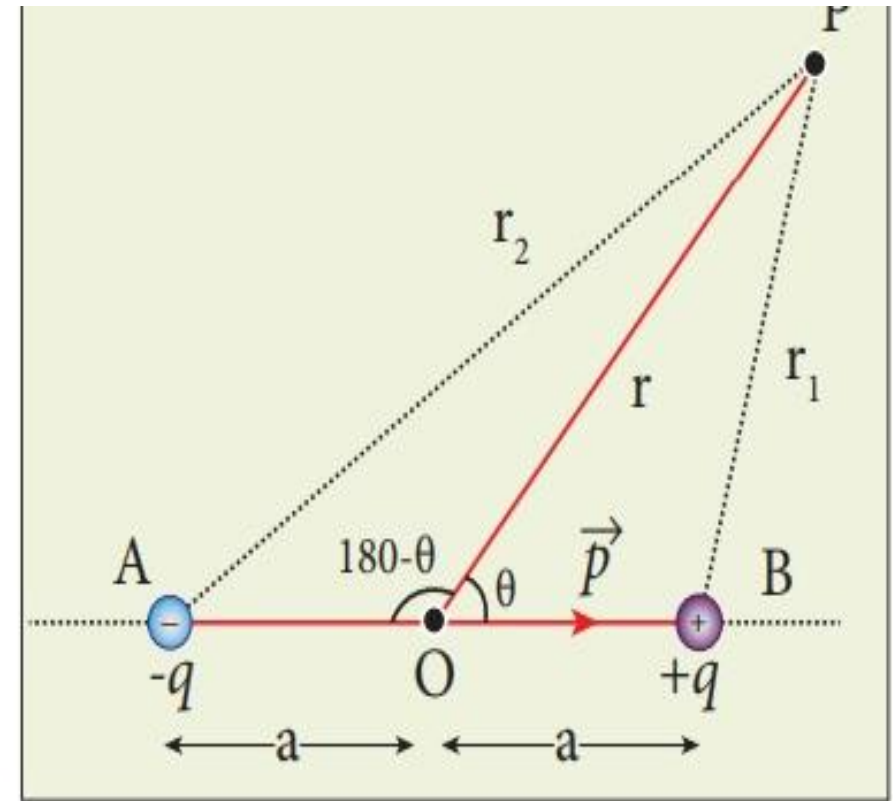


Figure 1.26 Potential due to electric dipole

Assume the point P is far from the dipole $r \gg a$. so neglect $\frac{a^2}{r^2}$

$r_1^2 = r^2 (1 - 2 \frac{a}{r} \cos \theta)$, take square root on both sides,

$r_1 = r (1 - 2 \frac{a}{r} \cos \theta)^{1/2}$, take reciprocal on both sides,

$\frac{1}{r_1} = \frac{1}{r} (1 - 2 \frac{a}{r} \cos \theta)^{-1/2}$, apply binomial

$\frac{1}{r_1} = \frac{1}{r} (1 - 2x - 1/2 \frac{a}{r} \cos \theta)$, $[(1+x)^n = 1+nx]$

$\frac{1}{r_1} = \frac{1}{r} (1 + \frac{a}{r} \cos \theta)$

Similarly consider ΔOAP ,

$$r_2^2 = r^2 + a^2 - 2ra\cos(180-\theta)$$

$$\cos(180-\theta) = -\cos\theta$$

$r_2^2 = r^2 + a^2 + 2ra\cos\theta$, repeat the same steps as we did before

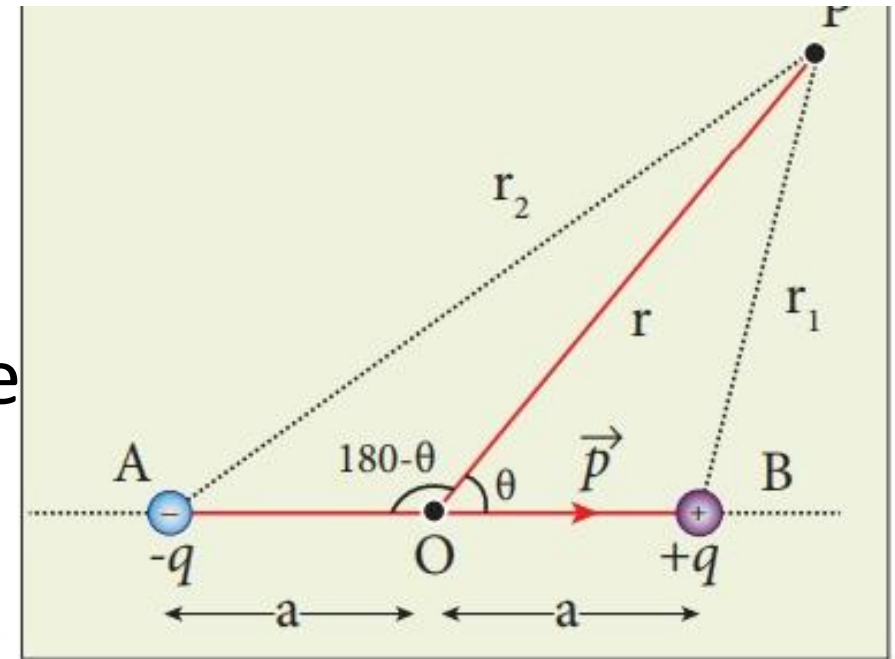


Figure 1.26 Potential due to electric dipole

Assume the point P is far from the dipole $r \gg a$. so neglect $\frac{a^2}{r^2}$

$$r_2^2 = r^2 \left(1 + 2 \frac{a}{r} \cos \theta\right), \text{ take square root on both sides,}$$

$$r_2 = r \left(1 + 2 \frac{a}{r} \cos \theta\right)^{1/2}, \text{ take reciprocal on both sides,}$$

$$\frac{1}{r_2} = \frac{1}{r} \left(1 + 2 \frac{a}{r} \cos \theta\right)^{-1/2}, \text{ apply binomial}$$

$$\frac{1}{r_2} = \frac{1}{r} \left(1 + 2x - \frac{1}{2} \frac{a}{r} \cos \theta\right), \quad [(1+x)^{-n} = 1 - nx]$$

$$\frac{1}{r_2} = \frac{1}{r} \left(1 - \frac{a}{r} \cos \theta\right)$$

Substituting $\frac{1}{r_1}$ and $\frac{1}{r_2}$ in V

$$V = \frac{1}{4\pi\epsilon_0} q \left[\frac{1}{r} \left(1 + \frac{a}{r} \cos\theta \right) - \frac{1}{r} \left(1 - \frac{a}{r} \cos\theta \right) \right]$$

$$= \frac{1}{4\pi\epsilon_0} q \frac{1}{r} \left[\left(1 + \frac{a}{r} \cos\theta \right) - \left(1 - \frac{a}{r} \cos\theta \right) \right]$$

$$= \frac{1}{4\pi\epsilon_0} q \frac{1}{r} \left[\left(2 \frac{a}{r} \cos\theta \right) \right]$$

$$= \frac{1}{4\pi\epsilon_0} q \frac{1}{r} \left[\left(2 \frac{a}{r} \cos\theta \right) \right]$$

$$= \frac{1}{4\pi\epsilon_0} \frac{2qa}{r^2} \cos\theta$$

$P=2qa$, so V becomes

$$V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2} \cos\theta$$

Write $p \cos\theta = \underbrace{\vec{p} \cdot \hat{r}}_p$

$$V = \frac{1}{4\pi\epsilon_0} \frac{\vec{p} \cdot \hat{r}}{r^2}$$

Special cases:

Case(i): if the point P lies on the axial line and at the side of $+q$ as in the figure.



$$\theta = 0, \cos 0 = 1, V = \frac{1}{4\pi\epsilon_0} \frac{p}{r^2}$$

Case(ii): if the point P lies on the axial line and at the side of -q as in the figure.



$$\theta = 180, \cos 180 = -1$$

$$V = \frac{-1}{4\pi\epsilon_0} \frac{p}{r^2}$$

Case(iii): if the point P lies on the equatorial line of the dipole, then

$$\theta = 90, \cos \theta = 0, \boxed{V = 0}$$

