

APPLICATIONS OF GAUSS LAW

Case (i) ELECTRIC FIELD DUE TO AN INFINITELY LONG CHARGED WIRE

ELECTRIC FIELD DUE TO AN INFINITELY LONG CHARGED WIRE:

In this topic we are going to use the following concepts:

Gauss's law:

$$\Phi_E = \oint \vec{E} \cdot \vec{dA} = \oint E dA \cos \theta = \frac{Q_{enclosed}}{\epsilon_0}$$

Symmetric nature of the Gaussian surface. That is every where on the surface the electric field remains the same

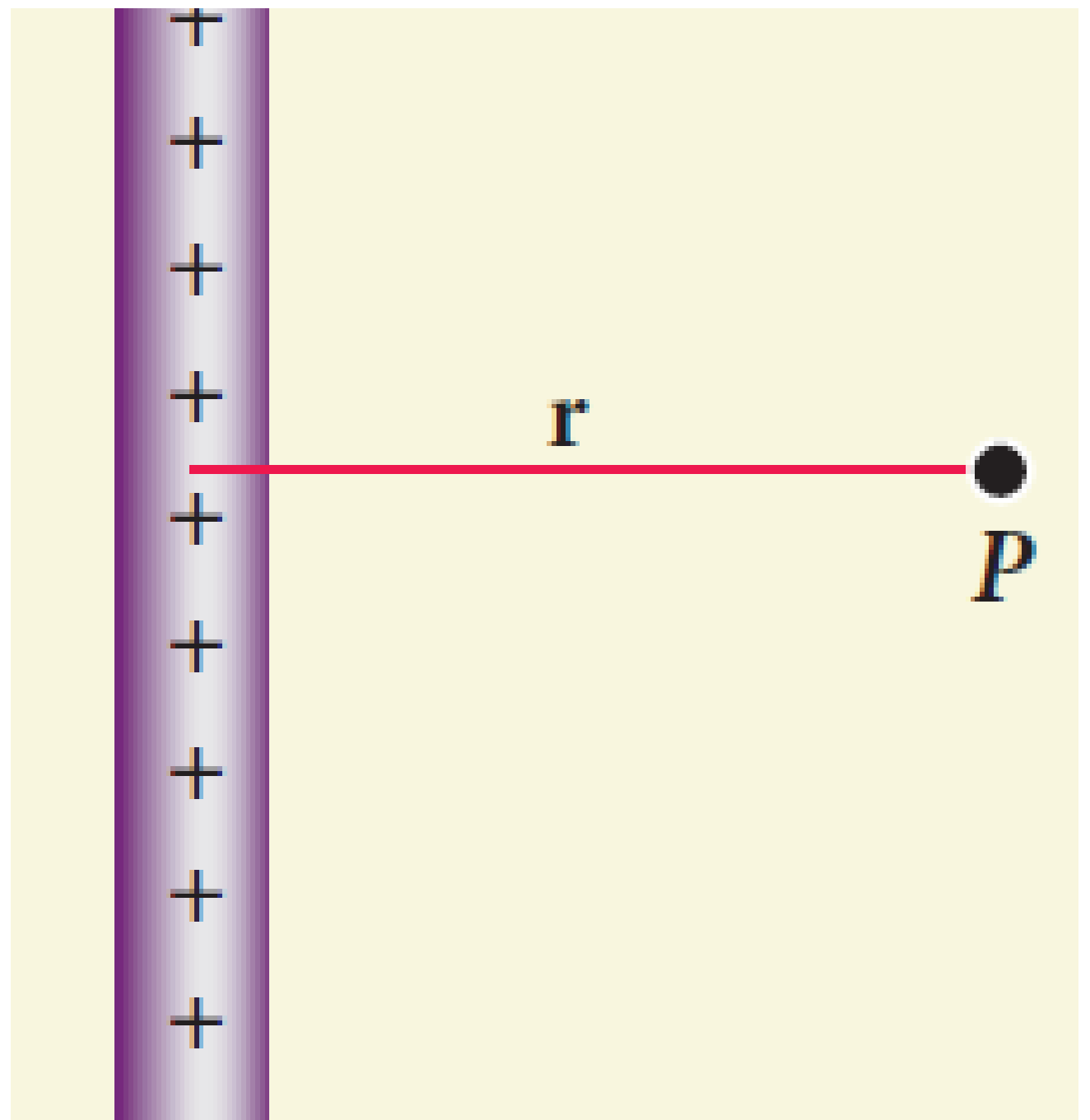
Consider an infinitely long wire of linear charge density λ

Linear charge density is the charge per unit length.

$$\lambda = q/L$$

There will be an electric field around the wire.

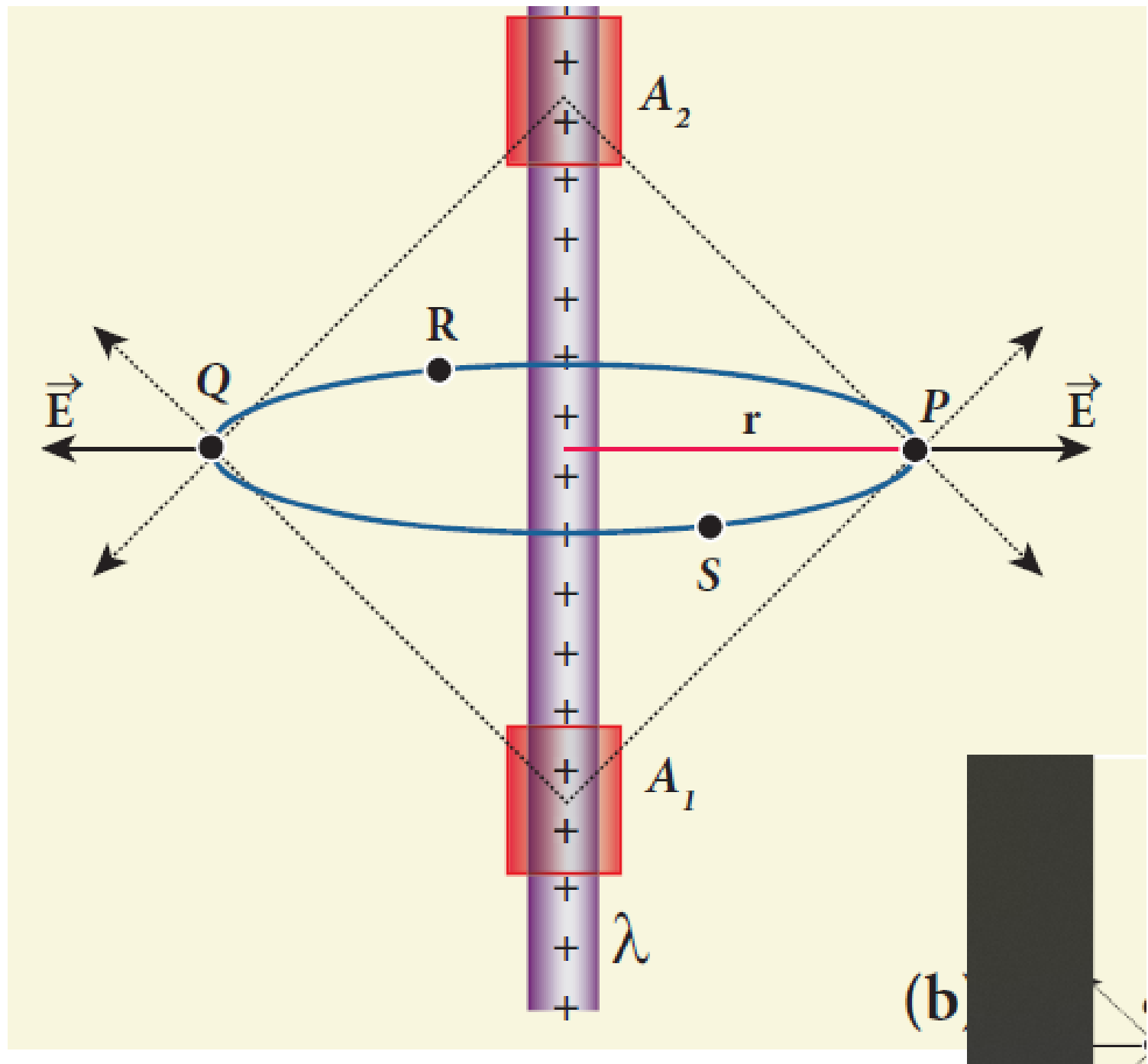
There is a point P which is at a perpendicular distance r from the wire



Choose two small elements A_1 and A_2 which are equi distance from P.

The electric fields from these two elements point outward and they are equal in magnitude.

Like that you can consider various points and if you join all the points it will show cylindrical symmetry as shown in the next figure.



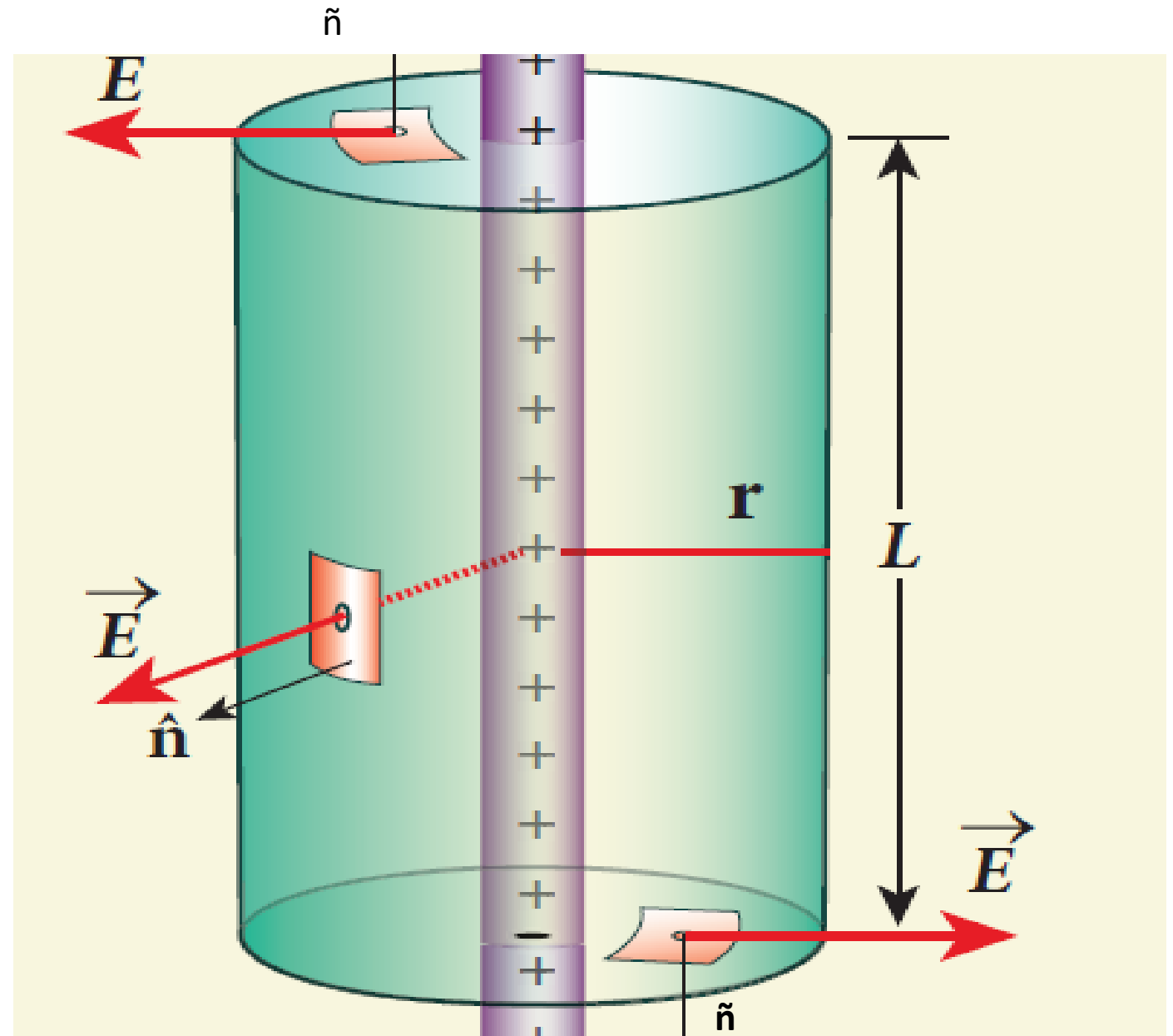
Consider the cylindrical Gaussian surface of radius r and length L .

divide the surface into three :

**CURVED SURFACE, TOP SURFACE
AND BOTTOM SURFACE.**

Calculate the electric field due to the entire surfaces as follows:

$$\begin{aligned}\Phi_E &= \oint \vec{E} \cdot d\vec{A} \\ &= \int_{\text{curved surface}} \vec{E} \cdot d\vec{A} + \int_{\text{top surface}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom surface}} \vec{E} \cdot d\vec{A}\end{aligned}$$



In the top and bottom surfaces the area vector and the electric fields are perpendicular to each other. So the integrations become zero.

In the curved surface the area vector and the electric fields are parallel.

$$\phi_E = \int E \cdot dA = \frac{Q_{enclosed}}{\epsilon_0}, \quad \lambda = Q/L \text{ and } Q = \lambda L$$

curved surface

Here due to symmetry E is constant

$$\phi_E = E \int dA = \frac{\lambda L}{\epsilon_0}$$

The total surface area of the cylinder is $2\pi rL$

substitute the values in the previous equation

$$\phi_E = E \int dA = \frac{\lambda L}{\epsilon_0}$$

$$\phi_E = E 2\pi r L = \frac{\lambda L}{\epsilon_0}$$

$$E = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r}$$

In vector form, $\vec{E} = \frac{1}{2\pi\epsilon_0} \frac{\lambda}{r} \hat{r}$

So the electric field varies inversely with the distance.

Important points:

if $\lambda > 0$ ie, if the charge is positive, the electric field is radially outward

if $\lambda < 0$ ie, if the charge is negative, the electric field is radially inward

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E dA \cos\theta = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$\Phi_E = \oint \vec{E} \cdot d\vec{A}$$

$$= \int_{\text{curved surface}} \vec{E} \cdot d\vec{A} + \int_{\text{top surface}} \vec{E} \cdot d\vec{A} + \int_{\text{bottom surface}} \vec{E} \cdot d\vec{A}$$

curved surface **top surface** **bottom surface**

$$\Phi_E = \int E \cdot dA = \frac{Q_{\text{enclosed}}}{\epsilon_0}, \quad \lambda = Q/L \text{ and } Q = \lambda L$$

curved surface

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