

ELECTROSTATICS

Electric Charge is an intrinsic property of elementary particles of matter which gives rise to electric force between various objects.

Electric charge is a scalar quantity and its SI unit is Coulomb.

Charge on proton = 1.6×10^{-19} C

Charge on electron = -1.6×10^{-19} C

Like charges repel each other and unlike charges attract each other.

Charging Of a Body

Sign Convention:-

1. The charge developed on glass rod when rubbed with silk is called positive charge since the charges are transferred from glass rod to silk.
2. The charge developed on ebonite rod when rubbed with wool/cat skin is called negative charge since the charges are transferred from cat skin to ebonite rod.

There are three methods to charge an object:-

1. Charge by Friction
2. Charge by Conduction
3. Charge by Induction

1. By rubbing or by friction:

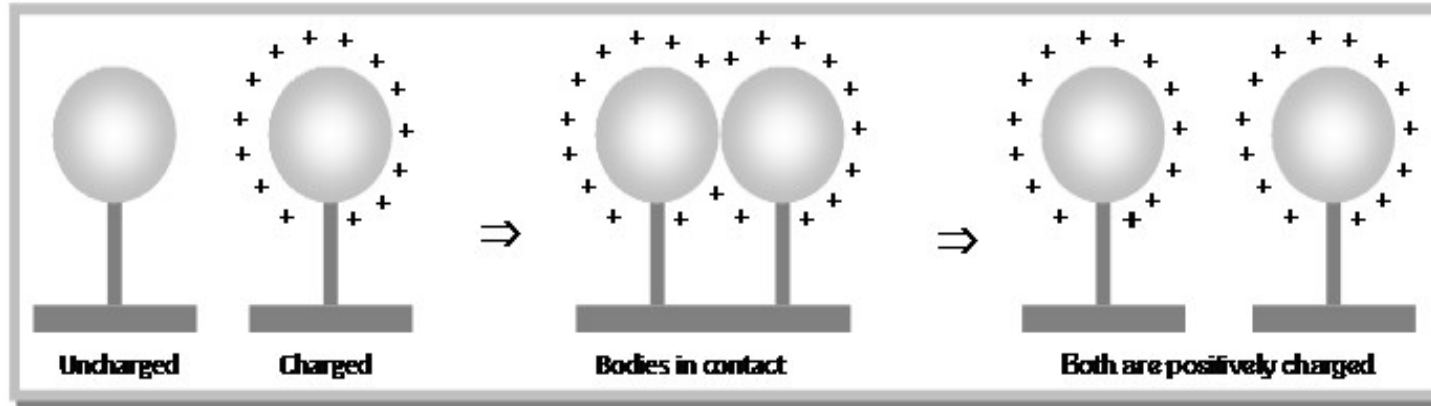
- When two different bodies rubbed against each other, electrons are transferred from the material with lower work function to a material with a higher work function.
- A body with lower work function will acquire a positive charge whereas a body with higher work function will acquire a negative charge.

Work function:

- It is a minimum amount of energy required to eject an electron from a metal surface

2. Charging by conduction:

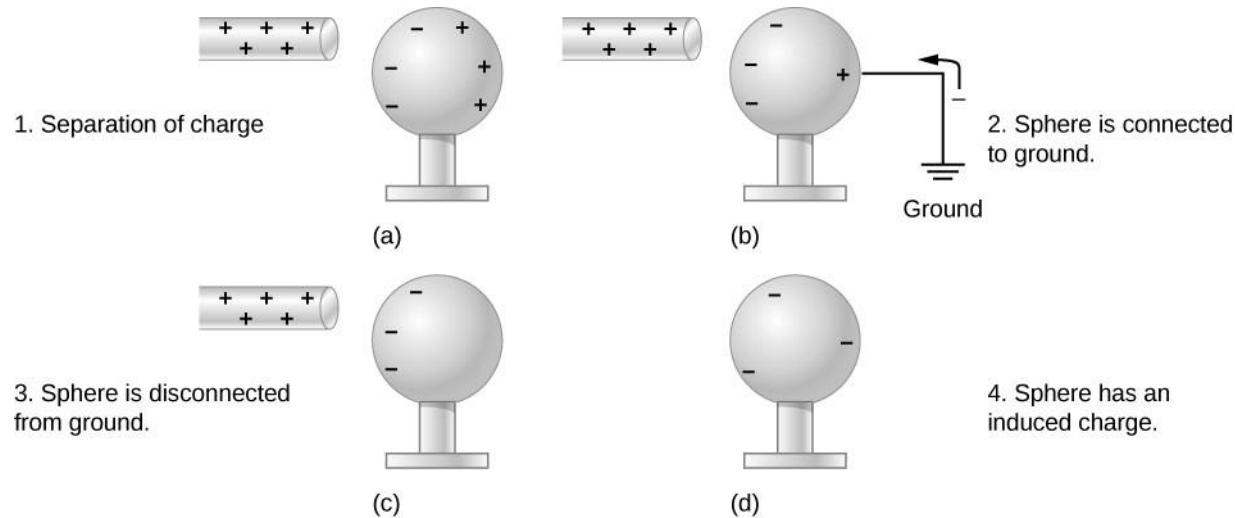
- Charging by Conduction method the objects physically come in contact with each other as shown in figure



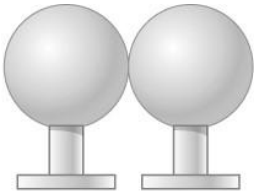
3. Charging by electrostatic induction

- When we place a charged body near to an uncharged body, an opposite charges induced on the near surface of uncharged body.

With one Sphere

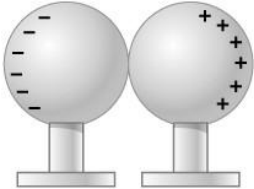


With Two Spheres



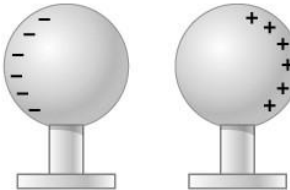
(a)

A charged rod...



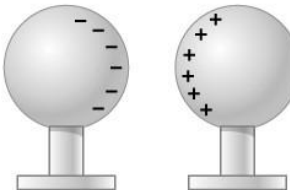
... causes separation of charge

(b)



The spheres are separated.

(c)



Each sphere is now charged:
one positive, one negative

(d)

- Properties of charge:

1. Quantization of charge:- The quantization of electric charge means that the total charge of a body is always an integral multiple of basic quantum of charge.

$$q = \pm ne$$

- *where $n = 0, 1, 2 \dots \dots \dots$* $e = 1.6 \times 10^{-19}C$

Cause of Quantization:- During rubbing only an integral number of electrons can be transferred from one body to another.

- 3. While dealing with the macroscopic charge we can ignore quantization of charge because the basic charge $e = 1.6 \times 10^{-19}C$ is very small as compared to the charge on macroscopic object.

2. Additivity of Electric Charge:-

Additivity of Electric Charge of electric charge means that the total charge of the system is the algebraic sum of all the individual charges located at different points inside the system.

If a system contains charges q_1 , q_2 , q_3 ----- q_n then its total charge is

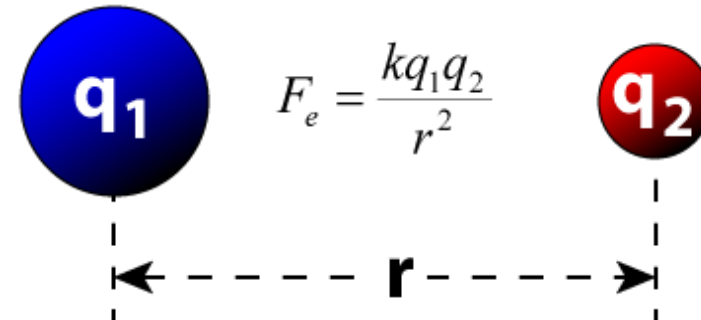
$$q = q_1 + q_2 + q_3 + \text{-----} q_n$$

3. Conservation of Charge:-

Conservation of charge states that the electric charge can neither be created nor be destroyed, they can only be transferred from one body to another. The total charge of an isolated system remains constant.

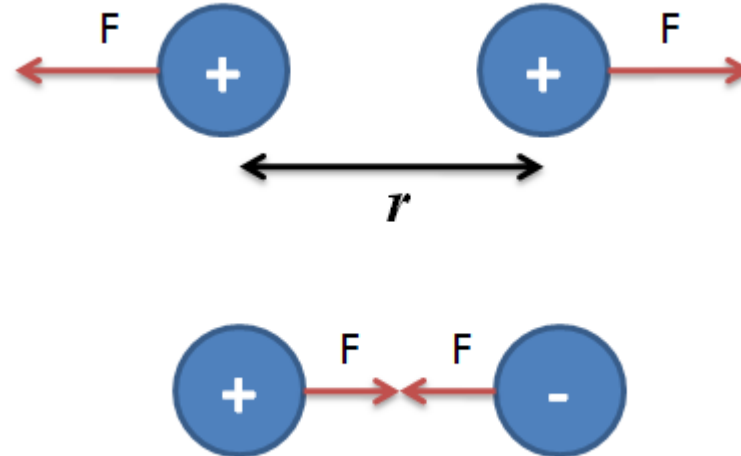
Coulomb's Law

- Charles-Augustin de **Coulomb**, a French physicist in 1784, measured the force between two point charges and found that the force of attraction or repulsion between two stationary point charges directly proportional to the product of charges (magnitudes only) and inversely proportional to the square of the distance between the charges. This force act along the line joining the two charges.
- Generally, the statement, let's say there are two charges q_1 and q_2 . The distance between the charges is 'r', and the force of attraction/repulsion between them is 'F'. Then
 - $F \propto q_1 q_2$
 - $F \propto 1/r^2$
 - $F = k q_1 q_2 / r^2$
 - where k is proportionality constant and equals to $1/4 \pi \epsilon_0$, the value of k comes $9 \times 10^9 \text{ Nm}^2/\text{C}^2$ **and** the symbol ϵ_0 is called epsilon naught and it signifies permittivity of a vacuum.
 - $\epsilon_0 = 1/4 \pi k$ is equal $8.854 \times 10^{-12} \text{ C}^2 \text{ N}^{-1} \text{ m}^{-2}$



Coulomb's Law

Coulomb also stated that like charges repel each other and unlike charges attract each other. This means charges of same sign will push each other with repulsive forces while charges with opposite signs will pull each other with attractive force.



Vector Form of Coulomb's Law

- $\mathbf{F}_{12}$ = force on charge q_1 due to q_2
- $\mathbf{F}_{21}$ = force on charge q_2 due to q_1
- $\mathbf{r}_{12}$ = unit vector pointing from q_2 to q_1
- $\mathbf{r}_{21}$ = unit vector pointing from q_1 to q_2

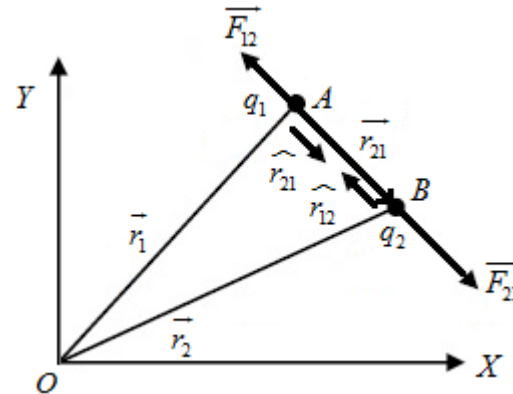
$$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12}$$

$$\hat{\mathbf{r}}_{12} = \frac{\mathbf{r}_{12}}{r_{12}}$$

$$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{21}^2} \hat{\mathbf{r}}_{21}$$

$$\hat{\mathbf{r}}_{21} = \frac{\mathbf{r}_{21}}{r_{21}}$$

$$\hat{\mathbf{r}}_{21} = -\hat{\mathbf{r}}_{12}$$

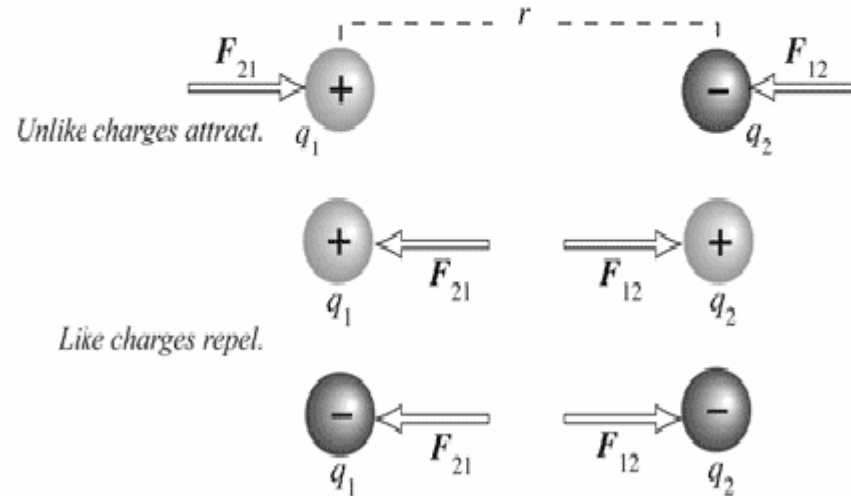


- This proves that Coulomb's Law obeys Newton's Third Law i.e. every action has its equal and opposite reaction
- As the coulomb force act along $\mathbf{F}_{12}$ or $\mathbf{F}_{21}$ i.e. along the line joining the Centre of two charges hence they are central forces.

Vector Form of Coulomb's Law

$$F = k \frac{q_1 q_2}{r^2}$$

F = electrostatic force
 q = electric charge
 r = distance between charge centers
 k = Coulomb constant
 $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$



1 Coulomb or 1 C is defined as the charge when placed at a distance of 1m from another charge of space magnitude in a vacuum and experiences a repulsion force of magnitude $9 \times 10^9 \text{ N}$.

Limitations of Coulomb's Law

It is valid only under the following conditions

1. The electric charge must be at rest.
2. The electric charge must be a point charge i.e. the extension of charges must be much smaller than the separation between the two charges.
3. The separation between the two charges must be greater than Nuclear size(10^{-15}m), because for distance less than 10^{-15}m , the strong nuclear forces dominates over the electrostatics force.

Dielectric Constant & Relative Electrical Permittivity

- **Dielectric constant:-** The dielectric constant (k) of a medium can be defined as the ratio of the force between two charges separated by some distance apart in free space to the force between the same two charges separated by the same distance apart in that medium.

$$\text{So, } k = F_0 / F_m$$

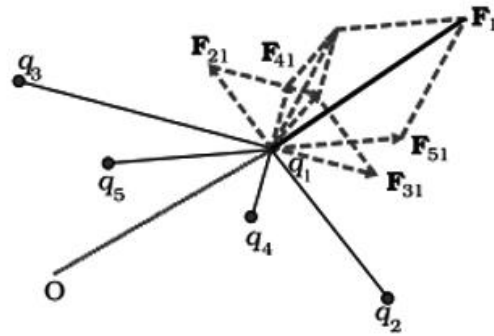
Here, F_0 and F_m are the magnitudes of the force between them in free space and in a medium respectively.

- **Relative Electrical Permittivity (ϵ_r):-**

The relative permittivity (ϵ_r) of a medium is defined as the ratio between its permittivity of the medium (ϵ) and the permittivity (ϵ_0) of the free space.

$$\epsilon_r = \epsilon / \epsilon_0$$

Superposition of Electrostatics Force



Principle of superposition states that when a number of charges are interacting the total force on a given charge is the vector sum of Force exerted on it due to all other charges. The force between the two charges is not affected by the presence of other charges.

$$\mathbf{F}_{12} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r_{12}^2} \hat{\mathbf{r}}_{12}$$

$$\mathbf{F}_{13} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_3}{r_{13}^2} \hat{\mathbf{r}}_{13}$$

$$\mathbf{F}_{1n} = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_n}{r_{1n}^2} \hat{\mathbf{r}}_{1n}$$

$$\mathbf{F}_1 = \mathbf{F}_{12} + \mathbf{F}_{13} + \dots + \mathbf{F}_{1n} = \frac{q_1}{4\pi\epsilon_0} \sum_{i=2}^n \frac{q_i}{r_{1i}^2} \hat{\mathbf{r}}_{1i}$$

Comparing Gravitational and Electrostatics Force

Similarities

1. Both forces obey inverse square law i.e $F \propto 1/r^2$
2. Both are central forces i.e. they act along the line joining the centre of the two bodies.
3. Both forces are proportional to product of masses or charges.
4. Both are conservative forces.
5. Both forces can operate in vacuum.

Dissimilarities

1. Gravitational force is attractive while electrostatic force may be attractive or repulsive.
2. Gravitational force does not depend on the nature of medium while electrostatic force depend on nature of medium between two charges.
3. Electrostatics are much stronger than gravitational force.

Electric Field

The electric field is defined as the force experienced by a unit positive test charge placed at that point without disturbing the position of source charge. The electric field is a vector quantity and it denoted by **E**. The standard unit of the electric field is **Newton/ Coulomb** or **N/C**.

i.e. $E=F/q_0$, there is a difficulty in defining the electric field by the above equation, the test charge q_0 may disturb the charge distribution of source charge, hence changes the electric field which we want to measure. The test charge q_0 must be small enough so that it does not change the value of E . It is better to define electric field as force per unit test charge acting on a vanishingly small positive test charge placed at that point. Hence

$$\mathbf{E} = \lim_{q \rightarrow 0} \frac{\mathbf{F}}{q}$$

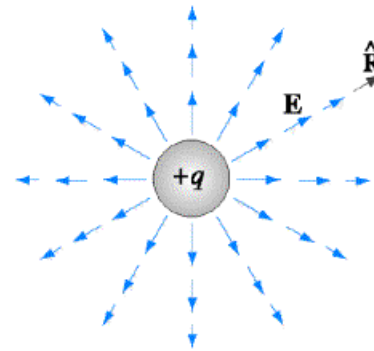
Let q be the source charge and q_0 be the test charge placed at a distance r from the source charge. The force between them is given by

$$F=kq_0 q/r^2$$

$$E=F/q_0$$

$$E= Kq_0 q/r^2 q_0$$

$$E=kq/r^2$$



Electric Field

- Electric field is a vector expressed in terms of the electric force and the charge

$$\vec{E} \equiv \frac{\vec{F}_e}{q}$$

$$\vec{F}_e = q\vec{E}$$

$$\vec{E} = \frac{1}{4\pi\epsilon_0} \frac{q}{r_{PC}^2} \hat{r}_{PC}$$

- If $Q > 0$, E points away from Q
- If $Q < 0$, E points toward Q

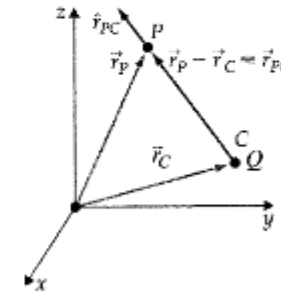
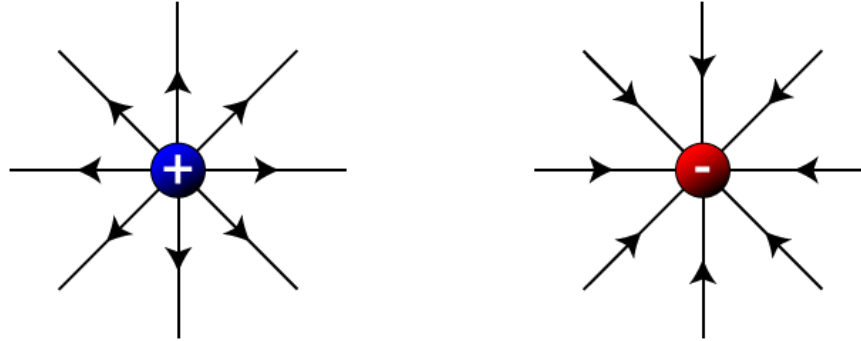


Figure E2.1
A diagram illustrating the definition of the vector $\vec{r}_{PC} \equiv \vec{r}_P - \vec{r}_C$ and the unit vector $\hat{r}_{PC}$.

Electric Field



Direction of electric field, radially outward for positive blue charge and radially inward for negative charge

If the electric field is positive, then its direction is radially outward, and if the electric field is negative, then its direction is radially inward. The electric field is also called **Electric Field Intensity** or **Electric Field Strength** as it determines the strength of electric field.

Electric Force

Electric field produced by charge Q at a distance ' r ' is E , and we place another charge q in it. The force exerted by charge Q on charge q be " F " and is equal to

$$F = \frac{kQq}{r^2} \hat{r}$$

$$F = qE$$



Direction of Electric Field in case of unit positive charge



Direction of Electric Field in case of unit negative charge

Properties of Electric Force

Electric force “F” and electric field “E” are the interlinked quantity and show several properties which are as follows:

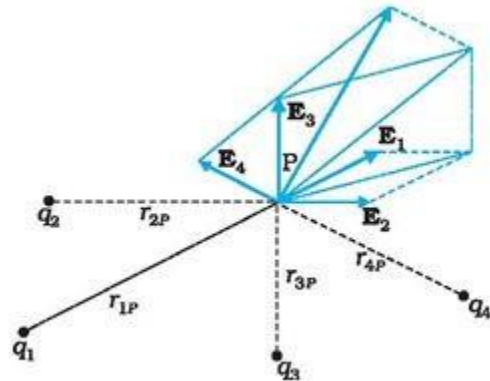
- The charge Q which produces electric field is called **Source Charge** and the charge q which tests the presence and effect of electric field on it is called **Test Charge**.
- The source charge Q must be stationary in order to make the electric field exertive when source charge Q will repel test charge q, then test charge will also try to move source charge Q and this may disturb the presence of electric field. To ensure presence of electric field we take q to be very small, or negligible with respect to Q so that electric field becomes exertive and can be represented as

$$\mathbf{E} = \lim_{q \rightarrow 0} \left(\frac{\mathbf{F}}{q} \right)$$

- The force F is proportional to q, which means F/q is a finite quantity and defines electric field
- The direction of electric field for a positive charge is radially outwards from the source charge, and direction of electric field for a negative charge is radially inwards from the source charge
- Electric field is same if the distance between charges are equal
- The electric field on any charge depends only on distance ‘r’. Electric field depends on symmetry of charge, if we have a sphere of charge Q, then electric field will be same throughout distance r

Electric Field due to Multiple Charges

- Consider a system having charges $q_1, q_2, q_3, \dots, q_n$ and let their position vectors with respect to origin be $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{r}_n$ respectively
- The electric field due to q_1 , on a unit positive test charge at position vector $\mathbf{r}_1$, will be independent of those of $q_2, q_3, \dots, q_n$. Thus we can apply the principle of superposition to proceed further with the process



Vector diagram depicting all the electric fields

Electric Field due to Multiple Charges contd...

Charges is given by the superposition theorem.

$$\vec{E}_1 = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots \dots \vec{E}_n$$

Electric field intensity due to the n^{th} charge is

$$\vec{E}_n = \frac{1}{4\pi\epsilon_0} \frac{q_n}{r_n^2} \hat{r}$$

Magnitude of the electric field intensity is given by the equation:

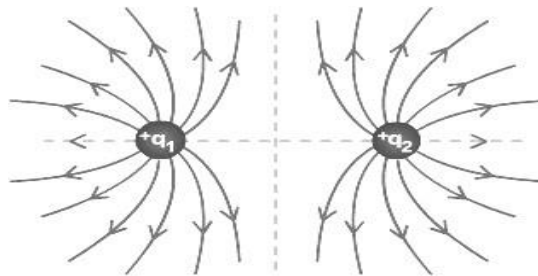
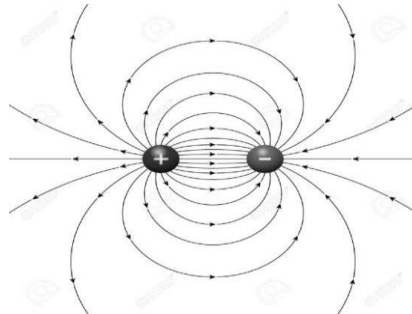
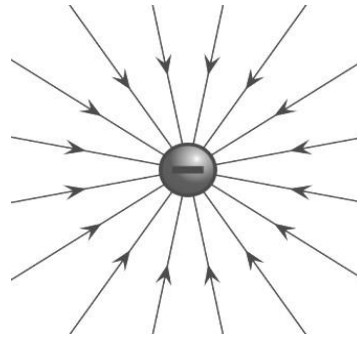
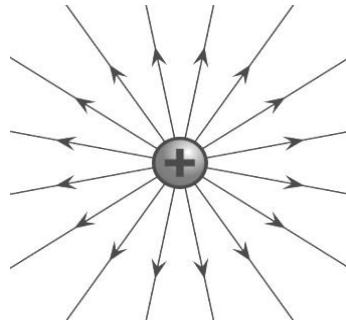
$$E_n = \frac{1}{4\pi\epsilon_0} \frac{q_n}{r_n^2}$$
$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \dots + \mathbf{E}_N = \frac{1}{4\pi\epsilon_0} \sum_{i=1}^N \frac{Q_i}{R_i^2} \hat{R}_i ,$$

Physical Significance of Electric Field

- The concept of electric field is convenient as it tells about the electrical environment around the charges
- Electric field helps in analyzing force on a unit charge at any point
- Electric field is a property of system of charges and is independent of test charge
- Electric field is defined at every point in space and varies accordingly
- The true and useful significance of electric field is that we can apply the concept of the electric field when charges are accelerated (in motion).

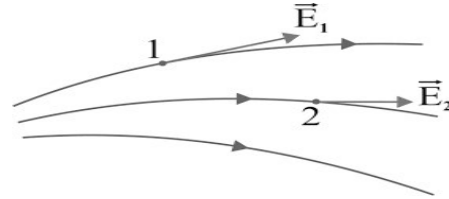
Electric field lines of force:

These are the imaginary lines on which a test charge moves under the action of an external electrostatic force.

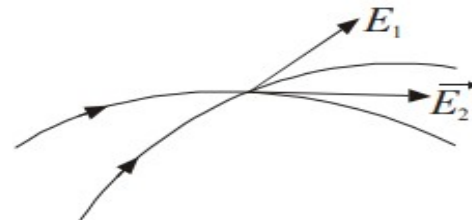


Properties of electric field lines:

1. These are the continuous curves which originate from the surface of positive charge and terminate on the surface of negative charge. And get discontinue on the surface of the charges. They do not form close loops.
2. Direction of electric field at any point can be determined by drawing a tangent on the electric field line passing through that point.



3. No two electric field lines can intersect each other. If it is so there would be two directions of electric field at the point of intersection which is not possible.



Properties of electric field lines:

4. Electric field lines are perpendicular to the surface of charge.
5. In the case of unlike charges electric field lines contract longitudinally.
6. In the case of like charges electric field lines exert lateral pressure.

Continuous Charge Distribution

Linear Charge Density

When the charge is non-uniformly distributed over the length of a conductor, it is called linear charge distribution. It is also called linear charge density and is denoted by the symbol λ (Lambda).

Mathematically linear charge density is

$$\lambda = \frac{dq}{dl}$$

The unit of linear charge density is C/m. If we consider a conductor of length 'L' with surface charge density λ and take an element dl on it, then small charge on it will be

$$dq = \lambda dl$$

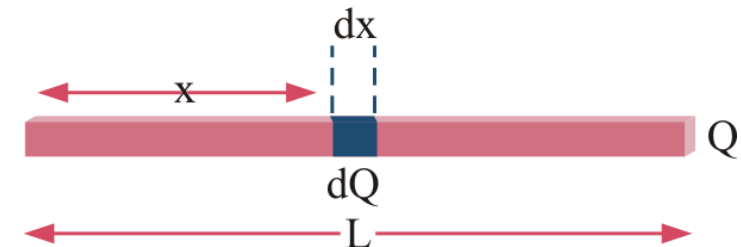
So, the electric field on small charge element dq will be

$$dE = \frac{k dq}{r^2}$$

$$dE = \frac{k \lambda dl}{r^2}$$

To calculate the net electric field we will integrate both sides with proper limit, that is

$$\int dE = \int_0^L \frac{k \lambda dl}{r^2}$$
$$\int dE = \frac{k}{r^2} \int_0^L \lambda dl \quad a^2 + b^2 = c^2$$



Surface Charge Density

When the charge is uniformly distributed over the surface of the conductor, it is called **Surface Charge Density** or **Surface Charge Distribution**. It is denoted by the symbol σ (sigma) symbol and is It is also defined as charge/ per unit area. Mathematically surface charge density is the unit is C / m².

$$\sigma = \frac{dq}{ds}$$

where dq is the small charge element over the small surface ds. So, the small charge on the conductor will be

$$dq = \sigma ds$$

The electric field due to small charge at some distance 'r' can be evaluated as

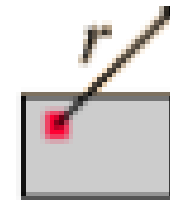
$$dE = \frac{k dq}{r^2}$$

$$dE = \frac{k \sigma ds}{r^2}$$

Integrating both sides with proper limits we get

$$\int dE = \int_0^S \frac{k \sigma ds}{r^2}$$

$$\int dE = \frac{k}{r^2} \int_0^S \sigma ds$$



$$\sigma dA = dQ$$

Area charge density

Volume Charge Density

When the charge is distributed over a volume of the conductor, it is called **Volume Charge Distribution**. It is denoted by symbol ρ (rho). In other words charge per unit volume is called **Volume Charge Density** and its unit is C/m^3 . Mathematically, volume charge density is

$$\rho = \frac{dq}{dv}$$

where dq is small charge element located in small volume dv . To find total charge we will integrate dq with proper limits. The electric field due to dq will be

$$dq = \rho dv$$

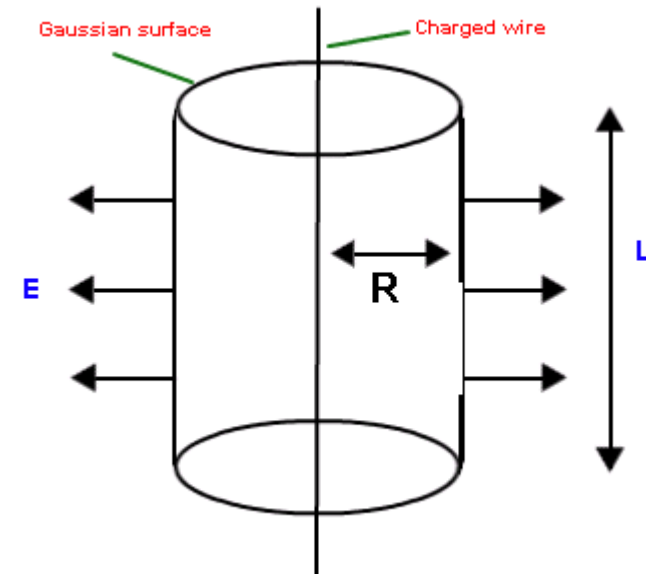
$$dE = \frac{k dq}{r^2}$$

$$dE = \frac{k \rho dv}{r^2}$$

Integrating both sides with proper limits we get

$$\int dE = \int_0^v \frac{\rho dv}{r^2}$$

$$\int dE = \frac{k}{r^2} \int_0^v \rho dv$$



Electric Dipole

A pair of equal and opposite charge separated by a small distance is called dipole.

Dipole moment measures the strength of an electric dipole, it's a vector quantity whose magnitude is either charge times the separation between the two opposite charges and the direction is along the dipole axis from the negative to the positive charge.

$$\mathbf{p} = q \times 2a$$

The SI. Unit of dipole is Cm.

Electric Field due to a Dipole on Axial Line

Let the point P be at distance r from the center of the dipole on the side of the charge q, as shown in Fig.

$$\mathbf{E}_{-q} = -\frac{q}{4\pi\epsilon_0(r+a)^2} \hat{\mathbf{p}}$$

Where p is the unit vector along the dipole axis from -q to +q.

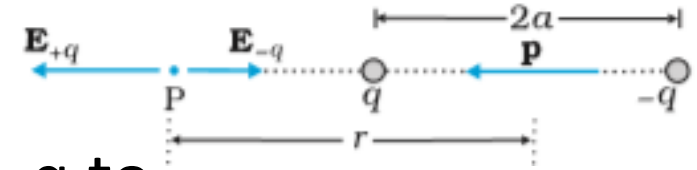
$$\mathbf{E}_{+q} = \frac{q}{4\pi\epsilon_0(r-a)^2} \hat{\mathbf{p}}$$

The total field at P is

$$\begin{aligned}\mathbf{E} &= \mathbf{E}_{+q} + \mathbf{E}_{-q} = \frac{q}{4\pi\epsilon_0} \left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2} \right] \hat{\mathbf{p}} \\ &= \frac{q}{4\pi\epsilon_0} \frac{4ar}{(r^2 - a^2)^2} \hat{\mathbf{p}} \quad E_C = \frac{1}{4\pi\epsilon_0} \left(\frac{2pr}{(r^2 - a^2)^2} \right)\end{aligned}$$

For $r \gg a$

$$\mathbf{E} = \frac{4qa}{4\pi\epsilon_0 r^3} \hat{\mathbf{p}} \quad (r \gg a) \quad E_C = \frac{1}{4\pi\epsilon_0} \left(\frac{2p}{r^3} \right)$$



Electric Field due to a Dipole on Equatorial Line

$$E_1 = E_2 = \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + a^2)}$$

As the magnitude of electric field are equal therefore the sine component of electric field is cancelled out, whereas the net electric field is due to the cosine components.

$$E = E_1 \cos\theta + E_2 \cos\theta$$

$$E = 2E_1 \cos\theta$$

$$E = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + a^2)} \cos\theta$$

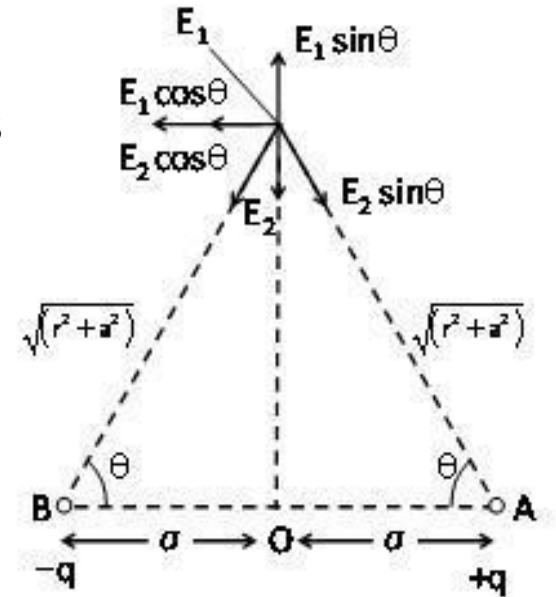
$$E = 2 \frac{1}{4\pi\epsilon_0} \frac{q}{(r^2 + a^2)} \frac{a}{\sqrt{(r^2 + a^2)}}$$

$$E = 2 \frac{1}{4\pi\epsilon_0} \frac{qa}{(r^2 + a^2)^{\frac{3}{2}}}$$

As $p = q \cdot 2a$

$$E = \frac{1}{4\pi\epsilon_0} \frac{p}{(r^2 + a^2)^{\frac{3}{2}}} \Rightarrow \text{If } r \gg 2a$$

$$E = \frac{1}{4\pi\epsilon_0} \left(\frac{p}{r^3} \right) \quad E_{axial} = 2E_{equatorial}$$



Torque on a Dipole in a Uniform Electric Field

- Let us assume an electric dipole is placed in a uniform magnetic field as shown in figure. Each charge of dipole experience a force qE in electric field. Since points of action of these forces are different, these equal and anti parallel forces give rise to a couple that rotate the dipole and make the dipole to align in the direction of field.
- The torque τ experienced by the dipole is $(qE) \times (2d \sin \theta)$, where $2d$ is the length of dipole and θ is the angle between dipole and field direction

$$\tau = (qE) \times (2d \sin \theta)$$

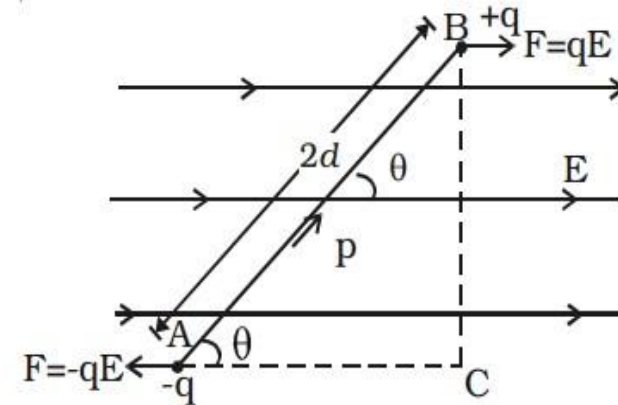
$$\tau = p E \sin \theta$$

$$\tau = \mathbf{p} \times \mathbf{E}$$

If $\theta = 0$, torque = 0, Stable equilibrium

If $\theta = 90$, torque = pE , Maximum

If $\theta = 180$, torque = 0, Unstable equilibrium



Dipole in a uniform field

Work done by a Dipole in Uniform Electric Field and Potential Energy

Suppose an external torque τ_{ext} is applied to neutralize this torque and rotates the dipole from an angle θ_0 to θ_1

at an infinitesimal angular speed and without angular acceleration. The amount of work done by the external torque will be given by

$$W = \int_{\theta_0}^{\theta_1} \tau_{\text{ext}}(\theta) d\theta = \int_{\theta_0}^{\theta_1} pE \sin \theta d\theta$$
$$= pE (\cos \theta_0 - \cos \theta_1)$$

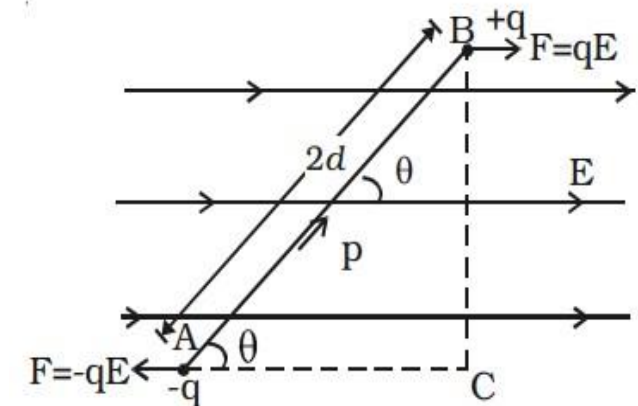
This work is stored as the potential energy of the system.

if $\theta_0 = \pi/2$ and $\theta_1 = \theta$

$$U(\theta) = pE (\cos(\pi/2) - \cos\theta) = -pE \cos\theta = -\mathbf{p} \cdot \mathbf{E}$$

when $\theta = 0$, i.e., $\mathbf{p}$ is aligned in the direction of $\mathbf{E}$, potential energy is minimum.

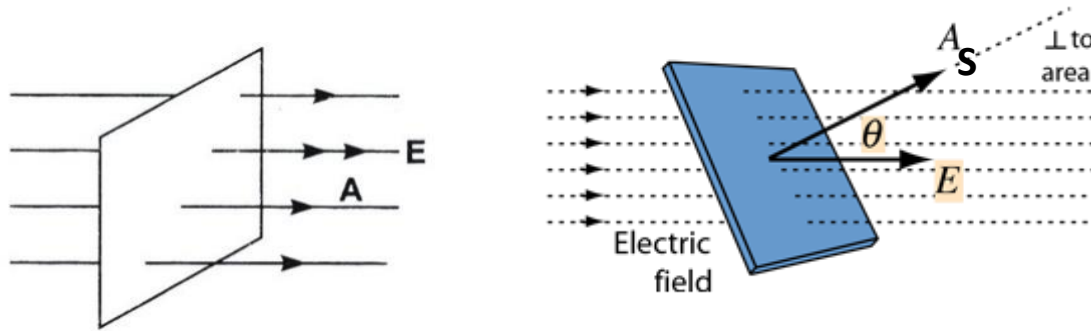
when $\theta = \pi$, i.e., $\mathbf{p}$ is aligned in the opposite direction of $\mathbf{E}$, potential energy is maximum



Dipole in a uniform field

Electric Flux

- **Introduction to Electric Flux**
- Electric field lines help in describing basic nature of the electric field in two-dimensional pictorial forms. What if we want to present electric field in a three-dimensional form? This can be done with the help of electric flux. Let's understand what the term flux actually means, with an example.



Electric Flux is defined as a number of electric field lines, passing per unit area. It is another physical quantity to measure the strength of electric field and frame the basics of electrostatics. It is also defined as the product of electric field and surface area projected in a direction perpendicular to the electric field.

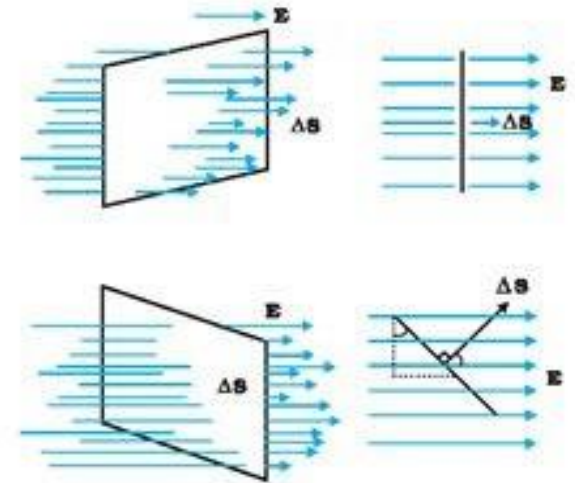
Electric Flux

- Since the strength of electric field is directly proportional to a number of lines passing per unit area, electric flux also signifies for determining the strength of electric field. The S.I unit of electric flux is Newton meters squared per **coulomb** ($\text{N m}^2 \text{C}^{-1}$). Electric field is denoted by the symbol Φ (phi) and is represented as
- $\Phi = E \Delta S$

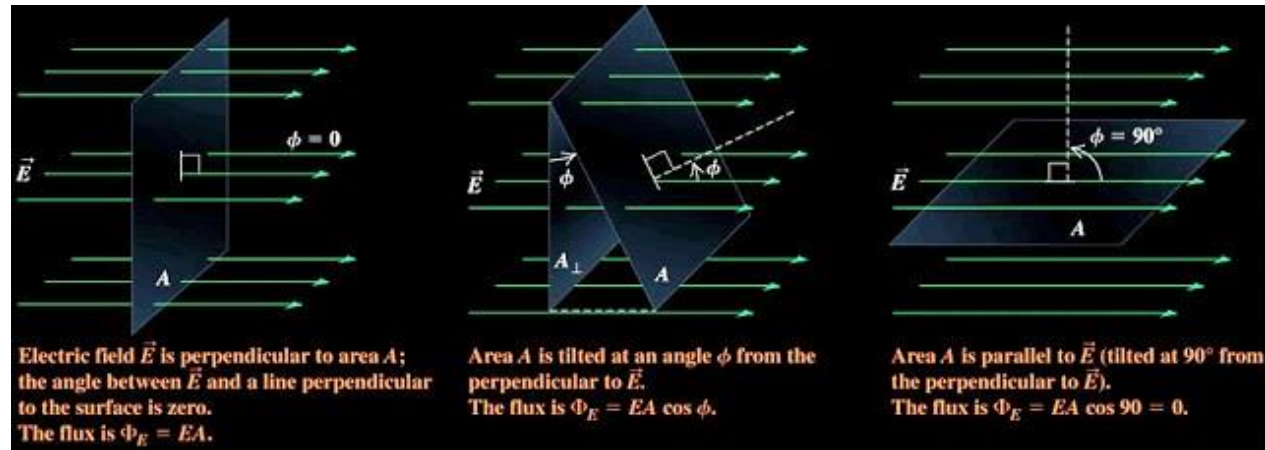
Electric flux varies with different orientation in electric field and area vector.

Consider an area with electric field E , in it. If we place, a planar object of surface area ΔS , then a number of electric field lines passing per unit area, will be proportional to $E \Delta S$. What if we accordingly tilt the planar object with angle θ ? Then electric flux will vary and depend on the relation:

$$\Phi = E \Delta S \cos \theta$$



Observations in Electric Flux

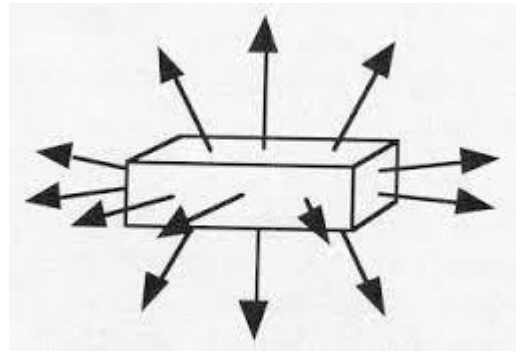


Electric Flux is a variable quantity as it depends on the direction of electric field and orientation of the planar object. Suppose you're asked to pass out water through a ring, on which position do you expect the maximum flow of water? This will be when the area of the ring would be perpendicular to the flow of water. The same thing holds for electric flux as well. Some observations related to electric flux are as follows:

- Electric flux becomes zero, when electric field is parallel to the surface area ΔS , as the angle becomes 90° and value of $\cos 90^\circ$ is zero
- The direction of ΔS is determined by area vector, as it has both magnitude and direction. Area vector's direction is always normal and out of the plane surface
- Electric flux becomes negative when electric field and area vector is antiparallel
- The direction of area vector is always out of the surface
- In case of curved surface, we evaluate ΔS by dividing a large surface area into small segments and integrate with proper limits

Applications of Electric Flux

- Electric Flux forms the basic foundation of electrostatics and has many applications like:
- Electric Flux helps in calculation of electric field
- Electric Flux simplifies evaluation of electric field in complex figures
- Gauss Theorem, one of the most useful theorems of electrostatics, is based mainly on electrostatics



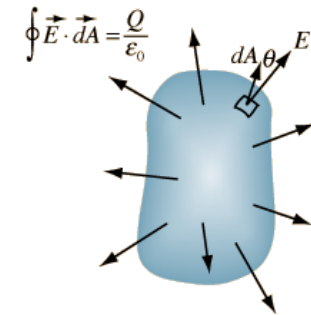
Direction of normal vector

Gauss's Law

- The **Gauss Law**, also known as Gauss theorem is a relation between an electric field with the distribution of charge in the system. The law was proposed by **Joseph- Louis Lagrange** in 1773 and later followed and formulated by **Carl Friedrich Gauss** in 1813.
- Gauss's Law states that the net electric flux is equal to $1/\epsilon_0$ times the charge enclosed in it. Mathematically Gauss Law is expressed as

$$\phi_E = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

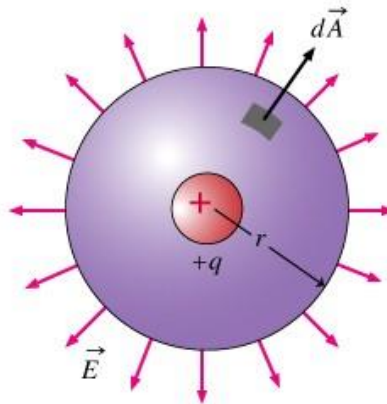
where Φ is the net electric flux and q is the charge enclosed in it.
 E is the electric field flowing through small area element ds . This form is used when we calculate electric flux in continuous charge distribution case. The surface, through which we calculate electric flux is called **Gaussian Surface**.



Gauss's Law

Proof of Gauss's Law

Consider a sphere of enclosed charge q inside it, placed at its center. The radius of the sphere is ' r '. To prove Gauss's Law, we need to find the net electric flux through the sphere. This can't be done directly as the surface area is non-uniform in nature, otherwise we could have multiplied electric field with a surface area of the sphere. So, to evaluate electric field we will divide the sphere into small surface elements.



Dividing a sphere into small surface area element dA

Proof of Gauss's Law

Now, each small surface area will feel same electric field due to charge q placed inside the sphere as each one is at equivalent distance ' r ' from charge q .

Adding further, these small surface elements will create small electric flux as

$$\phi_E = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

$$\phi_E = \oint \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} ds$$

$$\phi_E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} \oint ds$$

$$\phi_E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2} 4\pi r^2$$

$$\phi_E = \frac{q}{\epsilon_0}$$

Therefore electric flux due to enclosed charge is independent of the location of charge and the shape of the surface.

Significance of Gauss's Law

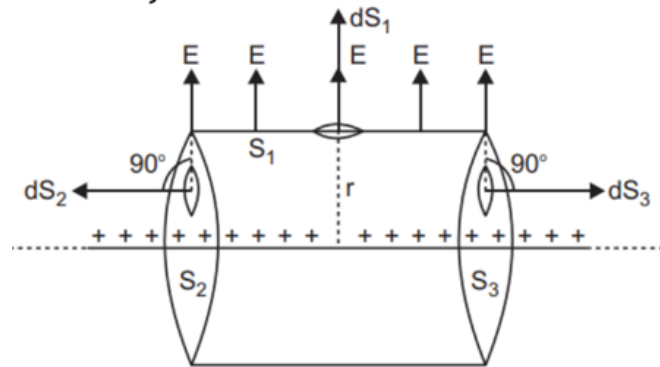
The most significant application of Gauss's Law is that it is not only limited to simple cases but can be used in a general form. We consider following points while applying Gauss's Law:

- Gauss's Law is always valid for any closed surface, irrespective of its shape that is, regular or irregular
- We consider only the sum of charges which are present inside the closed surface
- The inside charge and outside charge of the closed surface are responsible for electric field, but the electric flux will be due to inside charge only

Applications of Gauss's Law

- Electric Field due to Infinite Wire

Consider an infinitely long wire with linear charge density λ and length L . To calculate electric field, we assume a cylindrical Gaussian surface due to the symmetry of wire. As the electric field E is radial in direction; flux through the end of the cylindrical surface will be zero, as electric field and area vector are perpendicular to each other. The only flowing electric flux will be through the curved Gaussian surface. As the electric field is perpendicular to every point of the curved surface, its magnitude will be constant.



We consider a cylindrical Gaussian surface of radius r and length l

Electric Field due to Infinite Wire

$$\phi_E = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

$$\oint_{S_1} \vec{E} \cdot \vec{ds} + \oint_{S_2} \vec{E} \cdot \vec{ds} + \oint_{S_3} \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

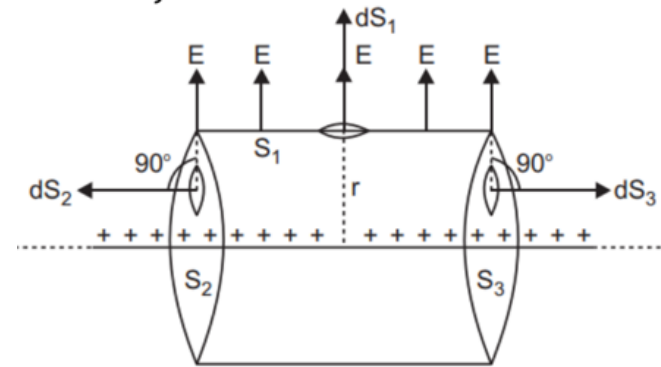
$$\oint_{S_1} E \, ds \cos 0^\circ + \oint_{S_2} E \, ds \cos 90^\circ + \oint_{S_3} E \, ds \cos 90^\circ = \frac{q}{\epsilon_0}$$

$$\oint_{S_1} E \, ds + 0 + 0 = \frac{q}{\epsilon_0}$$

$$E \cdot 2\pi r l = \frac{q}{\epsilon_0}$$

$$E = \frac{q}{2\pi r l \epsilon_0}$$

$$E = \frac{\lambda}{2\pi r \epsilon_0} \quad \text{here } \lambda = \frac{q}{l}$$



Coulomb's law using Gauss Theorem

Let us draw a spherical Gaussian surface of radius R around a charge and assume that charge is at the centre of the Gaussian surface.

According to Gauss law

$$\oint \vec{E} \cdot d\vec{s} = \frac{q}{\epsilon_0}$$

$$\oint E ds \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E 4\pi r^2 = \frac{q}{\epsilon_0}$$

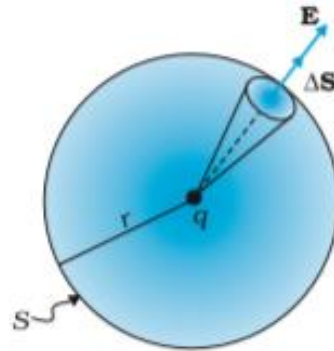
$$E = \frac{q}{4\pi r^2 \epsilon_0}$$

$$\text{therefore } E = \frac{1}{4\pi\epsilon_0} \frac{q}{r^2}$$

If a charge q_0 is available on the surface

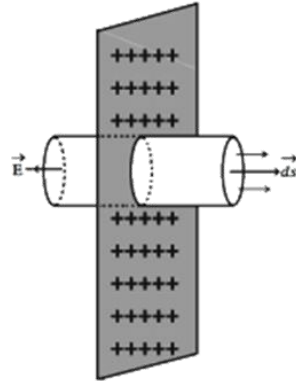
$$F = q_0 E$$

$$F = \frac{1}{4\pi\epsilon_0} \frac{qq_0}{r^2} \text{ which is coulomb's law}$$



Electric Field due to Infinite Thin Sheet

Imagine an infinite plane sheet, with surface charge density σ and cross-sectional area A. The position of the infinite plane sheet is given in the figure below:



The direction of the electric field due to infinite charge sheet will be perpendicular to the plane of the sheet. Let's consider cylindrical Gaussian surface, whose axis is normal to the plane of the sheet. The electric field E can be evaluated from Gauss's Law as

$$\oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

From continuous charge distribution charge q will be σA . Talking about net electric flux, we will consider electric flux only from the two ends of the assumed Gaussian surface. This is because the curved surface area and an electric field are normal to each other, thereby producing zero electric flux. So the net electric flux will be

$$\Phi = EA - (-EA)$$

$$\Phi = 2EA$$

Electric Field due to Infinite Thin Sheet

$$\phi_E = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

$$\oint E \, ds \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E = \frac{\sigma}{2 \epsilon_0} \text{ here } \sigma = \frac{dq}{ds} = \text{surface charge density}$$

$$\text{OR } \vec{E} = \frac{\sigma}{2 \epsilon_0} \hat{n}$$

Electric Field due thin Spherical Shell

Electric Field Outside the Spherical Shell

To find electric field outside the spherical shell, we take a point P outside the shell at a distance r from the center of the spherical shell. By symmetry, we take Gaussian spherical surface with radius r and center O. The Gaussian surface will pass through P, and experience a constant electric field all around as all points are equally distanced “ r ” from the center of the sphere. Then, According to Gauss’s Law

i. For a point outside the shell $r > R$

$$\phi_E = \oint \vec{E} \cdot \vec{ds} = \frac{q}{\epsilon_0}$$

$$\oint E \, ds \cos 0^\circ = \frac{q}{\epsilon_0}$$

$$E \times 4 \pi r^2 = \frac{\sigma}{\epsilon_0} 4 \pi R^2$$

$$E = \frac{\sigma R^2}{\epsilon_0 r^2}$$

ii. For a point on the shell ($r = R$)

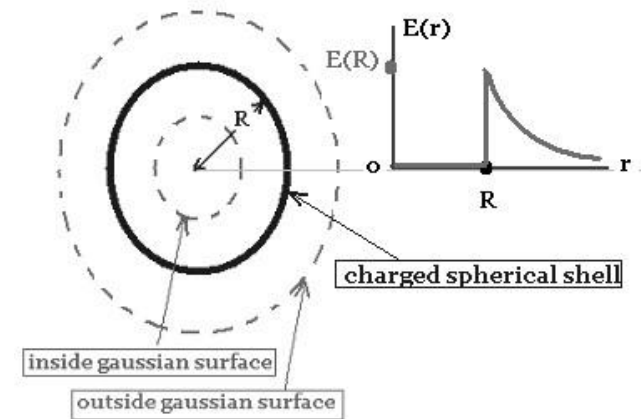
$$E = \sigma R^2 / \epsilon_0 r^2$$

$$E = \sigma R^2 / \epsilon_0 R^2$$

$$E = \sigma / \epsilon_0$$

iii. For a Point inside the shell $r < R$

$$q = 0 \text{ and } E = 0$$



Motion of a Charge Particle in Electric Field

When a charged particle move in uniform electric field it experiences a force

$$F = qE$$

$$ma = qE$$

$$a = qE/m$$

The distance travelled by the charged particle is given by

$$s = ut + \frac{1}{2}at^2$$

$$u = 0$$

$$v = l/t$$

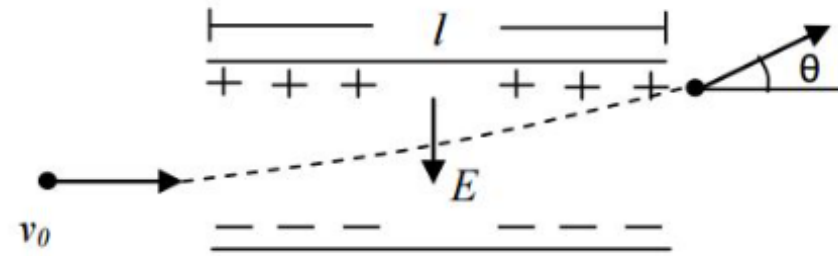
$$t = l/v$$

Put value of u and t in second motion equation

$$s = ut + \frac{1}{2}at^2$$

$$y = 0 + \frac{1}{2} * \frac{qE}{m} * \left(\frac{l}{v}\right)^2$$

$$y = \frac{qEl^2}{2mv^2}$$



Concept Map-Electrostatics

