

Work

When a force acts on an object and the object actually moves in the direction of force, then the work is said to be done by the force.

Work done by the force is equal to the product of the force and the displacement of the object in the direction of force.

If under a constant force F the object displaced through a distance s , then work done by the force

$$W = F * s = F s \cos \theta$$

where θ is the smaller angle between F and s .

Work is a scalar quantity, Its SI unit is joule and CGS unit is erg.

$$\therefore 1 \text{ joule} = 10^7 \text{ erg}$$

Its dimensional formula is $[ML^2T^{-2}]$.

Work done by a force is zero, if

(a) body is not displaced actually, i.e., $s = 0$

(b) body is displaced perpendicular to the direction of force, i.e.,

$$\theta = 90^\circ$$

Work done by a force is **positive** if angle between F and s is acute angle.

Work done by a force is **negative** if angle between F and s is obtuse angle.

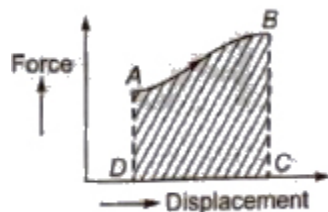
Work done by a constant force depends only on the initial and final Positions and not on the actual path followed between initial and final positions.

Work done in different conditions

(i) Work done by a variable force is given by

$$W = \int F * ds$$

It is equal to the area under the force-displacement graph along with proper sign.



$$\text{Work done} = \text{Area ABCDA}$$

(ii) Work done in displacing any body under the action of a number of forces is equal to the work done by the resultant force.

(iii) In equilibrium (static or dynamic), the resultant force is zero therefore resultant work done is zero.

(iv) If work done by a force during a round trip of a system is zero, then the force is conservative, otherwise it is called non-conservative force.

- Gravitational force, electrostatic force, magnetic force, etc are conservative forces. All the central forces are conservative forces.
- Frictional force, viscous force, etc are non-conservative forces.

(v) Work done by the force of gravity on a particle of mass m is given by $W = mgh$

where g is acceleration due to gravity and h is height through particle one displaced.

(vi) Work done in compressing or stretching a spring is given by

$$W = \frac{1}{2} kx^2$$

where k is spring constant and x is displacement from mean position.

(vii) When one end of a spring is attached to a fixed vertical support and a block attached to the free end moves on a horizontal

table from $x = x_1$ to $x = x_2$ then $W = \frac{1}{2} k (x_2^2 - x_1^2)$

(viii) Work done by the couple for an angular displacement θ is given by $W = \tau \cdot \theta$

where τ is the torque of the couple.

power

The time rate of work done by a body is called its power.

$$\text{Power} = \text{Rate of doing work} = \text{Work done} / \text{Time taken}$$

If under a constant force F a body is displaced through a distance s in time t , the power

$$P = W / t = F \cdot s / t$$

But $s / t = v$; uniform velocity with which body is displaced.

$$\therefore P = F \cdot v = F v \cos \theta$$

where θ is the smaller angle between F and v .

power is a scalar quantity. Its SI unit is watt and its dimensional formula is $[ML^2T^{-3}]$.

Its other units are kilowatt and horse power,

1 kilowatt = 1000 watt

1 horse power = 746 watt

Energy

Energy of a body is its capacity of doing work.

It is a scalar quantity.

Its SI unit is joule and CGS unit is erg. Its dimensional formula is $[ML^2T^{-2}]$.

There are several types of energies, such as mechanical energy (kinetic energy and potential energy), chemical energy, light energy, heat energy, sound energy, nuclear energy, electric energy etc.

Mechanical Energy

The sum of kinetic and potential energies at any point remains constant throughout the motion. It does not depend upon time. This is known as law of conservation of mechanical energy.

Mechanical energy is of two types:

1. Kinetic Energy

The energy possessed by any object by virtue of its motion is called its kinetic energy.

Kinetic energy of an object is given by

$$K = \frac{1}{2} mv^2 = \frac{p^2}{2m}$$

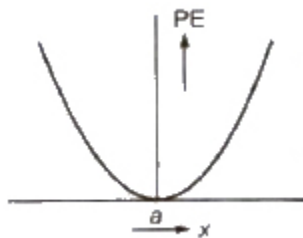
where m = mass of the object, U = velocity of the object and $p = mv$ = momentum of the object.

2. Potential Energy

The energy possessed by any object by virtue of its position or configuration is called its potential energy.

There are three important types of potential energies:

(i) **Gravitational Potential Energy** If a body of mass m is raised through a height h against gravity, then its gravitational potential energy = mgh ,



(ii) **Elastic Potential Energy** If a spring of spring constant k is stretched through a distance x , then elastic potential energy of the spring = $\frac{1}{2} kx^2$

The variation of potential energy with distance is shown in figure.

Potential energy is defined only for conservative forces. It does not exist for non-conservative forces.

Potential energy depends upon frame of reference.

(iii) **Electric Potential Energy** The electric potential energy of two point charges q_1 and q_2 separated by a distance r in vacuum is given by

$$U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$$

Here $\frac{1}{4\pi\epsilon_0} = 9.0 \times 10^{10} \text{ N-m}^2 / \text{C}^2$ constant.

Work-Energy Theorem

Work done by a force in displacing a body is equal to change in its kinetic energy.

$$W = \int_{v_1}^{v_2} F \cdot ds = \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2 = K_f - K_i = \Delta KE$$

where, K_i = initial kinetic energy

and K_f = final kinetic energy.

Regarding the work-energy theorem it is worth noting that

(i) If W_{net} is positive, then $K_f - K_i$ = positive, i.e., $K_f > K_i$ or kinetic energy will increase and vice-versa.

(ii) This theorem can be applied to non-inertial frames also. In a non-inertial frame it can be written as:

Work done by all the forces (including the Pseudo force) = change in kinetic energy in non-inertial frame.

Mass-Energy Equivalence

According to Einstein, the mass can be transformed into energy and vice – versa.

When Δm . mass disappears, then produced energy

$$E = \Delta mc^2$$

where c is the speed of light in vacuum.

Principle of Conservation of Energy

The sum of all kinds of energies in an isolated system remains constant at all times.

Principle of Conservation of Mechanical Energy

For conservative forces the sum of kinetic and potential energies of any object remains constant throughout the motion.

According to the quantum physics, mass and energy are not conserved separately but are conserved as a single entity called 'mass-energy'.

Collisions

Collision between two or more particles is the interaction for a short interval of time in which they apply relatively strong forces on each other.

In a collision physical contact of two bodies is not necessary. There are two types of collisions:

1. Elastic collision

The collision in which both the momentum and the kinetic energy of the system remains conserved are called elastic collisions.

In an elastic collision all the involved forces are conservative forces.

Total energy remains conserved.

2. Inelastic collision

The collision in which only the momentum remains conserved but kinetic energy does not remain conserved are called inelastic collisions.

In an inelastic collision some or all the involved forces are non-conservative forces.

Total energy of the system remains conserved.

If after the collision two bodies stick to each other, then the collision is said to be perfectly inelastic.

Coefficient of Restitution or Resilience

The ratio of relative velocity of separation after collision to the velocity of approach before collision is called coefficient of restitution resilience.

It is represented by e and it depends upon the material of the colliding bodies.

For a perfectly elastic collision, $e = 1$

For a perfectly inelastic collision, $e = 0$

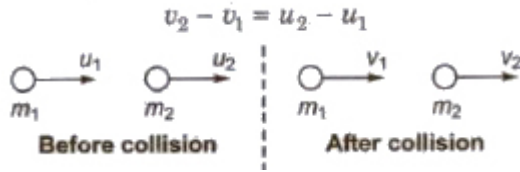
For all other collisions, $0 < e < 1$

One Dimensional or Head-on Collision

If the initial and final velocities of colliding bodies lie along the same line, then the collision is called one dimensional or head-on collision.

Inelastic One Dimensional Collision

Applying Newton's experimental law, we have



Velocities after collision

$$v_1 = (m_1 - m_2) u_1 + 2m_2 u_2 / (m_1 + m_2)$$

$$\text{and } v_2 = (m_2 - m_1) u_2 + 2m_1 u_1 / (m_1 + m_2)$$

When masses of two colliding bodies are equal, then after the collision, the bodies exchange their velocities.

$$v_1 = u_2 \text{ and } v_2 = u_1$$

If second body of same mass ($m_1 = m_2$) is at rest, then after collision first body comes to rest and second body starts moving with the initial velocity of first body.

$$v_1 = 0 \text{ and } v_2 = u_1$$

If a light body of mass m_1 collides with a very heavy body of mass m_2 at rest, then after collision.

$$v_1 = -u_1 \text{ and } v_2 = 0$$

It means light body will rebound with its own velocity and heavy body will continue to be at rest.

If a very heavy body of mass m_1 collides with a light body of mass m_2 ($m_1 \gg m_2$) at rest, then after collision

$$v_1 = u_1 \text{ and } v_2 = 2u_1$$

In Inelastic One Dimensional Collision

Loss of kinetic energy

$$\Delta E = m_1 m_2 / 2(m_1 + m_2) (u_1 - u_2)^2 (1 - e^2)$$

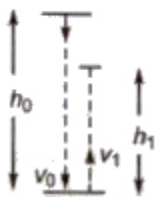
In Perfectly Inelastic One Dimensional Collision

Velocity of separation after collision = 0.

$$\text{Loss of kinetic energy} = m_1 m_2 (u_1 - u_2)^2 / 2(m_1 + m_2)$$

If a body is dropped from a height h_0 and it strikes the ground with velocity v_0 and after inelastic collision it rebounds with velocity v_1 and rises to a height h_1 , then

$$e = \frac{v_1}{v_0} = \sqrt{\frac{2gh_1}{2gh_0}} = \sqrt{\frac{h_1}{h_0}}$$



If after n collisions with the ground, the body rebounds with a velocity v_n and rises to a height h_n then

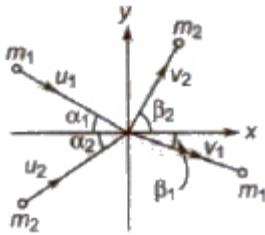
$$e^n = v_n / v_o = \sqrt{h_n / h_o}$$

Two Dimensional or Oblique Collision

If the initial and final velocities of colliding bodies do not lie along the same line, then the collision is called two dimensional or oblique Collision.

In horizontal direction,

$$m_1 u_1 \cos \alpha_1 + m_2 u_2 \cos \alpha_2 = m_1 v_1 \cos \beta_1 + m_2 v_2 \cos \beta_2$$



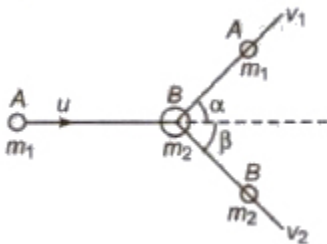
In vertical direction.

$$m_1 u_1 \sin \alpha_1 - m_2 u_2 \sin \alpha_2 = m_1 v_1 \sin \beta_1 - m_2 v_2 \sin \beta_2$$

If $m_1 = m_2$ and $\alpha_1 + \alpha_2 = 90^\circ$

then $\beta_1 + \beta_2 = 90^\circ$

If a particle A of mass m_1 moving along z-axis with a speed u makes an elastic collision with another stationary body B of mass m_2



From conservation law of momentum

$$m_1 u = m_1 v_1 \cos \alpha + m_2 v_2 \cos \beta$$

$$0 = m_1 v_1 \sin \alpha - m_2 v_2 \sin \beta$$