

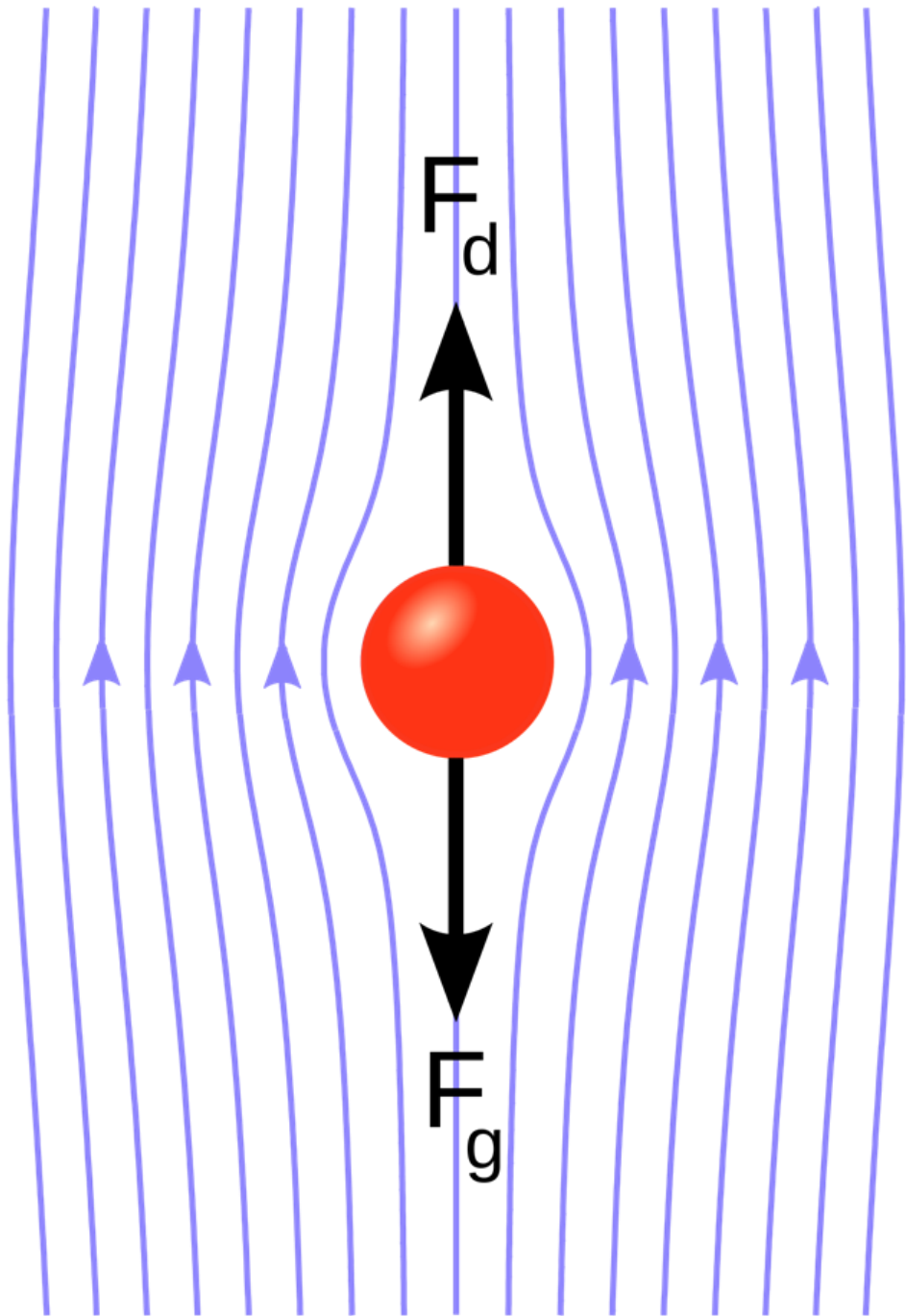
FLUID DYNAMICS

OBJECTIVES

TO LEARN ABOUT

- 1. **Stoke's law**
- 2. **Terminal velocity**
- 3. **Poiseuille's equation**
- 4. **Streamline flow and Turbulent flow**
- 5. **Critical velocity**

STOKE'S LAW



$$F_{drag} = -6\pi a \eta v$$

where a = sphere radius
 η = viscosity
 v = particle velocity

STOKE'S LAW

- It states that the viscous drag(F) of a spherical body of radius(r) moving with a terminal velocity(v) in a fluid of viscosity(η) is given by:

$$F=6\pi\eta rv$$

- This law can be derived dimensionally,

$$F \propto r^a v^b \eta^c$$
$$F = k r^a v^b \eta^c$$

Write dimensionally,

$$[MLT^{-2}] = [L]^a [LT^{-1}]^b [ML^{-1}T^{-1}]^c$$

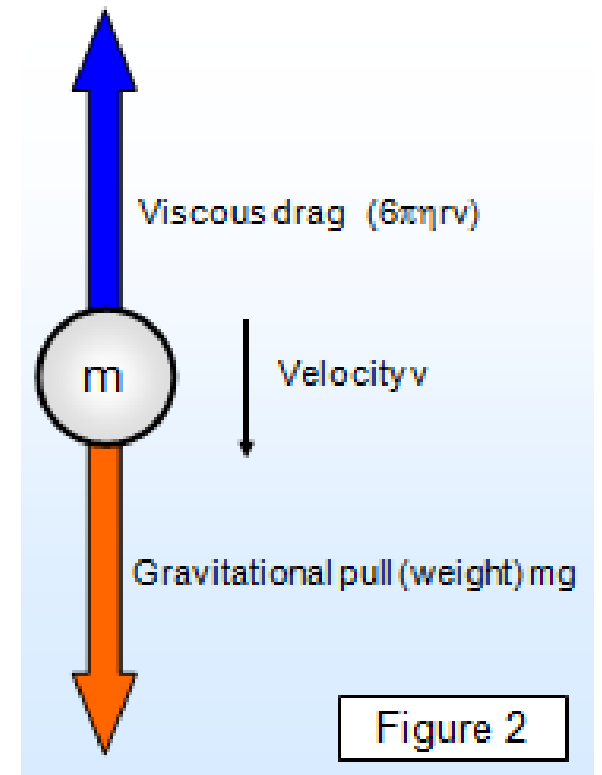
We can solve that $a=1$; $b=1$ and $c=1$

Therefore , $F=k rv\eta$

In SI units, $K=6\pi$,

So,

$$F=6\pi\eta rv$$



The object is
described as having
attained terminal
velocity



Force due to Air
Resistance

Force due to Gravity

TERMINAL VELOCITY

- When a body is dropped in a viscous fluid, initially it accelerates, and later the acceleration becomes zero. The velocity attained by the body with zero acceleration is called terminal velocity.
- Consider a spherical body of radius r falling through a viscous fluid of density σ and coefficient of viscosity η .

Various forces acting are:

1.Weight – W

2.Up thrust - U

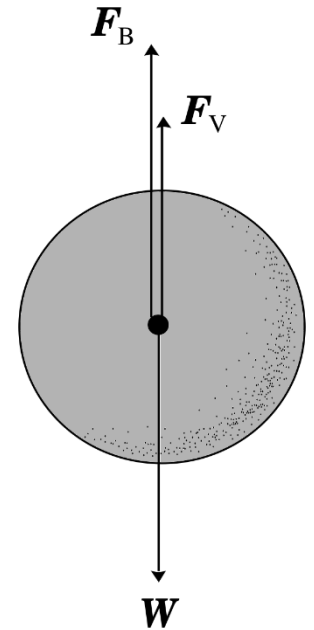
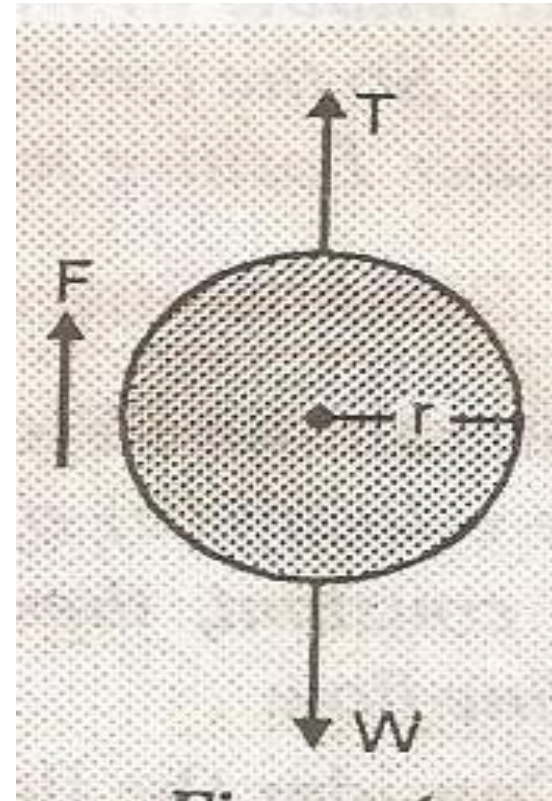
3.Viscous force - F

Net force = $W - F - U$

When the body attains terminal velocity,

$$W - F - U = 0$$

$$F = W - U$$



We know that,

Viscous Force ----- $F = 6\pi\eta r v$

Weight of the body ----- $W = mg$

$$= V\rho g$$

$$W = \frac{4}{3}\pi r^3 \rho g$$

Up thrust----- $U = \frac{4}{3}\pi r^3 \sigma g$

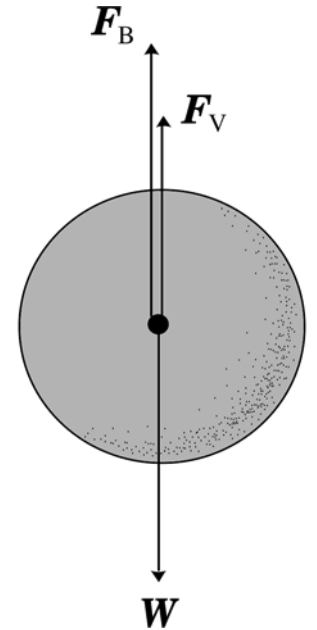
Hence

$$F = U + W$$

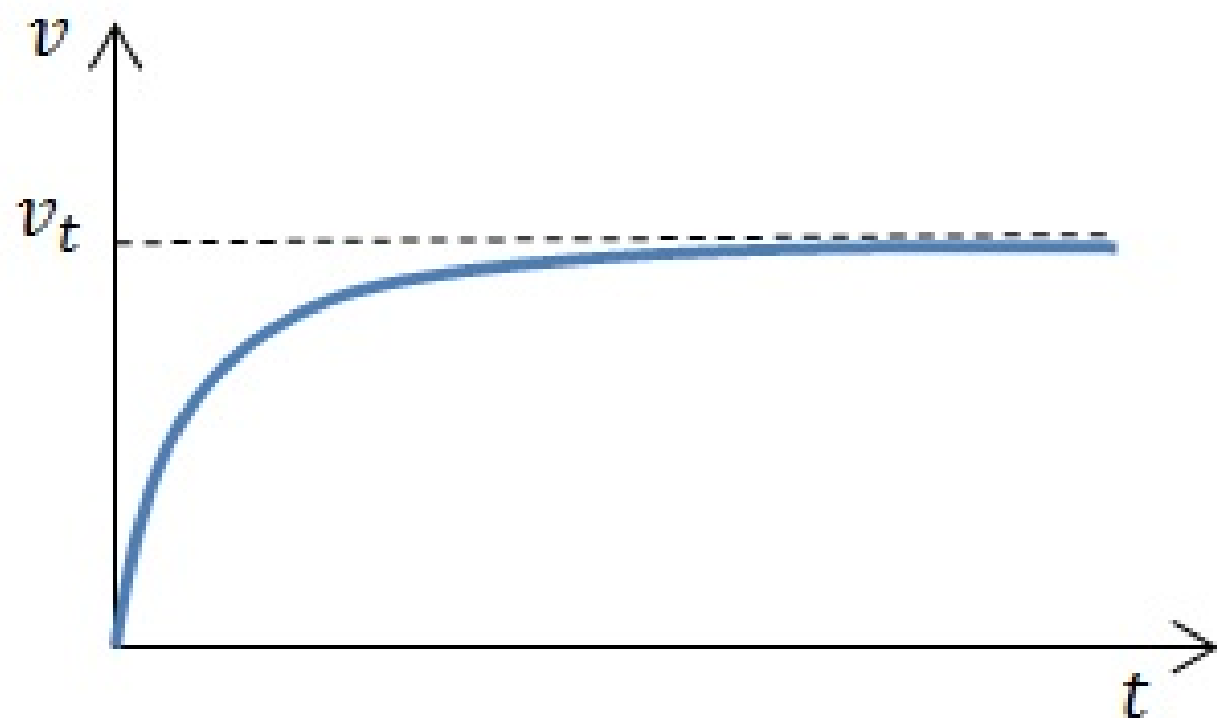
$$6\pi\eta r v = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

On simplifying

$$v = \frac{2}{9} r^2 \frac{(\rho - \sigma)g}{\eta}$$



$$v_t = \frac{2(\rho_s - \rho_l)g}{9\eta} r^2$$



POISEUILLE'S EQUATION

- If the pressure difference P across the two end of capillary tube of length l and radius r then volume of water coming out of tube,

$$V \propto P$$

$$V \propto r^4$$

$$V \propto \frac{1}{\eta}$$

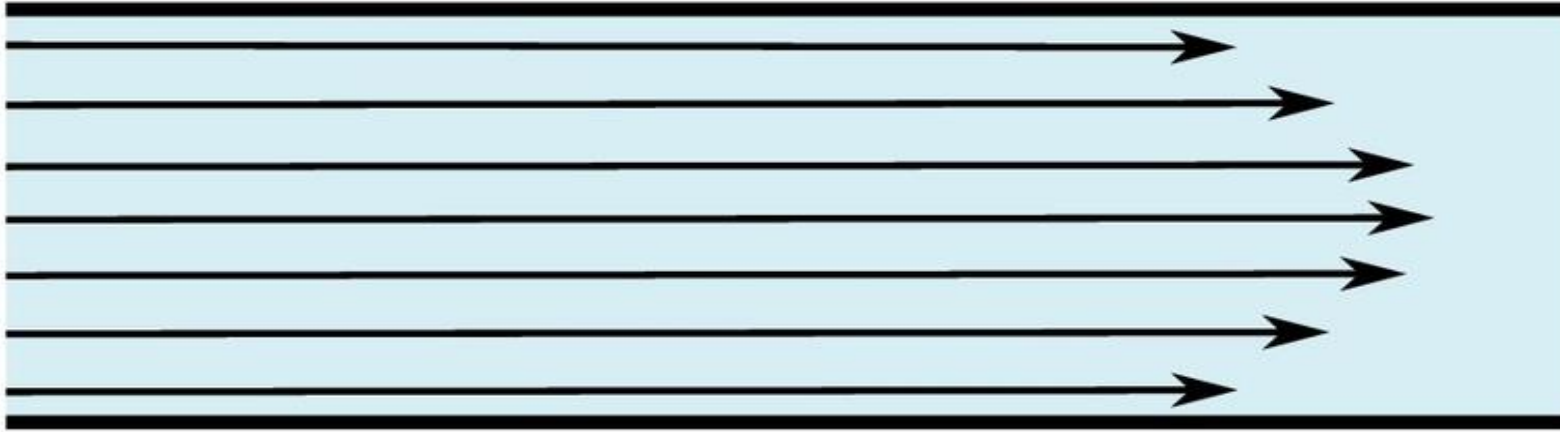
$$V \propto \frac{1}{l}$$

- Therefore----- $Q = V/t \propto P \frac{r^4}{\eta l}$

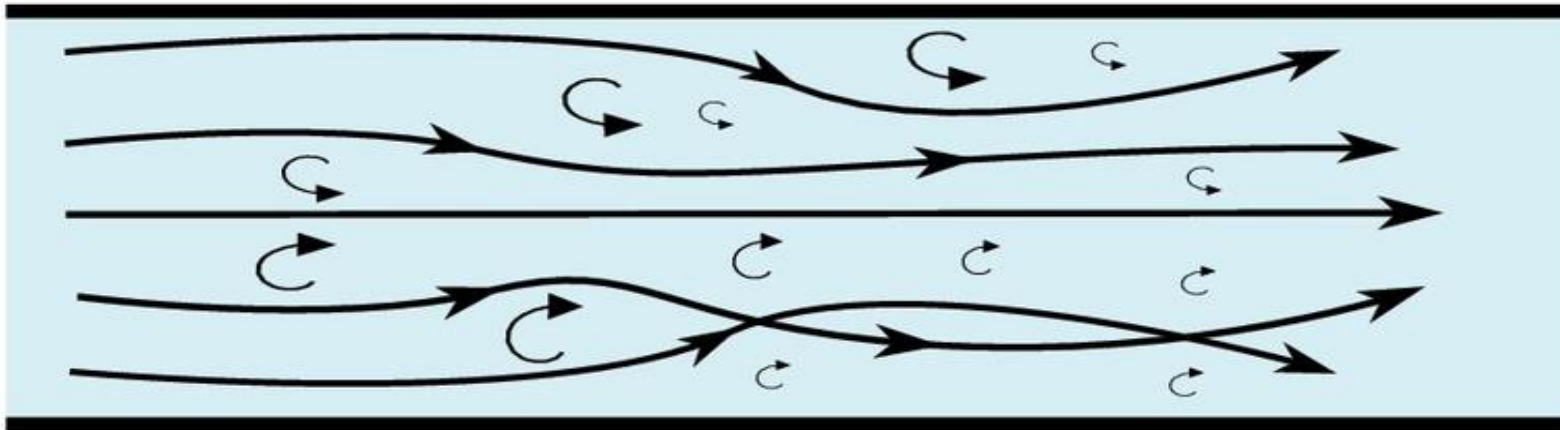
$$Q = k P \frac{r^4}{\eta l} \text{ (where } k = \pi/8 \text{)}$$

- $$Q = \frac{\pi P r^4}{8 \eta l}$$

laminar flow



turbulent flow



STREAMLINE FLOW

- If the velocity of every particle at a particular point at the time of crossing is constant then the flow is said to be steady or streamlined flow.
- A tangent at any point on the streamlined flow is in the direction of the velocity at that point.
- No two streamlines can intersect each other.

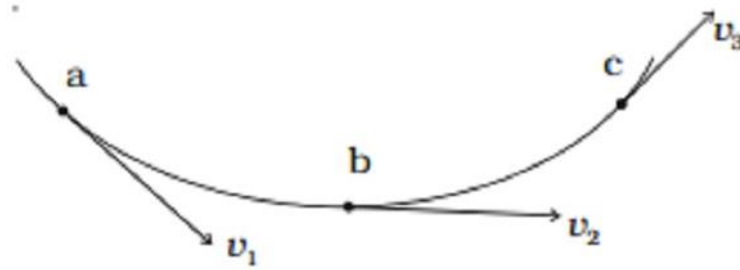


Fig. Streamline flow

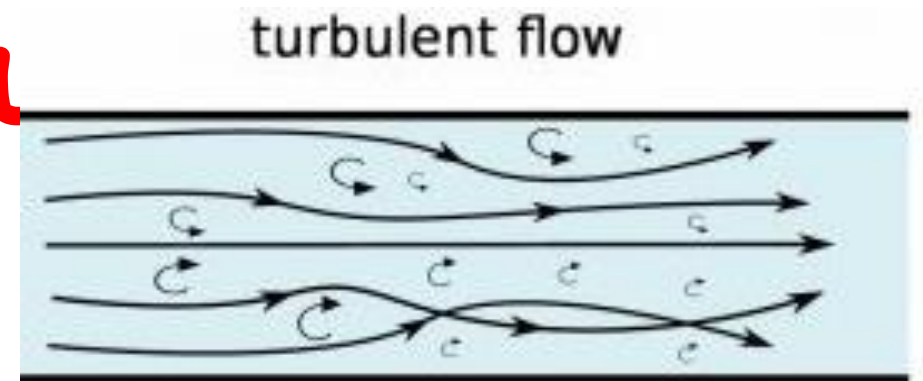
LAMINAR FLOW

- If the fluid flows over a horizontal surface in the form of layers of different velocities, then the flow is called laminar flow. It is also a steady flow.



TURBULENT FLOW

- The flow of fluid in which velocity of all particles crossing a given point is not same and motion of the fluid becomes disorderly or irregular is called turbulent flow.

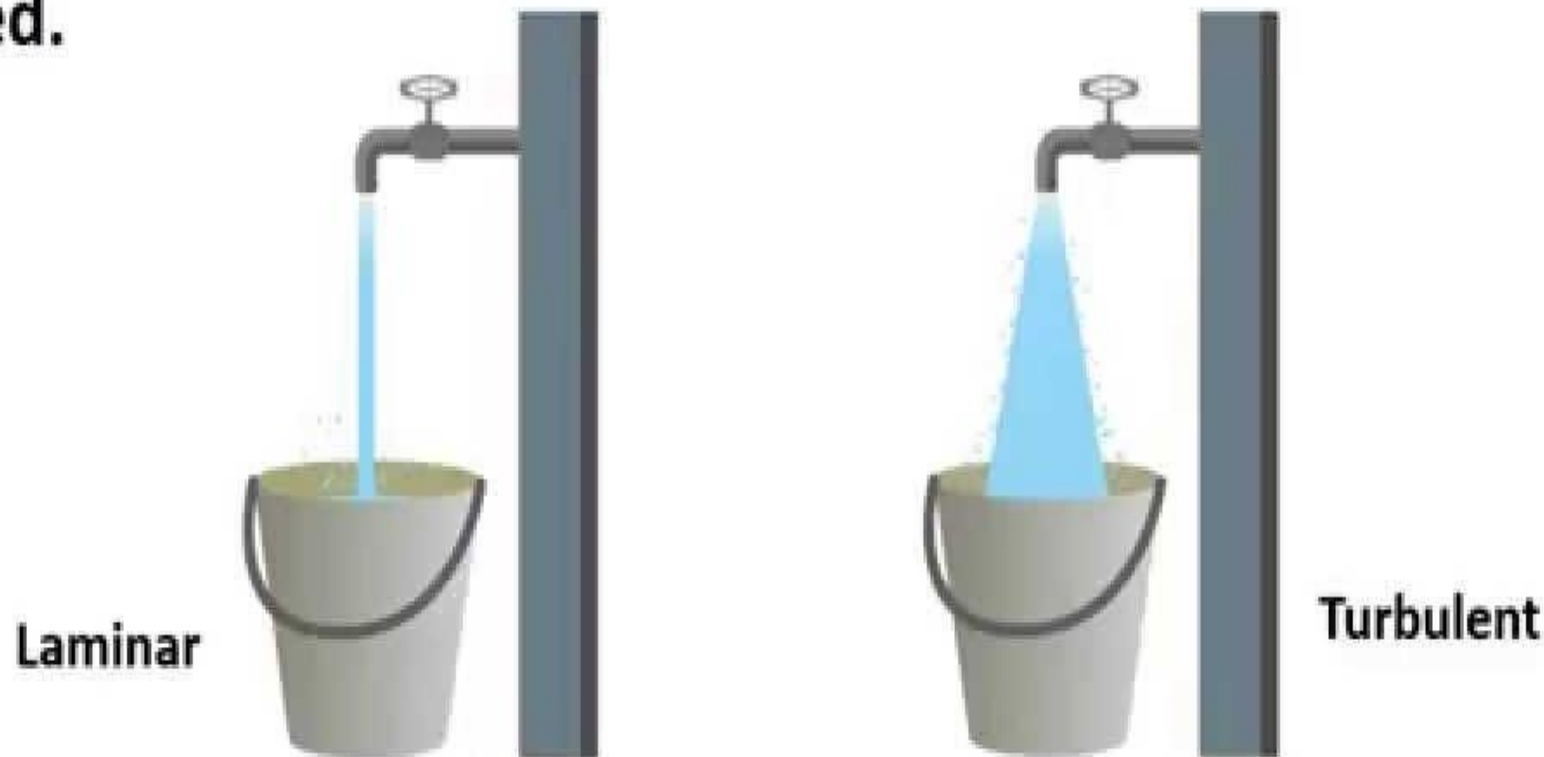


CRITICAL VELOCITY

- **The maximum velocity of the fluid up to which the flow is streamline flow is called critical velocity.**

Turbulent Flow

When **water tap** in a house is **opened slowly**, the **flow of water** remains **laminar initially** and becomes **turbulent** as **tap is fully opened**.



Laminar Flow

Turbulent Flow



REYNOLD'S NUMBER:

- According to Reynold , the critical velocity v_c of a liquid of density ρ flowing through a long narrow tube of diameter D .

$$v_c \propto \eta$$

$$v_c \propto \frac{1}{\rho}$$

$$v_c \propto \frac{1}{D};$$

$$v_c = R \frac{\eta}{\rho D}$$

where R is Reynold's number

- ❖ If $R > 2000$ it is turbulent flow
- ❖ If $R < 1000$ it is streamline flow
- ❖ Between 1000 and 2000 it is unstable flow.