

# **Class 10**

## **Mathematics**

### **Lesson 2: Polynomial**

**Topic : Zeroes and long division of Polynomial  
(Explanation Resource)**

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# Content

- Relationship between zeroes and coefficients of a quadratic polynomial.
- Relationship between zeroes and coefficients of a cubic polynomial.
- Examples.
- Division algorithm for polynomial.
- Examples of long division of polynomial.
- Practice Questions.

# Relationship between zeroes and coefficients of a quadratic polynomial.

If  $\alpha, \beta$  are zeros of a quadratic polynomial  $f(x) = ax^2 + bx + c$ , then

$$\alpha + \beta = \frac{-b}{a} = -\frac{\text{coefficient of } x}{\text{coefficient of } x^2}$$

$$\alpha\beta = \frac{c}{a} = \frac{\text{constant terms}}{\text{coefficient of } x^2}$$

Where  $\alpha$  and  $\beta$  are the zeroes of the quadratic polynomial.

$a$  is the coefficient of  $X^2$ .

$b$  is the coefficient of  $x$ .

$c$  is the constant.

# Relationship between zeroes and coefficients of a cubic polynomial.

If  $\alpha, \beta, \gamma$  are zeros of a cubic polynomial  $f(x) = ax^3 + bx^2 + cx + d$ , then

$$\alpha + \beta + \gamma = \frac{-b}{a} = -\frac{\text{coefficient of } x^2}{\text{coefficient of } x^3} \qquad \alpha\beta\gamma = \frac{-d}{a} = -\frac{\text{constant terms}}{\text{coefficient of } x^3}$$

$$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = \frac{\text{coefficient of } x}{\text{coefficient of } x^3}$$

Where  $\alpha, \beta$  and  $\gamma$  are the zeroes of the cubic polynomial.

$a$  is the coefficient of  $X^3$ .

$b$  is the coefficient of  $X^2$ .

$c$  is the coefficient of  $x$ .

$d$  is the constant.

# Example 1

We have,  $p(x) = 6x^2 - 3 - 7x$

$$\begin{aligned}\Rightarrow p(x) &= 6x^2 - 7x - 3 && \text{(In general form)} \\ &= 6x^2 - 9x + 2x - 3 = 3x(2x - 3) + 1(2x - 3) \\ &= (2x - 3)(3x + 1)\end{aligned}$$

The zeros of polynomial  $p(x)$  is given by

$$p(x) = 0 \quad \Rightarrow \quad (2x - 3)(3x + 1) = 0 \quad \Rightarrow \quad x = \frac{3}{2}, -\frac{1}{3}$$

Thus, the zeros of  $6x^2 - 7x - 3$  are  $\alpha = \frac{3}{2}$  and  $\beta = -\frac{1}{3}$

$$\text{Now, sum of the zeros} = \alpha + \beta = \frac{3}{2} - \frac{1}{3} = \frac{9 - 2}{6} = \frac{7}{6}$$

$$\text{and} \quad \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2} = \frac{-(-7)}{6} = \frac{7}{6}$$

$$\text{Therefore, sum of the zeros} = \frac{-(\text{Coefficient of } x)}{\text{Coefficient of } x^2}$$

$$\text{Again, product of zeros} = \alpha \cdot \beta = \frac{3}{2} \times \left(-\frac{1}{3}\right) = -\frac{1}{2}$$

$$\text{and} \quad \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{-3}{6} = -\frac{1}{2}$$

$$\text{Therefore, product of zeros} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

## Example 2

Let  $f(x) = 4x^2 - 3x - 1$

$$= 4x^2 - 4x + x - 1 \quad \text{[by splitting the middle term]}$$
$$= 4x(x - 1) + 1(x - 1)$$
$$= (x - 1)(4x + 1)$$

So, the value of  $4x^2 - 3x - 1$  is zero when  $x - 1 = 0$  or  $4x + 1 = 0$  i.e., when  $x = 1$  or  $x = -\frac{1}{4}$ .

So, the zeroes of  $4x^2 - 3x - 1$  are 1 and  $-\frac{1}{4}$ .

$$\therefore \text{Sum of zeroes} = 1 - \frac{1}{4} = \frac{3}{4} = \frac{-(-3)}{4}$$
$$= (-1) \left( \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2} \right)$$

and product of zeroes  $= (1) \left( -\frac{1}{4} \right) = -\frac{1}{4}$

$$= (-1)^2 \cdot \left( \frac{\text{Constant term}}{\text{Coefficient of } x^2} \right)$$

Hence, verified the relations between the zeroes and the coefficients of the polynomial.

## Example 3

$$\begin{aligned}\text{Let } p(u) &= 4u^2 + 8u \\ &= 4u(u+2)\end{aligned}$$

Equating both the factors with zero, we get

$$4u = 0 \quad \text{and} \quad u+2 = 0$$

$$\therefore u = 0 \quad \text{and} \quad u = -2$$

$\therefore$  Zeroes of  $p(u)$  are 0 and  $-2$

Here  $a = 4$ ,  $b = 8$  and  $c = 0$

$$\text{Sum of the zeroes} = 0 + (-2) = -2$$

$$\frac{-b}{a} = \frac{-8}{4} = -2$$

$$\Rightarrow \text{Sum of the zeroes} = \frac{-b}{a}$$

$$\text{Product of the zeroes} = 0 \times (-2) = 0$$

$$\frac{c}{a} = \frac{0}{4} = 0$$

$$\Rightarrow \text{Product of the zeroes} = \frac{c}{a}$$

# Example 4

$$p(t) = t^2 - 15$$

$$= t^2 + 0 - 15$$

$$= 1t^2 + 0(t) - 15$$

It is of the form  $at^2 + bt + c$

So,  $a = 1$ ,  $b = 0$ ,  $c = -15$

We have to verify

$$\text{Sum of zeroes} = - \frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$$

$$\text{i.e. } \alpha + \beta = - \frac{b}{a}$$

| L.H.S                     | R.H.S            |
|---------------------------|------------------|
| $\alpha + \beta$          | $-\frac{b}{a}$   |
| $= \sqrt{15} - \sqrt{15}$ | $= -\frac{0}{1}$ |
| $= 0$                     | $= 0$            |

Since, L.H.S = R.H.S

Hence relationship between zeroes & coefficient is verified

$$\text{Product of zeroes} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$$

$$\text{i.e. } \alpha \times \beta = \frac{c}{a}$$

| L.H.S                        | R.H.S             |
|------------------------------|-------------------|
| $\alpha \beta$               | $\frac{c}{a}$     |
| $= (\sqrt{15}) (-\sqrt{15})$ | $= \frac{-15}{1}$ |
| $= -(\sqrt{15})^2$           | $= -15$           |
| $= -15$                      |                   |

## Example 5

Hence,  $\frac{1}{2}$ , 1 and -2 are the zeroes of  $2x^3 + x^2 - 5x + 2$ . Hence **verified**.

Now we take  $\alpha = \frac{1}{2}$ ,  $\beta = 1$  and  $\gamma = -2$ .

$$\begin{aligned}\therefore \alpha + \beta + \gamma &= \frac{1}{2} + 1 + (-2) \\ &= \frac{3}{2} - 2 = \frac{-1}{2} = \frac{-b}{a}\end{aligned}$$

$$\begin{aligned}\alpha\beta + \beta\gamma + \gamma\alpha &= \frac{1}{2} \times 1 + 1 \times (-2) + (-2) \times \frac{1}{2} \\ &= \frac{1}{2} - 2 - 1 = \frac{-5}{2} = \frac{c}{a}\end{aligned}$$

$$\alpha\beta\gamma = \frac{1}{2} \times 1 \times (-2) = \frac{-2}{2} = -\frac{d}{a}.$$

Hence **verified**.

# Division algorithm for Polynomials

- If  $p(x)$  and  $g(x)$  are any two polynomials with  $g(x) \neq 0$ , then we can always find polynomials  $q(x)$  and  $r(x)$  such that,

$$P(x) = q(x) \cdot g(x) + r(x)$$

where  $r(x) \neq 0$  or  $\text{degree } r(x) < \text{degree } g(x)$

# Example 1

$$\begin{array}{r}
 3x^2 + 4x + 5 \\
 x+4 \overline{) 3x^3 + 16x^2 + 21x + 20} \\
 \underline{3x^3 + 12x^2} \phantom{+ 21x + 20} \\
 4x^2 + 21x + 20 \\
 \underline{4x^2 + 16x} \phantom{+ 20} \\
 5x + 20 \\
 \underline{5x + 20} \\
 0
 \end{array}$$

First term of  $q(x) = \frac{3x^3}{x} = 3x^2$

Second term of  $q(x) = \frac{4x^2}{x} = 4x$

Third term of  $q(x) = \frac{5x}{x} = 5$

## Example 2

$$\begin{array}{r} x^2 + 2x + 1 \\ 3x^2 - 5 \overline{) 3x^4 + 6x^3 - 2x^2 - 10x - 5} \\ \underline{3x^4 \phantom{+ 6x^3} - 5x^2} \phantom{- 10x - 5} \\ 6x^3 + 3x^2 - 10x - 5 \\ \underline{6x^3 \phantom{+ 3x^2} - 10x} \phantom{- 5} \\ 3x^2 - 5 \\ \underline{3x^2 - 5} \\ 0 \end{array}$$

# Example 3

By long division

$$\begin{array}{r} 3x^3 - x^2 - x - 4 \\ x - 1 \overline{) 3x^4 - 4x^3 - 3x - 1} \\ \underline{3x^4 - 3x^3} \phantom{- 3x - 1} \\ (-) \quad (+) \phantom{- 3x - 1} \\ \phantom{3x^4 - } -x^3 - 3x \phantom{- 1} \\ \phantom{3x^4 - } \underline{-x^3 + x^2} \phantom{- 1} \\ (+) \quad (-) \phantom{- 1} \\ \phantom{3x^4 - } \phantom{-x^3 - } -x^2 - 3x - 1 \\ \phantom{3x^4 - } \phantom{-x^3 - } \underline{-x^2 + x} \phantom{- 1} \\ (+) \quad (-) \phantom{- 1} \\ \phantom{3x^4 - } \phantom{-x^3 - } \phantom{-x^2 - } -4x - 1 \\ \phantom{3x^4 - } \phantom{-x^3 - } \phantom{-x^2 - } \underline{-4x + 4} \\ (+) \quad (-) \\ \phantom{3x^4 - } \phantom{-x^3 - } \phantom{-x^2 - } \phantom{-4x - } -5 \end{array}$$

# Practice Questions

- Find the zeroes of the following quadratic polynomials and verify the relationship between the zeroes and the coefficients.
  - 1)  $X^2+5x+6$
  - 2)  $2x^2-7x+3$
- Divide the polynomial  $p(x)$  by  $g(x)$  and find the quotient and remainder.
  - 1)  $p(x)= 6x^3+13x^2+x-2$  and  $g(x)=2x+1$
  - 2)  $p(x)= X^4+x^3+x^2-2x-3$  and  $g(x)= x^2-2$

