

SUBJECT: MATHEMATICS
TOPIC: POLYNOMIALS
CLASS: X

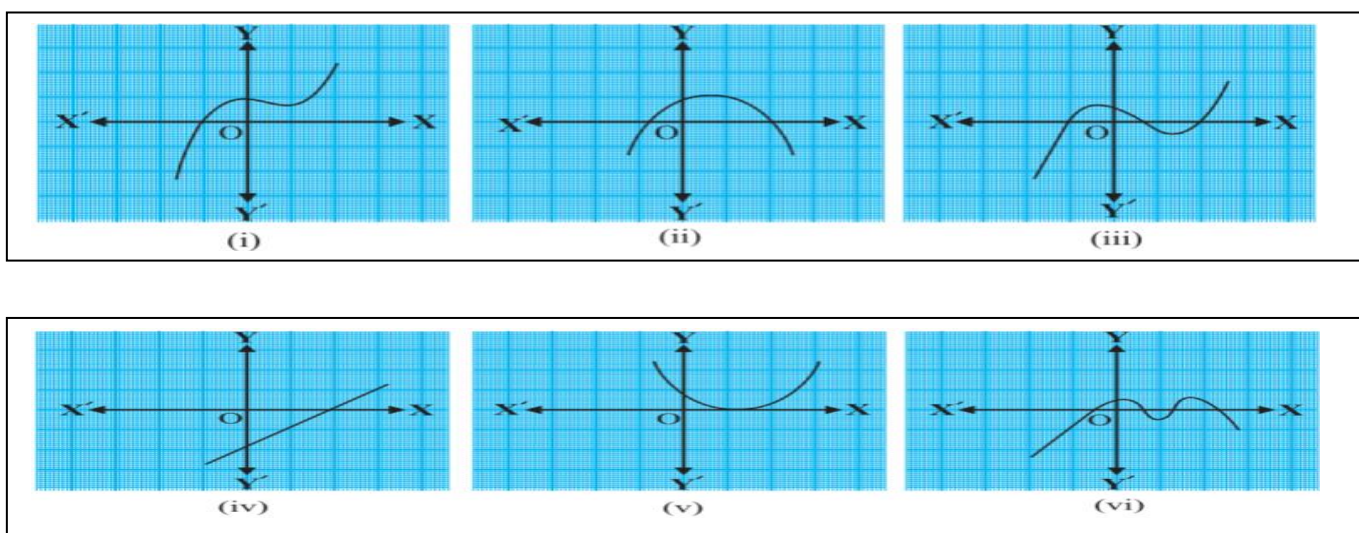
INTRODUCTION

A polynomial of degree 1 is called a linear polynomial. For example, $2x - 3$. A polynomial of degree 2 is called a quadratic polynomial. Eg $3x^2 - 2x - 10$. The word 'quadrate', which means 'square'. A polynomial of degree 3 is called a cubic polynomial. eg $4x^3 + 3x^2 - 5x - 1$. More generally, a real number k is said to be a zero of a polynomial $p(x)$, if $p(k) = 0$. Thus, the zero of a linear polynomial is related to its coefficients.

TYPE I

Geometrical Meaning of the Zeroes of a Polynomial: *The zeroes of a polynomial $p(x)$ are precisely the x -coordinates of the points, where the graph of $y = p(x)$ intersects the x -axis.*

Q1. Find the number of zeroes in the following graphs:



Ans(i) One

Ans(ii) Two

Ans(iii) Three

Ans(iv) One

Ans(v) One

Ans(vi) Four

TYPE II

Relationship between Zeroes and Coefficients of a Quadratic Polynomial:

Working Rule:

i) Find the zeroes of the given polynomial using splitting the middle term.

ii) Find a, b and c of the polynomial.

iii) Use the formula,

$$\text{Sum of the zeroes} = \square + \square = \square = -\frac{b}{a}$$

$$\text{Product of zeroes} = \square \times \square = \square = \frac{c}{a}$$

Hence verified.

Q Find the zeroes of the following quadratic polynomial, and verify the relationship between the zeroes and the coefficients:

1) $3x^2 - 10x - 8$

2) $x^2 + 7x + 10$

3) $x^2 - 3$

4) $4s^2 - 4s + 1$

5) $6x^2 - 3 - 7x$

6) $4u^2 + 8u$

7) $\sqrt{3}x^2 + 10x + 7\sqrt{3}$

8) $x^2 - 6\sqrt{2}x + 10$

1) Solution:

STEP I: $3x^2 - 10x - 8$ [multiply 3&8]

$$= 3x^2 - 12x + 2x - 8 \quad [\text{factors of 24 such that its sum is -10}]$$

$$= 3x(x-4) + 2(x-4)$$

$$= (3x+2)(x-4)$$

$$\therefore x = -\frac{2}{3} \text{ and } x = 4. \quad \left(-\frac{2}{3} \text{ and } 4 \text{ are the required zeroes of the given polynomial}\right)$$

STEP II: Here $a=3$, $b=-10$, $c=-8$

$$\text{Sum of the zeroes} = -\frac{2}{3} + 4 = \frac{10}{3} = -\frac{-10}{3} = -\frac{b}{a}$$

$$\text{Product of zeroes} = -\frac{2}{3} \times 4 = -\frac{8}{3} = \frac{-8}{3} = \frac{c}{a}$$

Hence verified.

2) Solution:

STEP I: $x^2 + 7x + 10$ [multiply 1&10]

$$= x^2 + 5x + 2x + 10 \quad [\text{factors of 10 such that its sum is 7}]$$

$$= x(x+5) + 2(x+5)$$

$$= (x+2)(x+5)$$

$$\therefore x = -2 \text{ and } x = -5. \quad (-2 \text{ and } -5 \text{ are the required zeroes of the given polynomial})$$

STEP II: Here $a=1$, $b=7$, $c=10$

$$\text{Sum of the zeroes} = -2 + (-5) = -7 = -\frac{7}{1} = -\frac{b}{a}$$

$$\text{Product of zeroes} = -2 \times -5 = 10 = \frac{10}{1} = \frac{c}{a}$$

Hence verified.

$$3)\text{Solution: } x^2 - 3$$

$$\text{STEP I: } x^2 - 3 \quad [\text{cannot use splitting middle term method}]$$

$$= x^2 - (\sqrt{3})^2 \quad [\text{arranging in the form } a^2 - b^2]$$

$$= (x + \sqrt{3})(x - \sqrt{3}) \quad [a^2 - b^2 = (a + b)(a - b)]$$

$$\therefore x = -\sqrt{3} \text{ and } x = \sqrt{3}. \quad (-\sqrt{3} \text{ and } \sqrt{3} \text{ are the required zeroes of the given polynomial})$$

$$\text{STEP II: Here } a=1, b=0, c=3$$

$$\text{Sum of the zeroes} = -\sqrt{3} + \sqrt{3} = 0 = -\frac{0}{1} = -\frac{b}{a}$$

$$\text{Product of zeroes} = -\sqrt{3} \times \sqrt{3} = 3 = \frac{3}{1} = \frac{c}{a}$$

Hence verified.

$$4)\text{Solution:}$$

$$\text{STEP I: } 4s^2 - 4s + 1 \quad [\text{multiply 4\&1}]$$

$$= 4s^2 - 2s - 2s + 1 \quad [\text{factors of 4 such that its sum is -4}]$$

$$= 2s(2s-1) - 1(2s-1)$$

$$= (2s-1)(2s-1)$$

$$\therefore s = \frac{1}{2} \text{ and } s = \frac{1}{2}. \quad (\frac{1}{2} \text{ and } \frac{1}{2} \text{ are the required zeroes of the given polynomial})$$

$$\text{STEP II: Here } a=4, b=-4, c=1$$

$$\text{Sum of the zeroes} = \frac{1}{2} + \frac{1}{2} = 1 = -\frac{-4}{4} = -\frac{b}{a}$$

$$\text{Product of zeroes} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \frac{1}{4} = \frac{c}{a}$$

Hence verified.

$$5)\text{Solution: } 6x^2 - 3 - 7x$$

$$\text{STEP I: } 6x^2 - 7x - 3 \quad [\text{multiply 6\&3}]$$

$$= 6x^2 - 9x + 2x - 3 \quad [\text{factors of 18 such that its sum is -7}]$$

$$= 3x(2x-3) + 1(2x-3)$$

$$=(3x+1)(2x-3)$$

$\therefore x = -\frac{1}{3}$ and $x = \frac{3}{2}$. ($-\frac{1}{3}$ and $\frac{3}{2}$ are the required zeroes of the given polynomial)

STEP II: Here $a=6$, $b= -7$, $c= -3$

$$\text{Sum of the zeroes} = -\frac{1}{3} + \frac{3}{2} = \frac{7}{6} = -\frac{-7}{6} = -\frac{b}{a}$$

$$\text{Product of zeroes} = -\frac{1}{3} \times \frac{3}{2} = -\frac{1}{2} = \frac{-3}{6} = \frac{c}{a}$$

Hence verified.

6)Solution:

STEP I: $4u^2 + 8u$ [cannot use splitting middle term method]

$$= 4u(u+2) \quad [\text{take out common factors}]$$

$\therefore u = 0$ and $u = -2$. (0 and -2 are the required zeroes of the given polynomial)

STEP II: Here $a=4$, $b= 8$, $c= 0$

$$\text{Sum of the zeroes} = 0 + (-2) = -2 = -\frac{8}{4} = -\frac{b}{a}$$

$$\text{Product of zeroes} = 0 \times -2 = 0 = \frac{0}{4} = \frac{c}{a}$$

Hence verified.

7)Solution:

STEP I: $\sqrt{3}x^2 + 10x + 7\sqrt{3}$ [multiply $\sqrt{3}$ & $7\sqrt{3}$]

$$= \sqrt{3}x^2 + 7x + 3x + 7\sqrt{3} \quad [\text{factors of } 21 \text{ such that its sum is } 10]$$

$$= x(\sqrt{3}x+7) + \sqrt{3}(\sqrt{3}x+7)$$

$$= (\sqrt{3}x+7)(x + \sqrt{3})$$

$\therefore x = -\frac{7}{\sqrt{3}}$ and $x = -\sqrt{3}$. ($-\frac{7}{\sqrt{3}}$ and $-\sqrt{3}$ are the required zeroes of the given polynomial)

STEP II: Here $a=\sqrt{3}$, $b= 10$, $c= 7\sqrt{3}$

$$\text{Sum of the zeroes} = -\frac{7}{\sqrt{3}} + -\sqrt{3} = -\frac{10}{\sqrt{3}} = -\frac{10}{\sqrt{3}} = -\frac{b}{a}$$

$$\text{Product of zeroes} = -\frac{7}{\sqrt{3}} \times -\sqrt{3} = 7 = \frac{7\sqrt{3}}{\sqrt{3}} = \frac{c}{a}$$

Hence verified.

8)Solution:

STEP I: $x^2 - 6\sqrt{2}x + 10$

[multiply 1 & 10]

$$= x^2 - 5\sqrt{2}x - \sqrt{2}x + 10$$

[factors of 10 such that its sum is $-6\sqrt{2}$]

$$= x(x - 5\sqrt{2}) - \sqrt{2}(x - 5\sqrt{2})$$

$$= (x - 5\sqrt{2})(x - \sqrt{2})$$

$\therefore x = 5\sqrt{2}$ and $x = \sqrt{2}$. ($5\sqrt{2}$ and $\sqrt{2}$ are the required zeroes of the given polynomial)

STEP II: Here $a=1$, $b= -6\sqrt{2}$, $c= 10$

$$\text{Sum of the zeroes} = 5\sqrt{2} + \sqrt{2} = 6\sqrt{2} = -\frac{-6\sqrt{2}}{1} = -\frac{b}{a}$$

$$\text{Product of zeroes} = 5\sqrt{2} \times \sqrt{2} = 10 = \frac{10}{1} = \frac{c}{a}$$

Hence verified.

TYPE III

Any Quadratic Polynomial is given by, $x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}$

Q Find a quadratic polynomial each with the given numbers as the sum and product of its zeroes respectively.

1) $1/4, -1$ 2) $7, 10$ 3) $\sqrt{2}, 1/3$ 4) $0, \sqrt{5}$ 5) $-2, -10$

****Dear students, remember that the first number represents SUM OF THE ZEROES while the second number represents PRODUCT OF THE ZEROES.**

1)Solution:

Any Quadratic Polynomial is given by, $x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}$

$$\therefore x^2 - \left(\frac{1}{4}\right)x + (-1)$$

$$\therefore x^2 - \frac{1}{4}x - 1$$

2)Solution:

Any Quadratic Polynomial is given by, $x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}$

$$\therefore x^2 - (7)x + (10)$$

$$\therefore x^2 - 7x + 10$$

3)Solution:

Any Quadratic Polynomial is given by, $x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}$

$$\therefore x^2 - (\sqrt{2})x + \left(\frac{1}{3}\right)$$

$$\therefore x^2 - \sqrt{2}x + \frac{1}{3}$$

4)Solution:

Any Quadratic Polynomial is given by, $x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}$

$$\therefore x^2 - (0)x + (\sqrt{5})$$

$$\therefore x^2 + \sqrt{5}$$

5)Solution:

Any Quadratic Polynomial is given by, $x^2 - (\text{sum of zeroes})x + \text{Product of zeroes}$

$$\therefore x^2 - (-2)x + (-10)$$

$$\therefore x^2 + 2x - 10$$

TYPE IV

Relationship between Zeroes and Coefficients of a Cubic Polynomial:

The general form of a cubic polynomial is ax^3+bx^2+cx+d

If α, β and γ are the zeroes of the cubic polynomial then,

$$\alpha + \beta + \gamma = -\frac{b}{a}$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$$

$$\alpha \beta \gamma = -\frac{d}{a}$$

Example 1 : Verify that 3, -1, $-\frac{1}{3}$ are the zeroes of the cubic polynomial $p(x) = 3x^3 - 5x^2 - 11x - 3$, and then verify the relationship between the zeroes and the coefficients.

Solution : Comparing the given polynomial with $ax^3 + bx^2 + cx + d$, we get

$a = 3, b = -5, c = -11, d = -3$. Further

$$p(3) = 3 \times 3^3 - (5 \times 3^2) - (11 \times 3) - 3 = 81 - 45 - 33 - 3 = 0,$$

$$p(-1) = 3 \times (-1)^3 - 5 \times (-1)^2 - 11 \times (-1) - 3 = -3 - 5 + 11 - 3 = 0,$$

$$p\left(-\frac{1}{3}\right) = 3 \times \left(-\frac{1}{3}\right)^3 - 5 \times \left(-\frac{1}{3}\right)^2 - 11 \times \left(-\frac{1}{3}\right) - 3,$$

$$= -\frac{1}{9} - \frac{5}{9} + \frac{11}{3} - 3 = -\frac{2}{3} + \frac{2}{3} = 0$$

Therefore, 3, -1 and $-\frac{1}{3}$ are the zeroes of $3x^3 - 5x^2 - 11x - 3$.

So, we take $\alpha = 3, \beta = -1$ and $\gamma = -\frac{1}{3}$.

Now,

$$\alpha + \beta + \gamma = 3 + (-1) + \left(-\frac{1}{3}\right) = 2 - \frac{1}{3} = \frac{5}{3} = \frac{-(-5)}{3} = \frac{-b}{a},$$

HW:

Q Verify that the numbers given alongside of the cubic polynomials below are their zeroes. Also verify the relationship between the zeroes and the coefficients in each case:

(i) $2x^3 + x^2 - 5x + 2$; $\frac{1}{2}, 1, -2$.

(ii) $x^3 - 4x^2 + 5x - 2$; $2, 1, 1$.

TYPE V

Division Algorithm for Polynomials:

If $p(x)$ and $g(x)$ are any two polynomials with $g(x) \neq 0$, then we can find polynomials $q(x)$ and $r(x)$ such that $p(x) = q(x) \times g(x) + r(x)$, where $r(x) = 0$ or degree of $r(x) <$ degree of $g(x)$.
This result is known as the Division Algorithm for polynomials.

Q1. Find all the zeroes of the polynomial $p(x) = 2x^4 - 3x^3 - 3x^2 + 6x - 2$, if two of the zeroes are $\sqrt{2}, -\sqrt{2}$.

Q2. Find all the zeroes of the polynomial $p(x) = x^4 - 3x^3 - x^2 + 9x - 6$, if two of the zeroes are $\sqrt{3}, -\sqrt{3}$.

Q3. Find all the zeroes of the polynomial $p(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$, if two of the zeroes are -2, -1.

Q4. Find all the zeroes of the polynomial $p(x) = 2x^4 - 2x^3 - 7x^2 + 3x + 6$, if two of the zeroes are $\sqrt{\frac{3}{2}}, -\sqrt{\frac{3}{2}}$.

Q5. Find all the zeroes of the polynomial $p(x) = 3x^4 + 6x^3 - 2x^2 - 10x - 5$, if two of the zeroes are $\sqrt{\frac{5}{3}}, -\sqrt{\frac{5}{3}}$.

TYPE VI

Division Algorithm for Polynomials:

In this type, we are going to come across questions pertaining to previous classes(VIII&IX).

Q1. What must be added to the polynomial $4x^4 + 2x^3 - 2x^2 + x - 1$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$.

Q2. What must be Subtracted from the polynomial $x^4 + 2x^3 - 13x^2 - 12x + 21$ so that the resulting polynomial is exactly divisible by $x^2 + 2x - 3$.

Q3. Divide the polynomial $p(x)$ by the polynomial $g(x)$ and find the quotient and remainder in each of the following :

(i) $p(x) = x^3 - 3x^2 + 5x - 3, \quad g(x) = x^2 - 2$

(ii) $p(x) = x^4 - 3x^2 + 4x + 5, \quad g(x) = x^2 + 1 - x$

(iii) $p(x) = x^4 - 5x + 6, \quad g(x) = 2 - x^2$

Q4. Check whether the first polynomial is a factor of the second polynomial by dividing the second polynomial by the first polynomial:

(i) $t^2 - 3, 2t^4 + 3t^3 - 2t^2 - 9t - 12$

(ii) $x^2 + 3x + 1, 3x^4 + 5x^3 - 7x^2 + 2x + 2$

(iii) $x^3 - 3x + 1, x^5 - 4x^3 + x^2 + 3x + 1$

Udertaking

I, Jagdish Gannavarapu mathematics teacher in Jain Heritage A Cambridge School, Nagpur hereby declare that the above teaching material(format) is purely drafted by me under the guidance of my Principal Mr.Praveen Sharma.