

SUBJECT: MATHEMATICS
TOPIC: REAL NUMBERS
CLASS: X

TYPE I

TYPE I: Problems on *Fundamental Theorem of Arithmetic*.

Working : (i) Find the factors using Prime Factorisation.

(ii) State the *Fundamental Theorem of Arithmetic*.

(iii) Write the missing factor(either 2 or 5).

(iv) So, there is no natural number n for which $__^n$ ends with the digit zero.

(Fundamental Theorem of Arithmetic):

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Examples:

Q1. Check whether there is any value of n for which 4^n ends with the digit zero.

Solution:

Step I: Prime factorisation of $4 = 2 \times 2$.

Step II: According to fundamental theorem of arithmetic,

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Step III: Prime factorisation of 4 does not contain 5 as one of the factors.

Step IV: So, there is no natural number n for which 4^n ends with the digit zero.

Q2. Check whether there is any value of n for which 6^n ends with the digit zero.

Solution:

Step I: Prime factorisation of $6 = 2 \times 3$.

Step II: According to fundamental theorem of arithmetic,

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Step III: Prime factorisation of 6 does not contain 5 as one of the factors.

Step IV: So, there is no natural number n for which 6^n ends with the digit zero.

Q3. Check whether there is any value of n for which 15^n ends with the digit zero.

Solution:

Step I: Prime factorisation of $15 = 3 \times 5$.

Step II: According to fundamental theorem of arithmetic,

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Step III: Prime factorisation of 15 does not contain 2 as one of the factors.

Step IV: So, there is no natural number n for which 15^n ends with the digit zero.

Q4. Check whether there is any value of n for which 21^n ends with the digit zero.

Solution:

Step I: Prime factorisation of $21 = 3 \times 7$.

Step II: According to fundamental theorem of arithmetic,

Every composite number can be expressed (factorised) as a product of primes, and this factorisation is unique, apart from the order in which the prime factors occur.

Step III: Prime factorisation of 4 does not contain 2 as well as 5 as its factors.

Step IV: So, there is no natural number n for which 21^n ends with the digit zero.

TYPE II

Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating repeating decimal expansion:

Working : (i) Find the factors using Prime Factorisation of the DENOMINATOR.

(ii) The prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates.

OR

The prime factorization of q is not of the form $2^n 5^m$, where n, m are non-negative integers.

Then, x has a decimal expansion which is non-terminating non-repeating.

(i) $\frac{13}{3125}$

(ii) $\frac{17}{8}$

(iii) $\frac{64}{455}$

(iv) $\frac{15}{1600}$

(v) $\frac{29}{343}$

(vi) $\frac{23}{2^3 5^2}$

(vii) $\frac{129}{2^2 5^7 7^5}$

(viii) $\frac{6}{15}$

(ix) $\frac{35}{50}$

(x) $\frac{77}{210}$

Q1) solution:

i) $3125 = 5 \times 5 \times 5 \times 5 \times 5 = 5^5 = 2^0 \times 5^5$

ii) The prime factorization of 3125 is of the form $2^n 5^m$, where n, m are non-negative integers

iii) Thus given rational number has a decimal expansion which terminates.

Qii) solution:

i) $8 = 2 \times 2 \times 2 = 2^3 = 2^3 \times 5^0$

ii) *The prime factorization of 8 is of the form $2^n 5^m$, where n, m are non-negative integers*

iii) Thus given rational number has a decimal expansion which terminates.

Qiii) solution:

i) $455 = 5 \times 7 \times 13 = 2^0 \times 5^0$

ii) *The prime factorization of 455 is not of the form $2^n 5^m$, where n, m are non-negative integers*

iii) Thus given rational number has a decimal expansion which is non-terminating non-repeating.

Qiv) solution:

i) $1600 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 = 2^6 \times 5^2$

ii) *The prime factorization of 1600 is of the form $2^n 5^m$, where n, m are non-negative integers*

iii) Thus given rational number has a decimal expansion which terminates.

Qv) solution:

i) $343 = 7 \times 7 \times 7 = 2^0 \times 5^0 \times 7^3$

ii) *The prime factorization of 343 is not of the form $2^n 5^m$, where n, m are non-negative integers*

iii) Thus given rational number has a decimal expansion which is non-terminating non-repeating.

Qvi) solution:

i) $2^3 \times 5^2$

ii) *The prime factorization of $2^3 \times 5^2$ is of the form $2^n 5^m$, where n, m are non-negative integers*

iii) Thus given rational number has a decimal expansion which terminates.

Qvii) solution:

i) $2^2 \times 5^7 \times 7^5$

ii) *The prime factorization of $2^2 \times 5^7 \times 7^5$*

is not of the form $2^n 5^m$, where n, m are non-negative integers

iii) Thus given rational number has a decimal expansion *which is non-terminating non-repeating*.

Qviii) solution:

i) $6/15$ can be **Reduced to** $2/5$

Now the denominator consists only $5 = 2^0 \times 5^1$

ii) *The prime factorization of 5 is of the form $2^n 5^m$, where n, m are non-negative integers*

iii) Thus given rational number has a decimal expansion which terminates.

Qix) solution:

i) $35/50$ can be **Reduced to** $7/10$

Now the denominator consists only $10 = 2^1 \times 5^1$

ii) *The prime factorization of 10 is of the form $2^n 5^m$, where n, m are non-negative integers*

iii) Thus given rational number has a decimal expansion which terminates.

Qviii) solution:

i) $77/210$ can be **Reduced to** $11/30$

Now the denominator consists only $30 = 2^1 \times 3^1 \times 5^1$

ii) *The prime factorization of 30 is not of the form $2^n 5^m$, where n, m are non-negative integers*

iii) Thus given rational number has a decimal expansion *which is non-terminating non-repeating*.

TYPE III:

Problems on Irrational numbers:

Q1. To prove $\sqrt{3}$ is an irrational number.

Solution:

Working : (i) Let $\sqrt{3}$ be a rational number.

$$(ii) \therefore \sqrt{3} = \frac{p}{q} \text{ (p, q are co-prime and } q \neq 0)$$

$$(iii) \therefore \sqrt{3}q = p.$$

$$(iv) \therefore (\sqrt{3}q)^2 = (p)^2.$$

$$(v) \therefore 3q^2 = p^2 \text{ _____ (1)}$$

Here, 3 is a factor of p^2 .

(vi) Again, let $p = 3r$ (r is some Integer)

$$(vii) (p)^2 = (3r)^2$$

$$(viii) p^2 = 9r^2$$

$$3q^2 = 9r^2 \text{ (using (1))}$$

$$\therefore p^2 = 3r^2$$

Here, 3 is also a factor of p^2 .

(ix) This contradicts our assumption.

(x) Thus, $\sqrt{3}$ is an irrational number.

Q2.To prove $\sqrt{2}$ is an irrational number.

Solution:

(i) Let $\sqrt{2}$ be a rational number.

$$(ii) \therefore \sqrt{2} = \frac{p}{q} \text{ (p, q are co-prime and } q \neq 0)$$

$$(iii) \therefore \sqrt{2}q = p.$$

$$(iv) \therefore (\sqrt{2}q)^2 = (p)^2.$$

$$(v) \therefore 2q^2 = p^2 \text{ (1)}$$

Here, 2 is a factor of p^2 .

(vi) Again, let $p = 2r$ (r is some Integer)

$$(vii) (p)^2 = (2r)^2$$

$$(viii) p^2 = 4r^2$$

$$(ix) 2m^2 = 4r^2 \text{ (using (1))}$$

$$(x) \therefore p^2 = 2r^2$$

Here, 2 is also a factor of p^2 .

(xi) This contradicts our assumption.

(xii) Thus, $\sqrt{2}$ is an irrational number.

Q3.To prove $2\sqrt{3}$ is an irrational number.

Solution:

(i) Let $\sqrt{3}$ be a rational number.

(ii) $\therefore \sqrt{3} = \frac{p}{q}$. (p, q are co-prime and $q \neq 0$)

(iii) $\therefore \sqrt{3}q = p$.

(iv) $\therefore (\sqrt{3}q)^2 = (p)^2$.

(v) $\therefore 3q^2 = p^2$ _____(1)

Here, 3 is a factor of p^2 .

(vi) Again, let $p = 3r$ (r is some Integer)

(vii) $(p)^2 = (3r)^2$

(viii) $p^2 = 9r^2$

(ix) $3q^2 = 9r^2$ (using (1))

(x) $\therefore p^2 = 3r^2$

Here, 3 is also a factor of p^2 .

(ix) This contradicts our assumption.

(x) Thus, $\sqrt{3}$ is an irrational number.

(xi) Product of a rational number and an irrational number is always an irrational number.

Q4. To prove $5\sqrt{3} + 2$ is an irrational number.

Solution:

(i) Let $5\sqrt{3} + 2$ be a rational number.

(ii) $\therefore 5\sqrt{3} + 2 = \frac{p}{q}$. (p, q are co-prime and $q \neq 0$)

(iii) $\therefore 5\sqrt{3} = \frac{p}{q} - 2$.

(iv) LHS = $5\sqrt{3}$ = irrational number

(v) RHS = $\frac{p}{q} - 2$ = rational number.

(vi) $\therefore$) LHS $\neq$ RHS

(vii) This contradicts our assumption.

(viii) Thus, $5\sqrt{3} + 2$ is an irrational number.

Q5. To prove $1/\sqrt{5}$ is an irrational number.

Solution: After rationalizing $1/\sqrt{5}$, we get $\sqrt{5}$

(i) Let $\sqrt{5}$ be a rational number.

(ii) $\therefore \sqrt{5} = \frac{p}{q}$. (p, q are co-prime and $q \neq 0$)

(iii) $\therefore \sqrt{5}q = p$.

(iv) $\therefore (\sqrt{5}q)^2 = (p)^2$.

(v) $\therefore 5q^2 = p^2$ _____(1)

Here, 5 is a factor of p^2 .

(vi) Again, let $p = 5r$ (r is some Integer)

(vii) $(p)^2 = (5r)^2$

(viii) $p^2 = 25r^2$

(ix) $5m^2 = 25r^2$ (using (1))

(x) $\therefore p^2 = 5r^2$

Here, 5 is also a factor of p^2 .

(xi) This contradicts our assumption.

(xii) Thus, $\sqrt{5}$ is an irrational number.

Q6. Prove that $\sqrt{3} + \sqrt{5}$ is irrational.

Solution:

(i) Let $\sqrt{3} + \sqrt{5}$ be a rational number.

(ii) $\therefore \sqrt{3} + \sqrt{5} = \frac{p}{q}$. (p, q are co-prime and $q \neq 0$)

(iii) LHS = $\sqrt{3} + \sqrt{5}$ = irrational number.

(iv) RHS = $\frac{p}{q}$ = rational number.

(v) $\therefore$) LHS $\neq$ RHS

(vi) This contradicts our assumption.

(vii) Thus, $\sqrt{3} + \sqrt{5}$ is an irrational number.

TYPE IV: Problems on Composite numbers

Composite numbers: It is a positive integer that is factorable into more than two.

Eg: $6=1 \times 2 \times 3$

Q1. Explain why $3 \times 5 \times 7 + 7$ is a composite number.

Q2. Explain why $7 \times 11 \times 13 + 13$ and $7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$ are composite numbers.

Note: Here, you need to simplify and check if it satisfies definition of composite number.

Q1. Solution:

$$\begin{aligned} &3 \times 5 \times 7 + 7 \\ &= 7(3 \times 5 \times 1 + 1) \\ &= 7(15 + 1) \\ &= 7 \times 16 \\ &= 7 \times 2^4 \times 1 \end{aligned}$$

Thus, it is a composite number.

Q2. Solution:

$$\begin{aligned} &7 \times 11 \times 13 + 13 \\ &= 13(7 \times 11 \times 1 + 1) \\ &= 13(77 + 1) \\ &= 13 \times 78 \\ &= 13 \times 13 \times 2 \times 3. \end{aligned}$$

Thus, it is a composite number. ($\because$ factors are 1, 2, 3 and 13)

$$\begin{aligned} &7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5 \\ &= 5(7 \times 6 \times 4 \times 3 \times 2 + 1) \\ &= 5(1008 + 1) \\ &= 5 \times 1009 \\ &= 5 \times 1009 \times 1 \end{aligned}$$

Thus, it is a composite number. ($\because$ factors are 1, 5 and 1009)

TYPE V: Problems on HCF and LCM

Euclid's division algorithm: $a = bq + r$.

Note: The stage at which $r = 0$, thus HCF so obtained is **b**.

Q1. Use Euclid's division algorithm to find the HCF of 441, 567, 693.

solution: To find HCF of 441, 567 and 693.

STEP I: Find HCF of 567 and 693.

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0$$

$$\text{HCF} = 63$$

STEP II: Find HCF of 63 and 441.

$$441 = 63 \times 7 + 0$$

$$\text{HCF} = 63$$

Thus, HCF of 441, 567 and 693 is 63.

Q2. Find the HCF and LCM of 6, 72 and 120, using the prime factorization method.

Solution:

STEP I: Find HCF of 72 and 120.

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0$$

$$\text{HCF} = 63$$

STEP II: Find HCF of 63 and 441.

$$441 = 63 \times 7 + 0$$

$$\text{HCF} = 63$$

Thus, HCF of 441, 567 and 693 is 63.

Q3. Find the HCF of 96 and 404 by the prime factorisation method. Hence, find their LCM.

Solution:

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$404 = 2 \times 2 \times 101$$

$$\text{HCF} = 2 \times 2 = 4$$

$$\text{LCM} = 9696.$$

WORD PROBLEMS

Note: Read the question twice and try to understand if its implying towards some minimum number or maximum number.(ie HCF or LCM).

Q1. Three sets of English, Hindi and Mathematics books have to be stacked in such a way that all the books are stored topic wise and height of each stack is the same. The number of English books is 96, the number of Hindi books is 240 and that of mathematics is 336. If the thickness of the books is same then determine the number stacks of the respective books.

Solution:

Note: Here we need to find minimum number of books to satisfy the given condition. (HCF)

Step I: Finding HCF of 96,240 and 336 using Euclid's Division Algorithm.

$$336 = 240 \times 1 + 96$$

$$240 = 96 \times 2 + 48$$

$$96 = 48 \times 2 + 0$$

$$\text{HCF}(336,240) = 48$$

Now to find HCF of 48 and 96

$$96 = 48 \times 2 + 0$$

$$\text{HCF}(96, 48) = 48$$

Step II:

$$\text{Number of stacks of English books} = \frac{96}{48} = 2$$

$$\text{Number of stacks of Hindi books} = \frac{240}{48} = 5$$

$$\text{Number of stacks of Maths books} = \frac{336}{48} = 7$$

Q2. A sweetseller has 420 *kaju barfis* and 130 *badam barfis*. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the maximum number of *barfis* that can be placed in each stack for this purpose?

Solution:

Note: Here we need to find minimum number of sweets to satisfy the given condition. (HCF)
Finding HCF of 420 and 130 using Euclid's Division Algorithm.

$$420 = 130 \times 3 + 30$$

$$130 = 30 \times 4 + 10$$

$$30 = 10 \times 3 + 0$$

$$\therefore \text{HCF} = 10$$

$\therefore$ The sweetseller can make stacks of 10 for both kinds of *barfi*.

Q3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution:

Note: Here we need to find minimum number of soldiers in order to maximum number of columns. (HCF)

Finding HCF of 616 and 32 using Euclid's Division Algorithm.

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0.$$

$$\therefore \text{HCF} = 4$$

$\therefore$ The maximum number of columns in which they can march is 4.

Q4. There is a circular path around a sports field. Sonia takes 18 minutes to drive one round of the field, while Ravi takes 12 minutes for the same. Suppose they both start at the same point and at the same time, and go in the same direction. After how many minutes will they meet again at the starting point?

Solution:

Note: Here we need to find minimum number of minutes so that they meet again.

Finding LCM of 18 and 12.

$$18 = 2 \times 3 \times 3$$

$$12 = 2 \times 2 \times 3$$

$$\therefore \text{LCM} = 2 \times 2 \times 3 \times 3 = 36$$

$\therefore$ Sonia and Ravi will meet again at the starting point in 36 minutes.

TYPE V

Problems based on the formula $\text{HCF} \times \text{LCM} = \text{Product of two numbers}$.

Q1. Given that $\text{HCF}(306, 657) = 9$, find $\text{LCM}(306, 657)$.

Solution:

Step I: Use the formula $\text{HCF} \times \text{LCM} = \text{Product of two numbers}$

Step II: $9 \times \text{LCM} = 306 \times 657$

$$\therefore \text{LCM} = \frac{306 \times 657}{9}$$

$$\therefore \text{LCM} = 22338$$

TYPE V

Q1. Show that every positive even integer is of the form $2q$, and that every positive odd integer is of the form $2q + 1$, where q is some integer.

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then

$$a = bq + r \quad \text{such that } 0 < r < b.$$

Step II: Here $b = 2$, $0 < r < 2$ (possible values of $r = 0, 1$)

Step III:

$$\therefore a = 2q + 0 = 2q \quad \text{is an positive even integer.}$$

$$\therefore a = 2q + 1 = 2q + 1 \quad \text{is an odd positive integer.}$$

Q2. Show that any positive integer is of the form $3q$, $3q+1$ or $3q+2$, where q is some integer.

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then

$$a = bq + r \quad \text{such that } 0 < r < b.$$

Step II: Here $b = 3$, $0 < r < 3$ (possible values of $r = 0, 1, 2$)

Step III:

$$\therefore a = 3q + 0 = 3q$$

$$\therefore a = 3q + 1$$

$$\therefore a = 3q + 2 \quad \text{is any positive integer.}$$

Q3. Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$, where q is some integer.

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then

$$a = bq + r \quad \text{such that } 0 < r < b.$$

Step II: Here $b = 4$, $0 < r < 4$ (possible values of $r = 0, 1, 2, 3$)

Step III:

$$\therefore a = 4q + 1 \quad (\text{adding 1 to a multiple of 4 gives an positive odd integer})$$

$$\therefore a = 4q + 3 \quad (\text{adding 3 to a multiple of 4 gives an positive odd integer})$$

Q4. Use Euclid's division lemma to show that the square of any positive integer is either of the form $3m$ or $3m + 1$ for some integer m .

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then

$$a = bq + r \quad \text{such that } 0 < r < b.$$

Step II: Here $b = 3$, $0 < r < 3$ (possible values of $r = 0, 1, 2$)

Step III:

$$\therefore a = 3q + 0 = 3q$$

$$\therefore a = 3q + 1$$

$$\therefore a = 3q + 2 \quad \text{is any positive integer.}$$

Step IV:

Case i:

$$(a)^2 = (3q)^2$$

$$(a)^2 = 9q^2 = 3 \times 3q^2$$

$$(a)^2 = 3m \quad \text{where } m = 3q^2$$

Case ii:

$$(a)^2 = (3q + 1)^2$$

$$(a)^2 = 9q^2 + 6q + 1 = 3(3q^2 + 2q) + 1$$

$$(a)^2 = 3m + 1 \quad \text{where } m = 3q^2 + 2q$$

Case iii:

$$(a)^2 = (3q + 2)^2$$

$$(a)^2 = 9q^2 + 12q + 4 = 9q^2 + 12q + 3 + 1 = 3(3q^2 + 4q + 1) + 1$$

$$(a)^2 = 3m + 1 \quad \text{where } m = 3q^2 + 4q + 1$$

Q5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then

$a = bq + r$ such that $0 < r < b$.

Step II: Here $b = 3$, $0 < r < 3$ (possible values of $r = 0, 1, 2$)

Step III:

$$\therefore a = 3q + 0 = 3q$$

$$\therefore a = 3q + 1$$

$$\therefore a = 3q + 2 \quad \text{is any positive integer.}$$

Step IV:

Case i:

$$(a)^3 = (3q)^3$$

$$(a)^3 = 27q^3 = 9 \times 3q^3$$

$$(a)^3 = 9m \quad \text{where } m = 3q^3$$

Case ii:

$$(a)^3 = (3q + 1)^3$$

$$(a)^3 = 27q^3 + 27q^2 + 9q + 1 = 9(3q^3 + 3q^2 + q) + 1$$

$$(a)^3 = 9m + 1 \quad \text{where } m = 3q^3 + 3q^2 + q$$

Case iii:

$$(a)^3 = (3q + 2)^3$$

$$(a)^3 = 27q^3 + 54q^2 + 36q + 8 = 9(3q^3 + 6q^2 + 4q) + 8$$

$$(a)^3 = 3m + 8 \quad \text{where } m = 3q^3 + 6q^2 + 4q$$

Q6. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then

$$a = bq + r \quad \text{such that } 0 < r < b.$$

Step II: Here $b = 6$, $0 < r < 6$ (possible values of $r = 0, 1, 2, 3, 4, 5$)

Step III:

$$\therefore a = 6q + 1 \quad (\text{adding 1 to a multiple of 6 gives an positive odd integer})$$

$$\therefore a = 4q + 3 \quad (\text{adding 3 to a multiple of 6 gives an positive odd integer})$$

$$\therefore a = 6q + 5 \quad (\text{adding 3 to a multiple of 6 gives an positive odd integer})$$

Q7. Show that cube of any positive integer is of the form $4m$, $4m + 1$ or $4m + 3$, for some integer m .

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then

$$a = bq + r \quad \text{such that } 0 < r < b.$$

Step II: Here $b = 4$, $0 < r < 4$ (possible values of $r = 0, 1, 2, 3$)

Step III:

$$a = 4q + 1 \quad \text{is any positive integer.}$$

$$a = 4q + 2 \quad \text{is any positive integer.}$$

$$a = 4q + 3 \quad \text{is any positive integer.}$$

Step IV:

Case i:

$$(a)^3 = (4q)^3$$

$$(a)^3 = 64q^3 = 4 \times 16q^3$$

$$(a)^3 = 4m \quad \text{where } m = 16q^3$$

Case ii:

$$(a)^3 = (4q + 1)^3$$

$$(a)^3 = 64q^3 + 48q^2 + 12q + 1 = 4(16q^3 + 12q^2 + 3q) + 1$$

$$(a)^2 = 4m + 1 \quad \text{where } m = 16q^3 + 12q^2 + 3q$$

Case iii:

$$(a)^3 = (4q + 2)^3$$

$$(a)^3 = 64q^3 + 96q^2 + 48q + 8 = 4(16q^3 + 24q^2 + 12q + 2)$$

$$(a)^3 = 4m \quad \text{where } m = 16q^3 + 24q^2 + 12q + 2$$

Case iv:

$$(a)^3 = (4q + 3)^3$$

$$(a)^3 = 64q^3 + 144q^2 + 108q + 27 = 4(16q^3 + 36q^2 + 27q + 6) + 3$$

$$(a)^3 = 4m + 3 \quad \text{where } m = 16q^3 + 36q^2 + 27q + 6$$

Q8. Show that the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q .

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then

$a = bq + r$ such that $0 < r < b$.

Step II: Here $b = 5$, $0 < r < 5$ (possible values of $r = 0, 1, 2, 3, 4$)

Step III:

$a = 5m + 1$ is any positive integer.

$a = 5m + 2$ is any positive integer.

$a = 5m + 3$ is any positive integer.

$a = 5m + 4$ is any positive integer.

Step IV:

Case i:

$$(a)^2 = (5m)^2$$

$$(a)^2 = 25m^2 = 5 \times 5m^2$$

$$(a)^2 = 5q \quad \text{where } q = 5m^2$$

Case ii:

$$(a)^2 = (5m + 1)^2$$

$$(a)^2 = 25m^2 + 10m + 1 = 5(5m^2 + 2m) + 1$$

$$(a)^2 = 5q + 1 \quad \text{where } q = 5m^2 + 2m$$

Case iii:

$$(a)^2 = (5m + 2)^2$$

$$(a)^2 = 25m^2 + 20m + 4 = 5(5m^2 + 4m) + 4$$

$$(a)^2 = 5q + 4 \quad \text{where } q = 5m^2 + 4m$$

Case iv:

$$(a)^2 = (5m + 3)^2$$

$$(a)^2 = 25m^2 + 30m + 9 = 25m^2 + 30m + 5 + 4 = 5(5m^2 + 6m + 1) + 4$$

$$(a)^2 = 5q + 4 \quad \text{where } q = 5m^2 + 6m + 1$$

Case v:

$$(a)^2 = (5m + 4)^2$$

$$(a)^2 = 25m^2 + 40m + 16 = 25m^2 + 30m + 15 + 1 = 5(5m^2 + 10m + 3) + 1$$

$$(a)^2 = 5q + 1 \quad \text{where } q = 5m^2 + 10m + 3$$

Thus, the square of any positive integer cannot be of the form $5q + 2$ or $5q + 3$ for any integer q .

Q9. Show that the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m .

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then

$a = bq + r$ such that $0 < r < b$.

Step II: Here $b = 6$, $0 < r < 6$ (possible values of $r = 0, 1, 2, 3, 4, 5$)

Step III:

$a = 6q + 1$ is any positive integer.

$a = 6q + 2$ is any positive integer.

$a = 6q + 3$ is any positive integer.

$a = 6q + 4$ is any positive integer.

$a = 6q + 5$ is any positive integer.

Case i:

$$(a)^2 = (6q)^2$$

$$(a)^2 = 36^2 = 6 \times 6q^2$$

$$(a)^2 = 6m \quad \text{where } m = 6q^2$$

Case ii:

$$(a)^2 = (6q + 1)^2$$

$$(a)^2 = 36q^2 + 12q + 1 = 6(6q^2 + 2q) + 1$$

$$(a)^2 = 6m + 1 \quad \text{where } m = 6q^2 + 2q$$

Case iii:

$$(a)^2 = (6q + 2)^2$$

$$(a)^2 = 36q^2 + 24q + 4 = 6(6q^2 + 4q) + 4$$

$$(a)^2 = 6m + 4 \quad \text{where } m = 6q^2 + 4q$$

Case iv:

$$(a)^2 = (6q + 3)^2$$

$$(a)^2 = 36q^2 + 36q + 9 = 36q^2 + 36q + 6 + 3 = 6(6q^2 + 6q + 1) + 3$$

$$(a)^2 = 6m + 3 \quad \text{where } m = 6q^2 + 6q + 1$$

Case v:

$$(a)^2 = (6q + 5)^2$$

$$(a)^2 = 36q^2 + 60q + 25 = 36q^2 + 60q + 24 + 1 = 6(6q^2 + 10q + 4) + 1$$

$$(a)^2 = 6m + 1 \quad \text{where } m = 6q^2 + 10q + 4$$

Thus, the square of any positive integer cannot be of the form $6m + 2$ or $6m + 5$ for any integer m .

Q10. If n is an odd integer, then show that $n^2 - 1$ is divisible by 8.

Solution:

Step I: According to Euclid's Division Lemma, if a any given positive integer then $a = bq + r$ such that $0 < r < b$.

Step II: Here $b = 4$, $0 < r < 4$ (possible values of $r = 0, 1, 2, 3$)

Step III:

$\therefore a = 4q + 1$ (adding 1 to a multiple of 4 gives an positive odd integer)

$\therefore a = 4q + 3$ (adding 3 to a multiple of 4 gives an positive odd integer)

Step IV:

Case i: here $a = n$

$$(n)^2 = (4q + 1)^2$$

$$(n)^2 = 16q^2 + 8q + 1$$

$$\therefore (n)^2 - 1 = 16q^2 + 8q + 1 - 1$$

$$\therefore (n)^2 - 1 = 16q^2 + 8q$$

$$\therefore (n)^2 - 1 = 8(2q^2 + q)$$

$$\therefore (n)^2 - 1 = 8m \quad \text{where } m = 2q^2 + q$$

Case ii: here $a = n$

$$(n)^2 = (4q + 3)^2$$

$$(n)^2 = 16q^2 + 24q + 9$$

$$\therefore (n)^2 - 1 = 16q^2 + 24q + 9 - 1$$

$$\therefore (n)^2 - 1 = 16q^2 + 24q + 8$$

$$\therefore (n)^2 - 1 = 8(2q^2 + 3q + 1)$$

$$\therefore (n)^2 - 1 = 8m \quad \text{where } m = 2q^2 + 3q + 1$$

Udertaking

I, Jagdish Gannavarapu mathematics teacher in Jain Heritage A Cambridge School, Nagpur hereby declare that the above teaching material(format) is purely drafted by me under the guidance of my Principal Mr.Praveen Sharma.

