

EQUATIONS REDUCIBLE TO A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Equations Reducible to a Pair of Linear Equations in Two Variables

- There are several situations which can be mathematically represented by two equations that are not linear to start with. But we alter them so that they are reduced to a pair of linear equations.
- In this section, we shall discuss the solution of such pairs of equations which are not linear but can be reduced to linear form by making some suitable substitutions.

Example 17 : Solve the pair of equations:

$$\frac{2}{x} + \frac{3}{y} = 13$$

$$\frac{5}{x} - \frac{4}{y} = -2$$

Solution : Let us write the given pair of equations as

$$2\left(\frac{1}{x}\right) + 3\left(\frac{1}{y}\right) = 13 \quad (1)$$

$$5\left(\frac{1}{x}\right) - 4\left(\frac{1}{y}\right) = -2 \quad (2)$$

These equations are not in the form $ax + by + c = 0$. However, if we substitute

$\frac{1}{x} = p$ and $\frac{1}{y} = q$ in Equations (1) and (2), we get

$$2p + 3q = 13 \quad (3)$$

$$5p - 4q = -2 \quad (4)$$

So, we have expressed the equations as a pair of linear equations. Now, you can use any method to solve these equations, and get $p = 2$, $q = 3$.

You know that $p = \frac{1}{x}$ and $q = \frac{1}{y}$.

Substitute the values of p and q to get

$$\frac{1}{x} = 2, \text{ i.e., } x = \frac{1}{2} \text{ and } \frac{1}{y} = 3, \text{ i.e., } y = \frac{1}{3}.$$

Example 18 : Solve the following pair of equations by reducing them to a pair of linear equations :

$$\frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

Solution : Let us put $\frac{1}{x-1} = p$ and $\frac{1}{y-2} = q$. Then the given equations

$$5\left(\frac{1}{x-1}\right) + \frac{1}{y-2} = 2 \quad (1)$$

$$6\left(\frac{1}{x-1}\right) - 3\left(\frac{1}{y-2}\right) = 1 \quad (2)$$

can be written as :

$$5p + q = 2 \quad (3)$$

$$6p - 3q = 1 \quad (4)$$

Equations (3) and (4) form a pair of linear equations in the general form. Now, you can use any method to solve these equations. We get $p = \frac{1}{3}$ and $q = \frac{1}{3}$.

Now, substituting $\frac{1}{x-1}$ for p , we have

$$\frac{1}{x-1} = \frac{1}{3},$$

i.e., $x - 1 = 3$, i.e., $x = 4$.

Similarly, substituting $\frac{1}{y-2}$ for q , we get

$$\frac{1}{y-2} = \frac{1}{3}$$

i.e., $3 = y - 2$, i.e., $y = 5$

Hence, $x = 4, y = 5$ is the required solution of the given pair of equations.

Verification : Substitute $x = 4$ and $y = 5$ in (1) and (2) to check whether they are satisfied.

Ex : A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.

- **The motion of the boat with respect to the river banks can be of two types.**
 - 1. One of the types is when the boat goes in the same direction as the direction of the river. This is what we know as the downstream motion.**
 - 2. However, when the boat goes in the opposite direction as that of the stream or the river, it is what we call the upstream motion.**

In the downstream motion, the boat and the water move in the same direction and thus the speed is more than what it will be when the two are moving opposite to each other.

In other words , the upstream motion is slower.

So in case of downstream the speeds will be added, while in case of upstream the speeds will be subtracted .

The speed of boat downstream is equal to the sum of the speed of the boat in still water and speed of the stream

and the speed of boat upstream is equal to the difference of the speed of the boat in still water and speed of the stream .((speed of boat is always greater then speed of stream))**

i.e. speed of boat-speed of stream

Ex : A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km down-stream. Determine the speed of the stream and that of the boat in still water.

Solution: Let the speed of the boat in still water = x km/hr

Let the speed of the stream = y km/hr

Downstream speed = $x + y$ km/hr

Upstream speed = $x - y$ km/hr



Downstream



Upstream



As we know ; distance=speed $\times$ time

so

$$time = \frac{distance}{speed}$$

Case 1: The boat goes 30 km upstream and 44 km downstream in 10 hours.

For upstream

Distance=30km

Speed =(x-y)km/h

Time for upstream= $\frac{30}{x-y}$

For downstream

Distance=44km

speed=(x+y)km/h

Time for downstream = $\frac{44}{x+y}$

According to question

Time for upstream+ Time for downstream =10h

$$\frac{30}{x-y} + \frac{44}{x+y} = 10$$

Case 2: The boat goes 40 km upstream and 55 km downstream in 13 hours.

For upstream

Distance=40km

Speed =(x-y)km/h

$$\text{Time for upstream} = \frac{40}{x-y}$$

According to question

Time for upstream+ Time for downstream =13h

$$\frac{40}{x-y} + \frac{55}{x+y} = 13$$

For downstream

Distance=55km

speed=(x+y)km/h

$$\text{Time for downstream} = \frac{55}{x+y}$$

- $\frac{30}{x-y} + \frac{44}{x+y} = 10$

$$30\left(\frac{1}{x-y}\right) + 44\left(\frac{1}{x+y}\right) = 10 \dots\dots\dots (i)$$

$$\frac{40}{x-y} + \frac{55}{x+y} = 13$$

$$40\left(\frac{1}{x-y}\right) + 55\left(\frac{1}{x+y}\right) = 13 \dots\dots\dots (ii)$$

- Let $\frac{1}{x-y} = u$; $\frac{1}{x+y} = v$

From (1) and (2),

$$30u + 44v = 10 \dots\dots\dots (3)$$

$$40u + 55v = 13 \dots\dots\dots (4)$$

Multiply equation (3) with 4 and equation (4) with 3, and subtract them

$$\begin{array}{r} 120u + 176v = 40 \\ \underline{120u + 165v = 39} \\ 11v = 1 \end{array}$$

$$v = \frac{1}{11}$$

$$\Rightarrow \frac{1}{y+x} = \frac{1}{11}$$

$$x+y=11 \dots\dots\dots(5)$$

From equation (3),

$$30u=10-44v$$

$$30u = 10 - 44 \times \frac{1}{11}$$

$$u = \frac{6}{30}$$

$$u = \frac{1}{5}$$

$$\Rightarrow x-y = 5 \dots\dots\dots(6)$$

Adding (5) and (6), we get,

$$x+y=11$$

$$x-y = 5$$

$$2x=16$$

$$x=8$$

From equation (5),

$$y = 11-x$$

$$y = 11-8=3$$

Speed of boat=8km/h

Speed of stream=3km/h

EXERCISE 3.6

- 1. Solve the following pairs of equations by reducing them to a pair of linear equations:

$$\begin{aligned} \text{(i)} \quad & \frac{2}{\sqrt{x}} + \frac{3}{\sqrt{y}} = 2 \\ & \frac{4}{\sqrt{x}} - \frac{9}{\sqrt{y}} = -1 \end{aligned}$$

Sol: $2\left(\frac{1}{\sqrt{x}}\right) + 3\left(\frac{1}{\sqrt{y}}\right) = 2$

$$4\left(\frac{1}{\sqrt{x}}\right) - 9\left(\frac{1}{\sqrt{y}}\right) = -1$$

put $\frac{1}{\sqrt{x}} = u$, $\frac{1}{\sqrt{y}} = v$

$$2u + 3v = 2 \dots \dots \dots \text{(i)}$$

$$4u - 9v = -1 \dots \dots \dots \text{(ii)}$$

multiply (i) by 3 and (i)+(ii)

$$3(2u + 3v = 2) \Rightarrow 6u + 9v = 6$$

$$6u + 9v = 6$$

$$4u - 9v = -1$$

$$10u = 5$$

$$u = \frac{5}{10}$$

$$u = \frac{1}{2} \Rightarrow \frac{1}{\sqrt{x}} = \frac{1}{2}$$

$$\sqrt{x} = 2$$

$$x = 4$$

put $u = \frac{1}{2}$ in (i); $2\left(\frac{1}{2}\right) + 3v = 2$

- $2 \left(\frac{1}{2}\right) + 3v = 2$

$$\frac{2}{2} + 3v = 2$$

$$3v = 2 - 1$$

$$3v = 1$$

$$v = \frac{1}{3}$$

$$\frac{1}{\sqrt{y}} = \frac{1}{3} \implies \sqrt{y} = 3$$

$$y = 9$$

$$(iv) \frac{5}{x-1} + \frac{1}{y-2} = 2$$

$$\frac{6}{x-1} - \frac{3}{y-2} = 1$$

Put $5\left(\frac{1}{x-1}\right) + \left(\frac{1}{y-2}\right) = 2$

$$6\left(\frac{1}{x-1}\right) - 3\left(\frac{1}{y-2}\right) = 1$$

let $\frac{1}{x-1} = u, \frac{1}{y-2} = v$

$$5u + v = 2 \dots \dots \dots (i)$$

$$6u - 3v = 1 \dots \dots \dots (ii)$$

Multiply (i) by 3 and add (i) + (ii)

$$3(5u + v = 2)$$

$$15u + 3v = 6$$

$$6u - 3v = 1$$

$$21u = 7$$

$$u = \frac{1}{3} \text{ put in (i) } 5\left(\frac{1}{3}\right) + v = 2$$

$$v = 2 - \frac{5}{3} \Rightarrow v = \frac{1}{3}$$

$$\frac{1}{x-1} = \frac{1}{3} \Rightarrow x-1 = 3$$

$$x = 4$$

$$\text{now, } \frac{1}{y-2} = \frac{1}{3}$$

$$y-2 = 3$$

$$y = 5$$

• V. $\frac{7x-2y}{xy} = 5$

$$\frac{8x+7y}{xy} = 15$$

Sol: $\frac{7x-2y}{xy} = 5$

$$\frac{8x+7y}{xy} = 15$$

$$\frac{7x}{xy} - \frac{2y}{xy} = 5$$

$$\frac{8x}{xy} + \frac{7y}{xy} = 15$$

$$\frac{7}{y} - \frac{2}{x} = 5$$

$$\frac{8}{y} + \frac{7}{x} = 15$$

(ii) 2 women and 5 men can together finish an embroidery work in 4 days, while 3 women and 6 men can finish it in 3 days. Find the time taken by 1 woman alone to finish the work, and also that taken by 1 man alone.

- As:

If , A person completes work in 2 day

Work completed in one day = $\frac{1}{2}$

A person completes work in 3 day

Work completed in one day = $\frac{1}{3}$

Then if;

A person completes work in x day

Work completed in one day = $\frac{1}{x}$

Sol:

Let the time taken by 1 woman alone to finish the work= x days

Let the time taken by 1 man alone to finish the work= y days

Work done by 1 woman in one day= $\frac{1}{x}$

Work done by 1 man in one day= $\frac{1}{y}$

2 women and 5 men complete the work in 4 days

Work finished by 2 women and 5 men in 1 day= $\frac{1}{4}$

$2 \times (\text{work finished by 1 woman in 1 day}) + 5 \times (\text{work finished by 1 man in 1 day}) = \frac{1}{4}$

$$2 \times \left(\frac{1}{x}\right) + 5 \times \left(\frac{1}{y}\right) = \frac{1}{4}$$

$$\frac{2}{x} + \frac{5}{y} = \frac{1}{4} \dots\dots\dots (i)$$

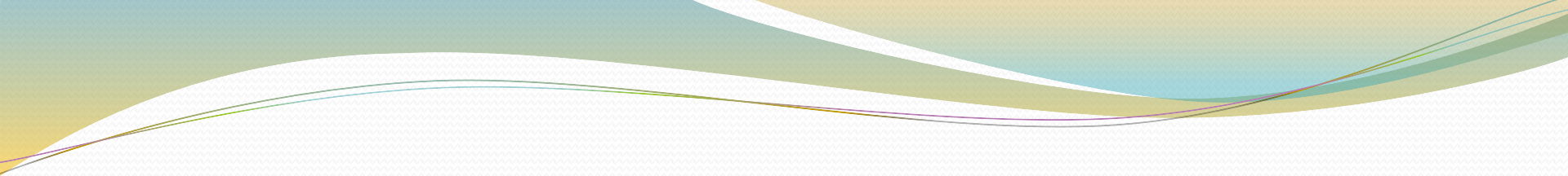
3 women and 6 men complete the work in 3 days

Work finished by 3 women and 6 men in 1 day = $\frac{1}{3}$

$3 \times (\text{work finished by 1 woman in 1 day}) + 6 \times (\text{work finished by 1 man in 1 day}) = \frac{1}{3}$

$$3 \times \left(\frac{1}{x}\right) + 6 \times \left(\frac{1}{y}\right) = \frac{1}{3}$$

$$\frac{3}{x} + \frac{6}{y} = \frac{1}{3} \dots\dots\dots \text{(ii)}$$



- $\frac{2}{x} + \frac{5}{y} = \frac{1}{4}$ (i)

- $\frac{3}{x} + \frac{6}{y} = \frac{1}{3}$ (ii)

Now solve these equations with suitable substitution

(iii) Roohi travels 300 km to her home partly by train and partly by bus. She takes 4 hours if she travels 60 km by train and the remaining by bus. If she travels 100 km by train and the remaining by bus, she takes 10 minutes longer. Find the speed of the train and the bus separately.

((do it by yourself)) form equations with given conditions and solve them

A diagram representing the first condition. It consists of three circles and two symbols. The first circle is teal and contains the text "60km by train". To its right is a teal plus sign. The second circle is yellow and contains the text "240km by bus". To its right is a yellow equals sign. The final circle is purple and contains the text "4 hours".

$$\begin{array}{c} 60\text{km} \\ \text{by} \\ \text{train} \end{array} + \begin{array}{c} 240\text{km} \\ \text{by bus} \end{array} = 4 \text{ hours}$$

A diagram representing the second condition. It consists of three circles and two symbols. The first circle is teal and contains the text "100km by train". To its right is a teal plus sign. The second circle is yellow and contains the text "200km by bus". To its right is a yellow equals sign. The final circle is purple and contains the text "4hours + 10minutes".

$$\begin{array}{c} 100\text{km} \\ \text{by} \\ \text{train} \end{array} + \begin{array}{c} 200\text{km} \\ \text{by bus} \end{array} = \begin{array}{c} 4\text{hours} \\ + \\ 10\text{minutes} \end{array}$$

Sol: let speed of train= x km

And speed of bus= y km

Distance=speed $\times$ time

Time = $\frac{\text{Distance}}{\text{speed}}$

Case 1:

By Train

Distance=

Speed=

Time=

Total time taken in first case= 4hours

Time taken by train + time taken by bus=4hours

By Bus

distance=

speed=

time=

Case 2:

By Train

Distance=

Speed=

Time=

Total time taken in first case= 4hours+10 minutes

Time taken by train + time taken by bus= 4hours+10 minutes

By Bus

distance=

speed=

time=