

(3) CROSS MULTIPLICATION METHOD

(3) CROSS MULTIPLICATION METHOD

- So far, you have learnt how to solve a pair of linear equations in two variables by
- Graphical
- Substitution and
- Elimination methods

Here, we introduce one more algebraic method to solve a pair of linear equations which for many reasons is a very useful method of solving these equations

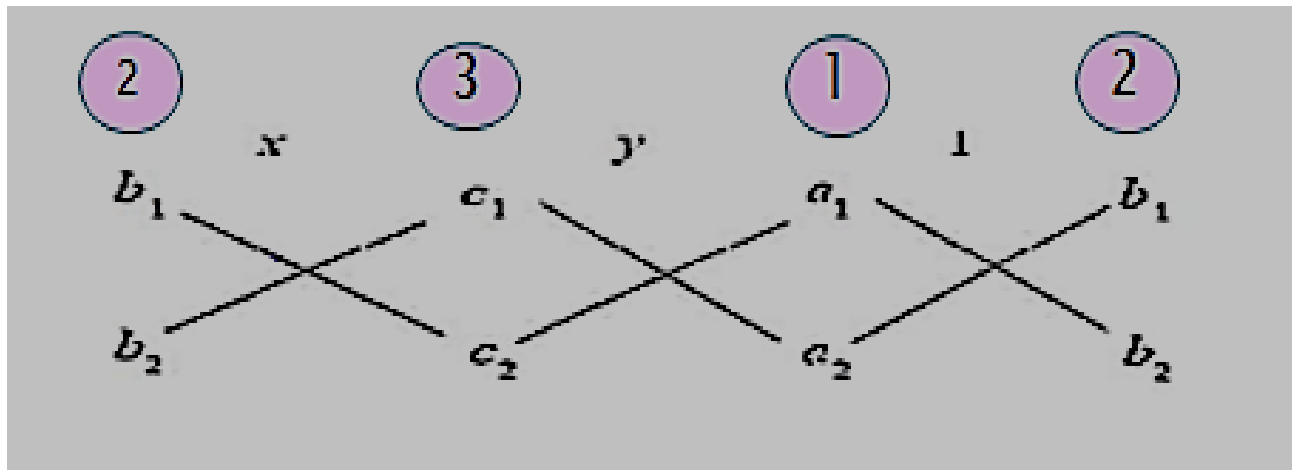
STEPS TO SOLVE THE PAIR OF LINEAR EQUATIONS BY METHOD OF CROSS-MULTIPLICATION.

Step1: Obtain two equations, and write them both in standard form

$$a_1x + b_1y + c_1 = 0 \dots \dots (i)$$

$$a_2x + b_2y + c_2 = 0 \dots \dots (ii)$$

Step2: To solve the pair of equations for X and Y using cross-multiplication, we will arrange the variable X and Y and their coefficients a_1, a_2, b_1, b_2 , and the constants c_1 and c_2 as shown below



x		y		1	
Coefficient of y	Constant term	Constant term	Coefficient of x	Coefficient of x	Coefficient of y
b_1	c_1	c_1	a_1	a_1	b_1
b_2	c_2	c_2	a_2	a_2	b_2

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{y}{c_1 a_2 - c_2 a_1} = \frac{1}{a_1 b_2 - a_2 b_1}$$

Step 3: now simplifying the above situation by equating them as given below

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{1}{a_1 b_2 - a_2 b_1}$$

$$\frac{y}{c_1 a_2 - c_2 a_1} = \frac{1}{a_1 b_2 - a_2 b_1}$$

$$x = \frac{b_1 c_2 - b_2 c_1}{a_1 b_2 - a_2 b_1}$$

$$y = \frac{c_1 a_2 - c_2 a_1}{a_1 b_2 - a_2 b_1}$$

Step 4: check the solution and write the solution set.

Point to be remember:

$$\frac{x}{b_1 c_2 - b_2 c_1} = \frac{y}{c_1 a_2 - c_2 a_1} = \frac{1}{b_2 a_1 - b_1 a_2}$$

From here, x and y are evaluated provided that $a_1 b_2 - a_2 b_1 \neq 0$. Therefore, this method is called as cross multiplication as depicted.

In such a method, the condition for consistency of pair of linear equation in two variables must be checked which are as follows:

1. If $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, then we get a unique solution and the pair of linear equations in two variables are consistent.
2. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, then there exists infinitely many solutions and the pair of lines are coincident and therefore, dependent and consistent.
3. If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, then there exists no solution and the pair of linear equations in two variables are said to be inconsistent.

Ex: Solve the following linear equations using cross multiplication method.

$$3x - 4y = 2 \dots\dots\dots (i)$$

$$y - 2x = 7 \dots\dots\dots (ii)$$

Sol: write them both in standard form

$$3x - 4y - 2 = 0 \dots\dots\dots (iii)$$

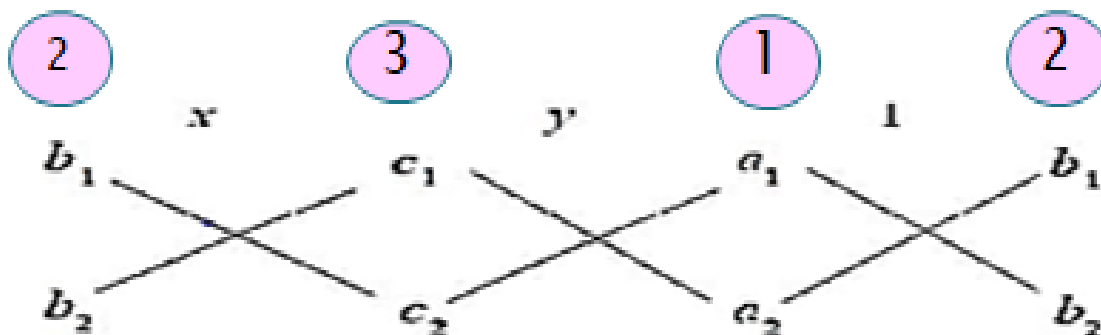
$$-2x + y - 7 = 0 \dots\dots\dots (iv)$$

Compare $3x - 4y - 2 = 0$ by $a_1x + b_1y + c_1 = 0$

and $-2x + y - 7 = 0$ by $a_2x + b_2y + c_2 = 0$

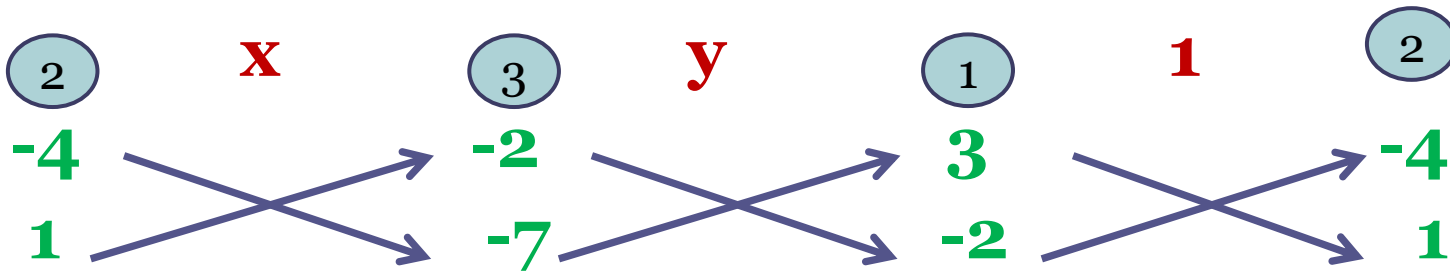
$$a_1 = 3, b_1 = -4, c_1 = -2,$$

$$a_2 = -2, b_2 = 1, c_2 = -7$$



$$a_1=3, b_1=-4, c_1=-2,$$

$$a_2=-2, b_2=1, c_2=-7$$



$$\frac{x}{28+2} = \frac{y}{4+21} = \frac{1}{3-8}$$

$$\Rightarrow \frac{x}{30} = \frac{y}{25} = -\frac{1}{5}$$

$$\Rightarrow x = -6, y = -5$$

$$\begin{aligned} \Rightarrow \frac{x}{30} &= \frac{-1}{5} ; \quad \frac{y}{25} = \frac{-1}{5} \\ \Rightarrow x &= \frac{-30}{5} ; \quad y = \frac{-25}{5} \end{aligned}$$

Example 14 : From a bus stand in Bangalore , if we buy 2 tickets to Malleswaram and 3 tickets to Yeshwanthpur, the total cost is `Rs.46; but if we buy 3 tickets to Malleswaram and 5 tickets to Yeshwanthpur the total cost is Rs. 74. Find the fares from the bus stand to Malleswaram, and to Yeshwanthpur.

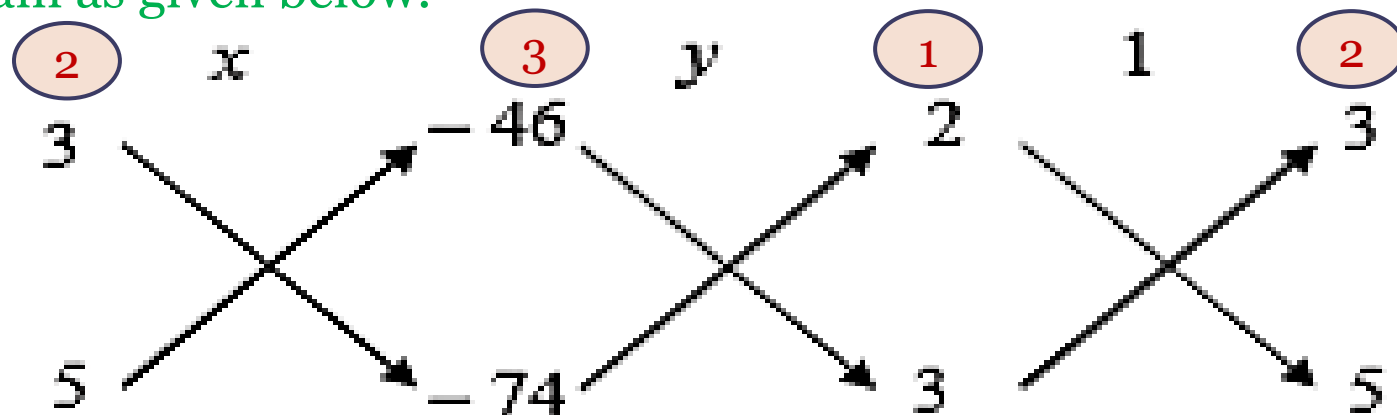
Sol : Let Rs. x be the fare from the bus stand in Bangalore to Malleswaram, and Rs. y to Yeshwanthpur.

From the given information, we have

$$2x + 3y = 46, \text{ i.e., } 2x + 3y - 46 = 0 \dots\dots\dots (1)$$

$$3x + 5y = 74, \text{ i.e., } 3x + 5y - 74 = 0 \dots\dots\dots (2)$$

To solve the equations by the cross-multiplication method, we draw the diagram as given below.



Then
$$\frac{x}{(3)(-74) - (5)(-46)} = \frac{y}{(-46)(3) - (-74)(2)} = \frac{1}{(2)(5) - (3)(3)}$$

i.e.,
$$\frac{x}{-222 + 230} = \frac{y}{-138 + 148} = \frac{1}{10 - 9}$$

i.e.,
$$\frac{x}{8} = \frac{y}{10} = \frac{1}{1}$$

i.e.,
$$\frac{x}{8} = \frac{1}{1} \text{ and } \frac{y}{10} = \frac{1}{1}$$

i.e.,
$$x = 8 \text{ and } y = 10$$

Hence, the fare from the bus stand in Bangalore to Malleswaram is ₹ 8 and the fare to Yeshwanthpur is ₹ 10.

Verification : You can check from the problem that the solution we have got is correct.

COMPARING THE RATIOS OF COEFFICIENTS OF A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

CONDITION	TYPE OF SOLUTION	GRAPHICAL REPRESENTATION	TYPE OF SYSTEM
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Unique solution	Intersecting lines	consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Infinitely many solutions	Coincident lines	consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	No solution	Parallel lines	inconsistent

Example 15 : For which values of p does the pair of equations given below has unique solution?

$$4x + py + 8 = 0$$

$$2x + 2y + 2 = 0$$

Solution : Here $a_1 = 4$, $a_2 = 2$, $b_1 = p$, $b_2 = 2$.

Now for the given pair to have a unique solution : $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$

i.e.,
$$\frac{4}{2} \neq \frac{p}{2}$$

i.e.,
$$p \neq 4$$

Therefore, for all values of p , except 4, the given pair of equations will have a unique solution.

Example 16 : For what values of k will the following pair of linear equations have infinitely many solutions?

$$kx + 3y - (k - 3) = 0 ; 12x + ky - k = 0$$

sol: given equations are

$$kx + 3y - (k - 3) = 0 \dots (i)$$

$$12x + ky - k = 0 \dots (ii)$$

$$a_1 = k, b_1 = 3, c_1 = -(k-3),$$

$$a_2 = 12, b_2 = k, c_2 = -k$$

$$\text{Here, } \frac{a_1}{a_2} = \frac{k}{12}, \frac{b_1}{b_2} = \frac{3}{k}, \frac{c_1}{c_2} = \frac{k-3}{k}$$

For a pair of linear equations to have infinitely many solutions

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

So, we need

$$\frac{k}{12} = \frac{3}{k} = \frac{k-3}{k}$$

or,

$$\frac{k}{12} = \frac{3}{k}$$

which gives $k^2 = 36$, i.e., $k = \pm 6$.

Also,

$$\frac{3}{k} = \frac{k-3}{k}$$

gives $3k = k^2 - 3k$, i.e., $6k = k^2$, which means $k = 0$ or $k = 6$.

Therefore, the value of k , that satisfies both the conditions, is $k = 6$. For this value, the pair of linear equations has infinitely many solutions.

EXERCISE 3.5

1. Which of the following pairs of linear equations has unique solution, no solution, or infinitely many solutions. In case there is a unique solution, find it by using cross multiplication method.

$$(i) \quad x - 3y - 3 = 0 \quad \dots(1)$$

$$3x - 9y - 2 = 0 \quad \dots(2)$$

$$\text{Here, } a_1 = 1, b_1 = -3, c_1 = -3$$

$$a_2 = 3, b_2 = -9, c_2 = -2$$

$$\therefore \frac{a_1}{a_2} = \frac{1}{3}, \frac{b_1}{b_2} = \frac{-3}{-9} = \frac{1}{3}$$

$$\text{and } \frac{c_1}{c_2} = \frac{-3}{-2} = \frac{3}{2}$$

$$\text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

Therefore, the pair of linear equations has **no solution**.

$$(ii) \quad 2x + y - 5 = 0 \quad \dots(1)$$

$$3x + 2y - 8 = 0 \quad \dots(2)$$

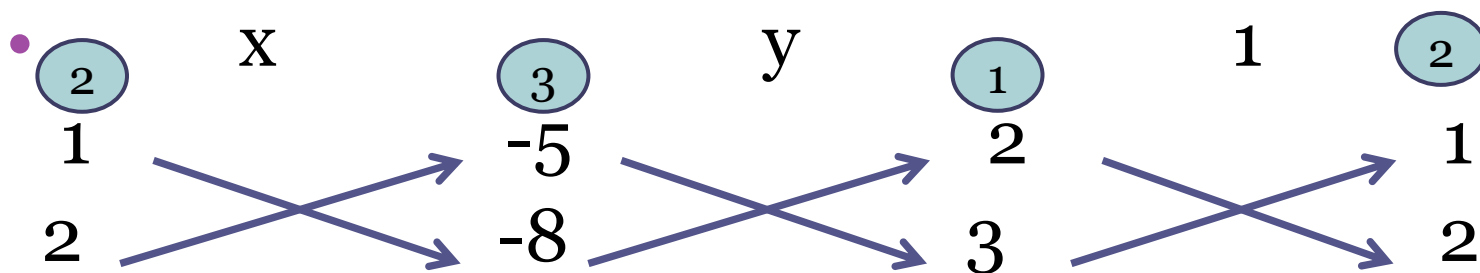
$$\text{Here, } a_1 = 2, b_1 = 1, c_1 = -5$$

$$a_2 = 3, b_2 = 2, c_2 = -8$$

$$\frac{a_1}{a_2} = \frac{2}{3} \text{ and } \frac{b_1}{b_2} = \frac{1}{2}$$

$$\text{Since } \frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

Therefore, the pair of linear equations has a unique solution.



Now by cross-multiplication method, we have:

$$\begin{aligned}
 & \frac{x}{\begin{array}{cc} 1 & -5 \\ 2 & -8 \end{array}} = \frac{y}{\begin{array}{cc} -5 & 2 \\ -8 & 3 \end{array}} = \frac{1}{\begin{array}{cc} 2 & 1 \\ 3 & 2 \end{array}} \\
 \Rightarrow & \frac{x}{-8 - (-10)} = \frac{y}{-15 - (-16)} = \frac{1}{4 - 3} \\
 \Rightarrow & \frac{x}{-8 + 10} = \frac{y}{-15 + 16} = \frac{1}{1} \\
 \Rightarrow & \frac{x}{2} = \frac{y}{1} = \frac{1}{1}
 \end{aligned}$$

Taking $\frac{x}{2} = \frac{1}{1}$ and $\frac{y}{1} = \frac{1}{1}$, we get:

$$x = 2 \text{ and } y = 1$$

Hence, the required solution is **$x = 2$** and **$y = 1$** .

$$\text{(iii) } 3x - 5y - 20 = 0 \quad \dots(1)$$

$$6x - 10y - 40 = 0 \quad \dots(2)$$

$$\text{Here, } a_1 = 3, b_1 = -5, c_1 = -20$$

$$a_2 = 6, b_2 = -10, c_2 = -40$$

$$\frac{a_1}{a_2} = \frac{3}{6} = \frac{1}{2}, \frac{b_1}{b_2} = \frac{-5}{-10} = \frac{1}{2}$$

$$\text{and } \frac{c_1}{c_2} = \frac{-20}{-40} = \frac{1}{2}$$

$$\text{Since } \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

Therefore, the pair of linear equations has
infinitely many solutions.

2. (i) For which values of a and b does the following pair of linear equations have an infinite number of solutions?

$$2x + 3y = 7$$

$$(a - b)x + (a + b)y = 3a + b - 2$$

- Sol: given equations are;

$$2x + 3y = 7$$

$$(a - b)x + (a + b)y = 3a + b - 2$$

Write in standard form

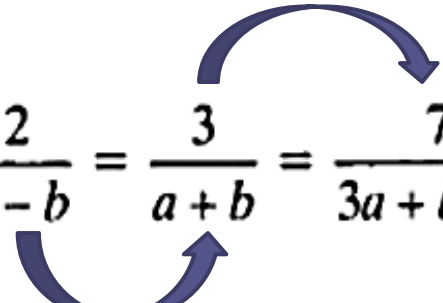
$$2x + 3y - 7 = 0 \dots\dots\dots(i)$$

$$(a - b)x + (a + b)y - (3a + b - 2) = 0 \dots\dots\dots(ii)$$

$$a_1 = 2, \quad b_1 = 3, \quad c_1 = -7,$$

$$a_2 = (a - b), \quad b_2 = (a + b), \quad c_2 = -(3a + b - 2)$$

For infinitely many solutions,

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} \Rightarrow \frac{2}{a - b} = \frac{3}{a + b} = \frac{7}{3a + b - 2}$$


Now, on comparing

$$\frac{2}{a-b} = \frac{3}{a+b} \Rightarrow 2(a+b) = 3(a-b) \Rightarrow 2a + 2b = 3a - 3b$$

$$\Rightarrow a - 5b = 0 \quad \dots (iii)$$

and on comparing

$$\frac{3}{a+b} = \frac{7}{3a+b-2} \Rightarrow 3(3a+b-2) = 7(a+b)$$

$$\Rightarrow 9a + 3b - 6 = 7a + 7b \Rightarrow 2a - 4b = 6$$

$$2a - 4b = 6$$

$$a - 2b = 3 \dots\dots\dots (iv)$$

now, Solve (iii) and (iv) for a and b

By cross multiplication method.

$$\begin{array}{ccccc} & a & & b & & -1 \\ -5 & & 0 & & 1 & & -5 \\ & \swarrow & \searrow & \swarrow & \searrow & \swarrow & \searrow \\ -2 & & 3 & & 1 & & -2 \end{array}$$

$$\frac{a}{-15 - 0} = \frac{b}{0 - 3} = \frac{-1}{-2 + 5} \Rightarrow \frac{a}{-15} = \frac{b}{-3} = \frac{-1}{3} \Rightarrow \boxed{a = 5} \text{ and } \boxed{b = 1}$$

(ii) For which value of k will the following pair of linear equations have no solution?

$$3x + y = 1$$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

- Sol: given equations are;

$$3x + y = 1$$

$$(2k - 1)x + (k - 1)y = 2k + 1$$

Write in standard form

$$3x + y - 1 = 0 \dots\dots\dots(i)$$

$$(2k - 1)x + (k - 1)y - (2k + 1) = 0 \dots\dots\dots(ii)$$

$$a_1 = 3, \quad b_1 = 1, \quad c_1 = -1,$$

$$a_2 = (2k - 1), \quad b_2 = (k - 1), \quad c_2 = -(2k + 1)$$

For no solution ; (parallel lines) $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\frac{3}{2k - 1} = \frac{1}{k - 1} \neq \frac{1}{2k + 1}$$

$$3k - 3 = 2k - 1$$

$$\frac{1}{k - 1} \neq \frac{1}{2k + 1}$$

$$\boxed{k = 2} \text{ and } \boxed{k \neq -2}$$

$$\Rightarrow 3(k - 1) = 2k - 1$$

$$\Rightarrow k = 2$$

$$\Rightarrow 2k + 1 \neq k - 1 \Rightarrow k \neq -2$$

$\Rightarrow$

and

Do it by yourself

3. Solve the following pair of linear equations by the substitution and cross-multiplication methods :

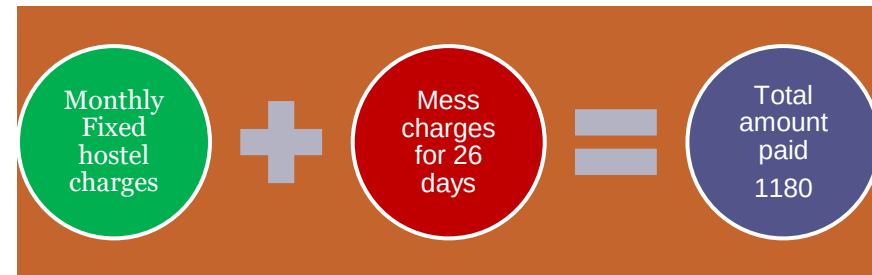
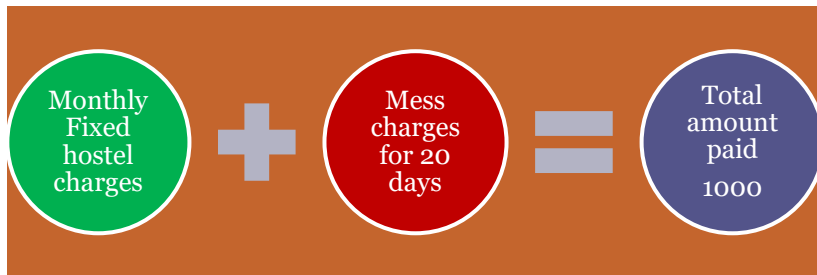
$$8x + 5y = 9$$

$$3x + 2y = 4$$

4. Form the pair of linear equations in the following problems and find their solutions (if they exist) by any algebraic method :

- (i) A part of monthly hostel charges is fixed and the remaining depends on the number of days one has taken food in the mess. When a student A takes food for 20 days she has to pay Rs.1000 as hostel charges whereas a student B, who takes food for 26 days, pays Rs.1180 as hostel charges. Find the fixed charges and the cost of food per day.

Sol:



- Let monthly fixed hostel charges be =x
Food charges(mess charges) per day= y (for one day)
 $x+20y=1000$(i)
 $x+26y=1180$(ii)

By cross multiplication method

$$\begin{array}{ccccccc}
 \textcircled{2} & x & \textcircled{3} & y & \textcircled{1} & -1 & \textcircled{2} \\
 20 & 1000 & 1 & 20 & & & \\
 & \swarrow \searrow & \swarrow \searrow & \swarrow \searrow & & & \\
 26 & 1180 & 1 & 26 & & &
 \end{array}$$

$$\frac{x}{23600 - 26000} = \frac{y}{1000 - 1180} = \frac{-1}{26 - 20}$$

$$\Rightarrow \frac{x}{-2400} = \frac{y}{-180} = \frac{-1}{6} \Rightarrow x = \frac{2400}{6} = 400 \Rightarrow x = 400$$

$$\text{and } y = \frac{180}{6} = 30 \Rightarrow y = 30$$

$\therefore$ Fixed monthly hostel charges = ₹ 400 and charges per day = ₹ 30

Do it by yourself

- (11) A fraction becomes $\frac{1}{3}$ when 1 is subtracted from the numerator and it becomes $\frac{1}{4}$ when 8 is added to its denominator. Find the fraction.