

TOPIC-6

ELIMINATION METHOD

- ▶ **Now let us consider another method of eliminating (i.e., removing) one variable. This is sometimes more convenient than the substitution method. Let us see how this method works.**

(2) ELIMINATION METHOD

- ▶ The main concept behind the elimination method is to create terms with opposite coefficients because they cancel each other when added. In the end, we should deal with a simple linear equation in one variable to solve, like a one-step equation in x or in y .

Steps to solve the pair of linear equations by method of Elimination

Step1: Obtain two equations, and write them both in standard form

$$\begin{cases} a_1x + b_1y + c_1 = 0 \dots \dots (i) \\ a_2x + b_2y + c_2 = 0 \dots \dots (ii) \end{cases}$$

Step2: make the coefficient of one pair of variable terms equal

((First multiply one or both the equations by some suitable non-zero constants (**appropriate number**) to make the coefficients of one variable (either x or y) numerically equal))

step3: Then add or subtract one equation from the other so that one variable gets eliminated. ((If in Step 3, we obtain a true statement involving no variable, then the original pair of equations has infinitely many solutions. If in Step 3, we obtain a false statement involving no variable, then the original pair of equations has no solution, i.e., it is inconsistent.))

Step4: solve the linear equation in one variable (X or Y) obtained in step 3

Step 5: substitute the value of X or Y obtained in step 4 into either of the original equations and solve for the other value.

Step6: check the solution and write the solution set.

For Example

- ▶ Let the two equations be

$$2x + 3y = 5 \dots\dots\dots (i)$$

$$3x - 5y = 10 \dots\dots\dots (ii)$$

If I want to make the coefficient of x equal

Multiply (i) by 3 and (ii) by 2

$$3(2x + 3y = 5) \longrightarrow 6x + 9y = 15$$

$$2(3x - 5y = 10) \longrightarrow 6x - 10y = 20$$

$$\begin{array}{r} 6x + 9y = 15 \\ - \quad \quad \quad + \quad \quad - \\ \hline \end{array}$$

$$19y = -5 \longrightarrow y = -5/19$$

► Let the two equations be

$$2x + 3y = 5 \dots\dots\dots(i)$$

$$3x - 5y = 10 \dots\dots\dots(ii)$$

If I want to make the coefficient of y equal

Multiply (i) by 5 and (ii) by 3

$$5(2x + 3y = 5) \quad \longrightarrow \quad 10x + 15y = 25$$

$$3(3x - 5y = 10) \quad \longrightarrow \quad 9x - 15y = 30$$

$$19x \quad = 55$$

$$x = 55 / 19$$

$$x = 55/19; y = -5/19$$

$$2x + 3y = 5 \dots (i)$$

$$2\left(\frac{55}{19}\right) + 3\left(\frac{-5}{19}\right) = 5$$

$$\frac{110}{19} - \frac{15}{19} = 5$$

$$\frac{95}{19} = 5$$

$$5 = 5$$

$$3x - 5y = 10 \dots (ii)$$

$$3\left(\frac{55}{19}\right) - 5\left(\frac{-5}{19}\right) = 10$$

$$\frac{165}{19} + \frac{25}{19} = 10$$

$$\frac{190}{19} = 10$$

$$10 = 10$$

1) Solve the system using elimination.

$$2x + 2y = 6$$

$$3x - y = 5$$

Multiply the bottom
equation by 2

$$2x + 2y = 6$$

$$\underline{(2)(3x - y = 5)}$$

$$\begin{array}{r} 2x + 2y = 6 \\ (+) \quad 6x - 2y = 10 \\ \hline 8x \quad = 16 \\ x = 2 \end{array}$$

$$2(2) + 2y = 6$$

$$4 + 2y = 6$$

$$2y = 2$$

$$y = 1$$

(2, 1)

Example 12 : Use elimination method to find all possible solutions of the following pair of linear equations :

$$2x + 3y = 8 \dots\dots\dots (1)$$

$$4x + 6y = 7 \dots\dots\dots (2)$$

Solution :

Step 1 : Multiply Equation (1) by 2 and Equation (2) by 1 to make the coefficients of x equal.

Then we get the equations as :

$$4x + 6y = 16 \dots\dots\dots (3)$$

$$4x + 6y = 7 \dots\dots\dots (4)$$

Step 2 : Subtracting Equation (4) from Equation (3),

$$\begin{array}{r} 4x + 6y = 16 \\ 4x + 6y = 7 \\ \hline \end{array}$$

$$0 = 9$$

Step 2 : Subtracting Equation (4) from Equation (3),
(4x - 4x) + (6y - 6y) = 16 - 7 i.e., $0 = 9$, which is a false statement.
Therefore, the pair of equations has no solution.

(v.imp) Ex11 : The ratio of incomes of two persons is 9 : 7 and the ratio of their expenditures is 4 : 3. If each of them manages to save ` 2000 per month, find their monthly incomes.

Solution : Let us denote the incomes of the two person by $9x$ and $7x$ and their expenditures by $4y$ and $3y$ respectively.

Then the equations formed in the situation is given by :

$$9x - 4y = 2000 \quad (1)$$

$$\text{and } 7x - 3y = 2000 \quad (2)$$

Step 1 : Multiply Equation (1) by 3 and Equation (2) by 4 on both sides to make the coefficients of y equal.

$$(9x - 4y = 2000) \times 3$$
$$\text{and } (7x - 3y = 2000) \times 4$$

$$(9x - 4y) \times 3 = (2000) \times 3$$
$$\text{and } (7x - 3y) \times 4 = (2000) \times 4$$

$$\text{Then we get the equations: } 27x - 12y = 6000 \dots\dots\dots(3)$$

$$28x - 12y = 8000 \dots\dots\dots(4)$$

Step 2 : Subtract Equation (3) from Equation (4) to eliminate y, because the coefficients of y are the same. So, we get

$$\begin{array}{r} 27x - 12y = 6000 \\ 28x - 12y = 8000 \\ \hline -x = -2000 \\ x = 2000 \end{array}$$

i.e., $x = 2000$

Step 3 : Substituting this value of x in (1),

we get $9(2000) - 4y = 2000$

i.e., $y = 4000$

(v.imp) **Example 13** : The sum of a two-digit number and the number obtained by reversing the digits is 66. If the digits of the number differ by 2, find the number. How many such numbers are there?

► **Solution** :

Ten's	Unit's
X	Y

- Let the ten's and the unit's digits in the first number be x and y , respectively. So, the original number $10x + y$

After reversing the digits

Ten's	Unit's
Y	X

- When the digits are reversed, x becomes the unit's digit and y becomes the ten's digit. This number, in the expanded notation is $10y + x$

According to the given condition

$$(10x + y) + (10y + x) = 66$$

$$\text{i.e.,} \quad 11(x + y) = 66$$

$$\text{i.e.,} \quad x + y = 6 \quad (1)$$

We are also given that the digits differ by 2,

$$\text{therefore, either } x - y = 2 \quad (2)$$

$$\text{or } y - x = 2 \quad (3)$$

- ▶ If $x - y = 2$, then solving (1) and (2) by elimination, we get $x = 4$ and $y = 2$. In this case, we get the number 42.
- ▶ If $y - x = 2$, then solving (1) and (3) by elimination, we get $x = 2$ and $y = 4$. In this case, we get the number 24.
- ▶ Thus, there are two such numbers 42 and 24.

EXERCISE 3.4

1. Solve the following pair of linear equations by the elimination method and the substitution method :

(i) $x + y = 5$ and $2x - 3y = 4$

(ii) $3x + 4y = 10$ and $2x - 2y = 2$

(iii) $3x - 5y - 4 = 0$ and $9x = 2y + 7$

(iv) $\frac{x}{2} + \frac{2y}{3} = -1$ and $x - \frac{y}{3} = 3$

EXERCISE 3.4

- ▶ 1. Solve the following pair of linear equations by the elimination method and the substitution method :
- ▶ BY ELIMINATION METHOD

(i) $x + y = 5$ and $2x - 3y = 4$

Sol: $x + y = 5$ (i)

and $2x - 3y = 4$(ii)

Multiply (i) by 2

$$2(x + y = 5)$$

$$2x + 2y = 10$$
.....(iii)

Subtract (ii) from (iii)

$$2x + 2y = 10$$

$$2x - 3y = 4$$

$$\begin{array}{r} - \quad + \quad - \\ 2x + 2y = 10 \\ 2x - 3y = 4 \\ \hline 5y = 6 \end{array}$$

$$\text{so, } y = \frac{6}{5}$$

now for x put $y = \frac{6}{5}$ in (i)

$$x + \frac{6}{5} = 5$$

$$x = 5 - \frac{6}{5}$$

$$x = \frac{25-6}{5}$$

$$x = \frac{19}{5}$$

Checking

$$x + y = 5 \dots\dots\dots(i)$$

$$\text{and } 2x - 3y = 4 \dots\dots\dots(ii)$$

$$x = \frac{19}{5} ; y = \frac{6}{5}$$

$$x + y = 5$$

$$\frac{19}{5} + \frac{6}{5} = 5$$

$$\frac{25}{5} = 5$$

$$5 = 5$$

$$2x - 3y = 4$$

$$2\left(\frac{19}{5}\right) - 3\left(\frac{6}{5}\right) = 4$$

$$\frac{38}{5} - \frac{18}{5} = 4$$

$$\frac{20}{5} = 4$$

$$4 = 4$$

By Substitution Method:

Equations are $x + y = 5$... (i)

and $2x - 3y = 4$... (ii)

From equation (i)

$$x = 5 - y$$

Putting this value in equation (ii), we have

$$2(5 - y) - 3y = 4 \Rightarrow 10 - 2y - 3y = 4$$

$$\Rightarrow 10 - 5y = 4 \Rightarrow 6 = 5y \Rightarrow \boxed{y = \frac{6}{5}}$$

Putting the value of y in equation (i), we get

$$x + \frac{6}{5} = 5 \Rightarrow x = 5 - \frac{6}{5} \Rightarrow \boxed{x = \frac{19}{5}}$$

(iv) By elimination method

$$\frac{x}{2} + \frac{2y}{3} = -1$$

$$3x + 4y = -6 \quad (1)$$

$$x - \frac{y}{3} = 3$$

$$3x - y = 9 \quad (2)$$

Subtracting equation (2) from equation (1), we obtain

$$5y = -15 \quad (3)$$

$$y = -3$$

Substituting this value in equation (1), we obtain

$$3x - 12 = -6$$

$$3x = 6$$

$$x = 2$$

Hence, $x = 2$, $y = -3$

By substitution method

From equation (2), we obtain

$$x = \frac{y + 9}{3} \quad (5)$$

Putting this value in equation (1), we obtain

$$3\left(\frac{y + 9}{3}\right) + 4y = -6$$

$$5y = -15$$

$$y = -3$$

Substituting the value in equation (5), we obtain

$$x = \frac{-3 + 9}{3} = 2$$

$$\therefore x = 2, y = -3$$

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces

to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

(ii) Five years ago, Nuri was thrice as old as Sonu. Ten years later, Nuri will be twice as old as Sonu. How old are Nuri and Sonu?

2. Form the pair of linear equations in the following problems, and find their solutions (if they exist) by the elimination method :

(i) If we add 1 to the numerator and subtract 1 from the denominator, a fraction reduces to 1. It becomes $\frac{1}{2}$ if we only add 1 to the denominator. What is the fraction?

Solution: let the numerator be =x
and denominator be =y

Let the fraction be $\frac{x}{y}$.

Then according to the question, we have:

$$\frac{x+1}{y-1} = 1$$

$$\Rightarrow x + 1 = y - 1$$

$$\Rightarrow x - y = -2 \quad \dots(1)$$

$$\text{and } \frac{x}{y+1} = \frac{1}{2}$$

$$\Rightarrow 2x = y + 1$$

$$\Rightarrow 2x - y = 1 \quad \dots(2)$$

Subtracting equation (1) from equation (2), we get:

$$\begin{array}{r} 2x - y = 1 \\ x - y = -2 \\ \hline x = 3 \end{array}$$

Substituting the value of x in equation (1), we get:

$$\begin{aligned} 3 - y &= -2 \\ \Rightarrow y &= 5 \end{aligned}$$

Thus, the fraction is $\frac{3}{5}$.

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