

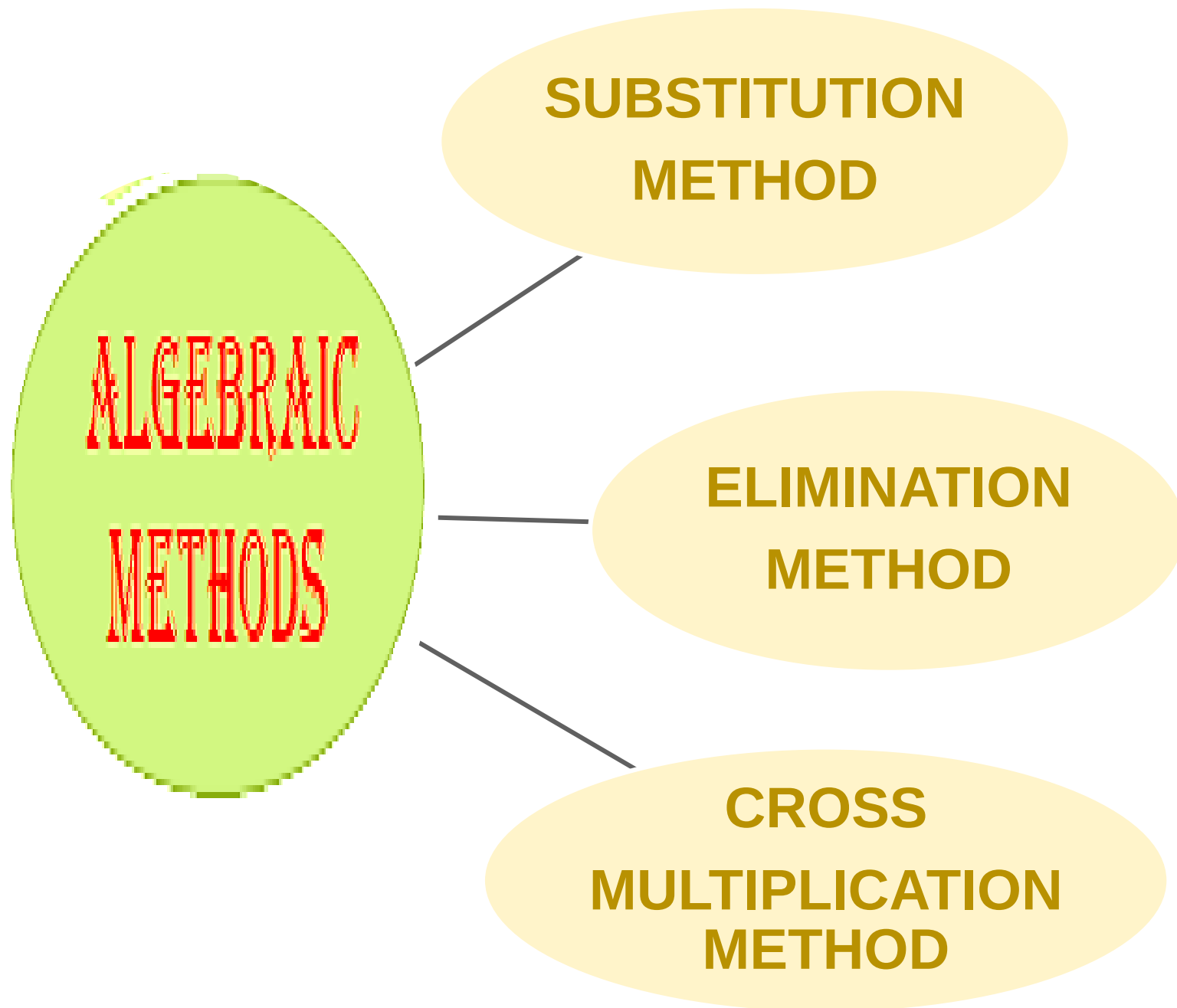
TOPIC-5

ALGEBRAIC METHODS OF
SOLVING A PAIR OF
LINEAR EQUATIONS

In the previous section, we discussed how to solve a pair of linear equations graphically.

The graphical method is not convenient in cases when the point representing the solution of the linear equations has non-integral coordinates like, $(\sqrt{3}, 2\sqrt{7})$, $(\frac{4}{13}, \frac{1}{19})$, $(-1.75, 3.3)$ etc.

There is every possibility of making mistakes while reading such coordinates. Is there any alternative method of finding the solution? There are several algebraic methods, which we shall now discuss.



FOR FINDING SOLUTION FOR CONSISTENT PAIR OF LINEAR EQUATIONS

The solution of a pair of linear equations

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

is of the form of ordered pair (x,y) which satisfies both the equations simultaneously.

Solution for a consistent pair of linear equations can be found out by using

I) SUBSTITUTION METHOD

II) ELIMINATION METHOD

III) CROSS-MULTIPLICATION METHOD

(1) SUBSTITUTION METHOD

The **substitution method** is the algebraic method to solve simultaneous linear equations. As the name says in this method, the value of one variable from one equation is substituted in the other equation.

In this way, a pair of the linear equation gets transformed into linear equation with only one variable, which can be easily solved.

STEPS TO SOLVE THE PAIR OF LINEAR EQUATIONS BY METHOD OF SUBSTITUTION.

Step1: Obtain two equations, and write them both in standard form

$$a_1x + b_1y + c_1 = 0 \dots \dots (i)$$

$$a_2x + b_2y + c_2 = 0 \dots \dots (ii)$$

Step 2: Choose either of the two equations, say (i) and find the value of one variable, say x in terms of y (*whichever is convenient*)

Step 3 : Substitute the value of x, obtained in the previous step in equation (ii) (*in the other equation*) and reduce it to an equation in one variable, i.e., in terms of y, which can be solved .

Step 4 : Solve the linear equation in one variable obtained in the previous step to get the value of y

Step 5 : Substitute the value of y(or x) obtained in Step 4 in any of the given equation (used in Step 1) to obtain the value of the other variable.

Step6: check the solution and write the solution set.

REMARK :

We have substituted the value of one variable by expressing it in terms of the other variable to solve the pair of linear equations. That is why the method is known as the substitution method.

EX : SOLVE THE FOLLOWING PAIR OF EQUATIONS BY SUBSTITUTION METHOD:

$$7X - 15Y = 2 \dots\dots\dots (1)$$

$$X + 2Y = 3 \dots\dots\dots (2)$$

Solution : **Step 1 :** We pick either of the equations and write one variable in terms of the other. Let us consider the Equation (2) :

$x + 2y = 3$ and write it as

$$x = 3 - 2y \dots\dots\dots (3)$$

Step 2 : Substitute the value of x in Equation (1). We get

$$7(3 - 2y) - 15y = 2$$

i.e., $21 - 14y - 15y = 2$

i.e., $-29y = -19$

Therefore, $y = \frac{19}{29}$

Step 3 : Substituting this value of y in Equation (3), we get

$$x = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$$

Therefore, the solution is $x = \frac{49}{29}$, $y = \frac{19}{29}$.

EXAMPLE 8 : SOLVE Q.1 OF EXERCISE 3.1 BY THE METHOD OF SUBSTITUTION. AFTAB TELLS HIS DAUGHTER, “SEVEN YEARS AGO, I WAS SEVEN TIMES AS OLD AS YOU WERE THEN. ALSO, THREE YEARS FROM NOW, I SHALL BE THREE TIMES AS OLD AS YOU WILL BE”.

Solution: Let the present age of Aftab and his daughter be x and y respectively.

Seven years ago, Age of Aftab = $x - 7$ and

Age of his daughter = $y - 7$

According to the first condition:

$$x - 7 = 7(y - 7)$$

$$x - 7 = 7y - 49$$

$$x - 7y = -42$$

$$x - 7y + 42 = 0 \quad \dots(i)$$

Again, Three years hence, Age of Aftab = $x + 3$ and Age of his daughter = $y + 3$

According to second given condition,

$$x + 3 = 3(y + 3)$$

$$x + 3 = 3y + 9$$

$$x - 3y = 6 \quad \dots(ii)$$

Using Equation (ii), we get $x = 3y + 6$.

Putting this value of x in Equation (i),

we get $(3y + 6) - 7y + 42 = 0$,

i.e., $4y = 48$

$$y = 12$$

Putting this value of y in Equation (ii),

we get $x = 3(12) + 6 = 42$

So, Aftab and his daughter are 42 and 12 years old, respectively.

Verify this answer by checking if it satisfies the conditions of the given problems.

EXAMPLE 9 : LET US CONSIDER EXAMPLE 2 IN SECTION 3.3, I.E., THE COST OF 2 PENCILS AND 3 ERASERS IS ` 9 AND THE COST OF 4 PENCILS AND 6 ERASERS IS ` 18. FIND THE COST OF EACH PENCIL AND EACH ERASER.

Solution : Let us denote the cost of 1 pencil by x and one eraser by y . Then the algebraic representation is given by the following equations:

$$2x + 3y = 9 \quad (1)$$

$$4x + 6y = 18 \quad (2)$$

We first express the value of x in terms of y from the equation

$2x + 3y = 9$, to get

$$x = \frac{9 - 3y}{2} \quad (3)$$

Now we substitute this value of x in Equation (2), to get

$$\frac{4(9 - 3y)}{2} + 6y = 18$$

$$\text{i.e.,} \quad 18 - 6y + 6y = 18$$

$$\text{i.e.,} \quad 18 = 18$$

REMARK :

This statement is true for all values of y . However, we do not get a specific value of y as a solution. Therefore, we cannot obtain a specific value of x .

This situation has arisen because both the given equations are the same. Therefore, Equations (1) and (2) have infinitely many solutions.

Observe that we have obtained the same solution graphically also. (on the next slide)

We cannot find a unique cost of a pencil and an eraser, because there are many common solutions, to the given situation.

EXAMPLE : ROMILA WENT TO A STATIONERY SHOP AND PURCHASED 2 PENCILS AND 3 ERASERS FOR 9. HER FRIEND SONALI SAW THE NEW VARIETY OF PENCILS AND ERASERS WITH ROMILA, AND SHE ALSO BOUGHT 4 PENCILS AND 6 ERASERS OF THE SAME KIND FOR 18. REPRESENT THIS SITUATION ALGEBRAICALLY AND GRAPHICALLY.

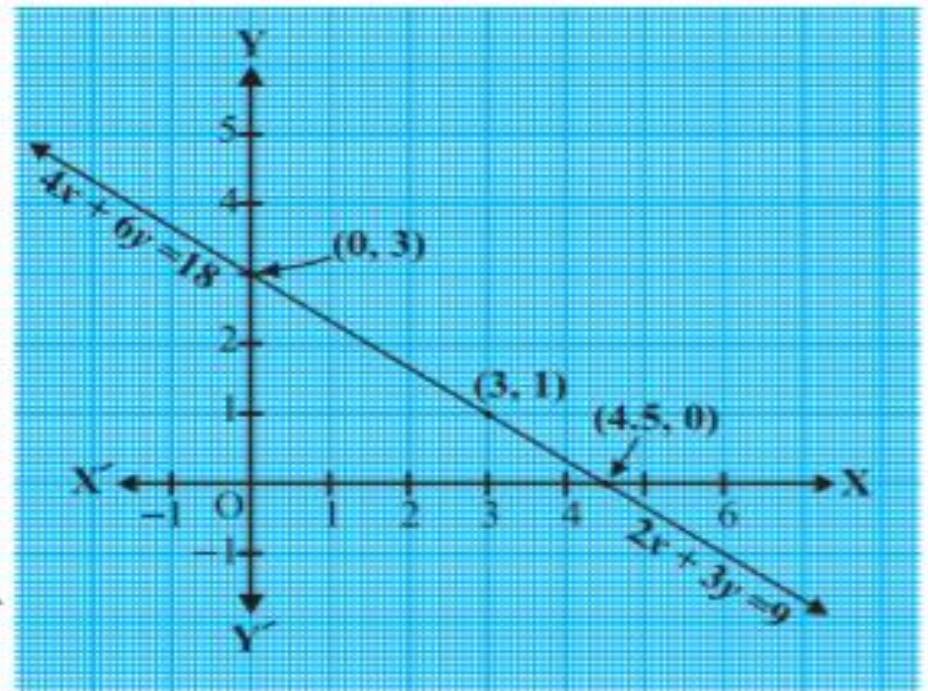
Solution : Let us denote the cost of 1 pencil by x and one eraser by y . Then the algebraic representation is given by the following equations:

$$2x + 3y = 9 \quad (1)$$

$$4x + 6y = 18 \quad (2)$$

x	0	4.5
$y = \frac{9 - 2x}{3}$	3	0

x	0	3
$y = \frac{18 - 4x}{6}$	3	1



EXAMPLE 10 : TWO RAILS ARE REPRESENTED BY THE EQUATIONS $X + 2Y - 4 = 0$ AND $2X + 4Y - 12 = 0$. REPRESENT THIS SITUATION GEOMETRICALLY

Solution : The pair of linear equations formed were:

$$x + 2y - 4 = 0 \dots\dots\dots (1)$$

$$2x + 4y - 12 = 0 \dots\dots\dots (2)$$

We express x in terms of y from Equation (1) to get

$$x = 4 - 2y$$

Now, we substitute this value of x in Equation (2) to get

$$2(4 - 2y) + 4y - 12 = 0 \text{ i.e., } 8 - 12 = 0 \text{ i.e., } -4 = 0$$

which is a false statement.

Therefore, the equations do not have a common solution.
So, the two rails will not cross each other.

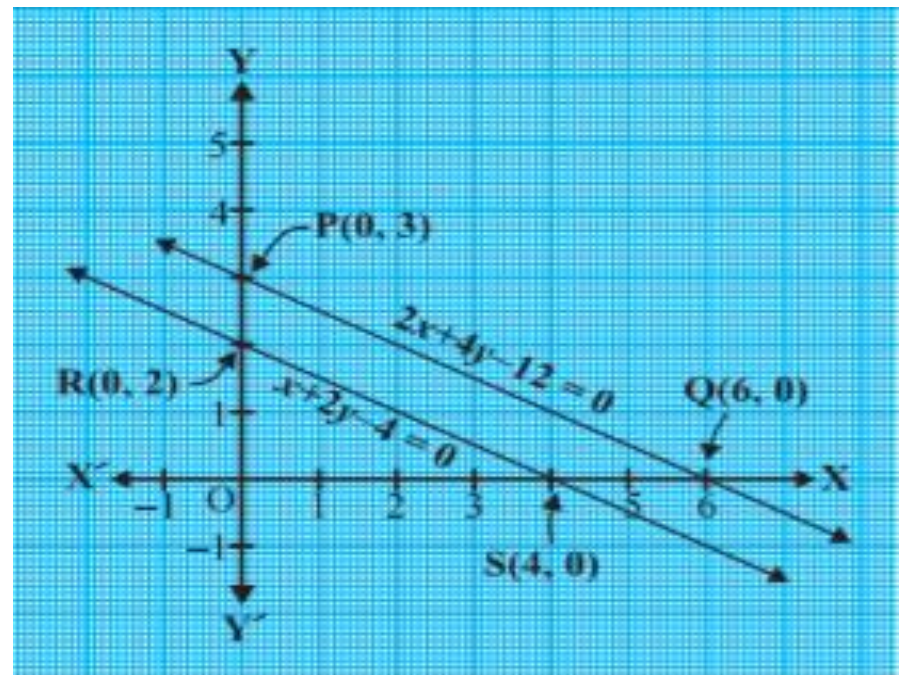
EXAMPLE 3 : TWO RAILS ARE REPRESENTED BY THE EQUATIONS $X + 2Y - 4 = 0$ AND $2X + 4Y - 12 = 0$. REPRESENT THIS SITUATION GEOMETRICALLY

Solution : Two solutions of each of the equations

$$x + 2y - 4 = 0 ; 2x + 4y - 12 = 0$$

x	0	4
$y = \frac{4-x}{2}$	2	0

x	0	6
$y = \frac{12-2x}{4}$	3	0



EXERCISE 3.3

1. Solve the following pair of linear equations by the substitution method.

(i) $x + y = 14$

$$x - y = 4$$

(iii) $3x - y = 3$

$$9x - 3y = 9$$

(v) $\sqrt{2}x + \sqrt{3}y = 0$

$$\sqrt{3}x - \sqrt{8}y = 0$$

(ii) $s - t = 3$

$$\frac{s}{3} + \frac{t}{2} = 6$$

(iv) $0.2x + 0.3y = 1.3$

$$0.4x + 0.5y = 2.3$$

(vi) $\frac{3x}{2} - \frac{5y}{3} = -2$

$$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$$

2. Solve $2x + 3y = 11$ and $2x - 4y = -24$ and hence find the value of ' m ' for which $y = mx + 3$.
3. Form the pair of linear equations for the following problems and find their solution by substitution method.
 - (i) The difference between two numbers is 26 and one number is three times the other. Find them.
 - (ii) The larger of two supplementary angles exceeds the smaller by 18 degrees. Find them.
 - (iii) The coach of a cricket team buys 7 bats and 6 balls for c 3800. Later, she buys 3 bats and 5 balls for c 1750. Find the cost of each bat and each ball.