

TOPIC 3

GRAPHICAL METHOD OF

FINDING SOLUTION OF A PAIR

OF LINEAR

EQUATIONS IN TWO

VARIABLES

SOLUTION OF PAIR OF LINEAR EQUATION IN TWO VARIABLES

- The value of variable which satisfies an equation is called its solution;
- Two linear equations in two variables when taken together are called simultaneous linear equations or system of linear equations or pair of linear equations in two variables
- $a_1x + b_1y + c_1 = 0$ and
 $a_2x + b_2y + c_2 = 0$

and solution of pair of linear equations in two variables is the ordered pair (x,y) which satisfies both the equations

METHODS OF SOLVING A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

- To solve a pair of linear equations in two variables we have two methods
 - Graphical method
 - Algebraic method.

GRAPHICAL METHOD OF FINDING SOLUTION OF A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

Step 1

Plot both the equations (two straight lines)

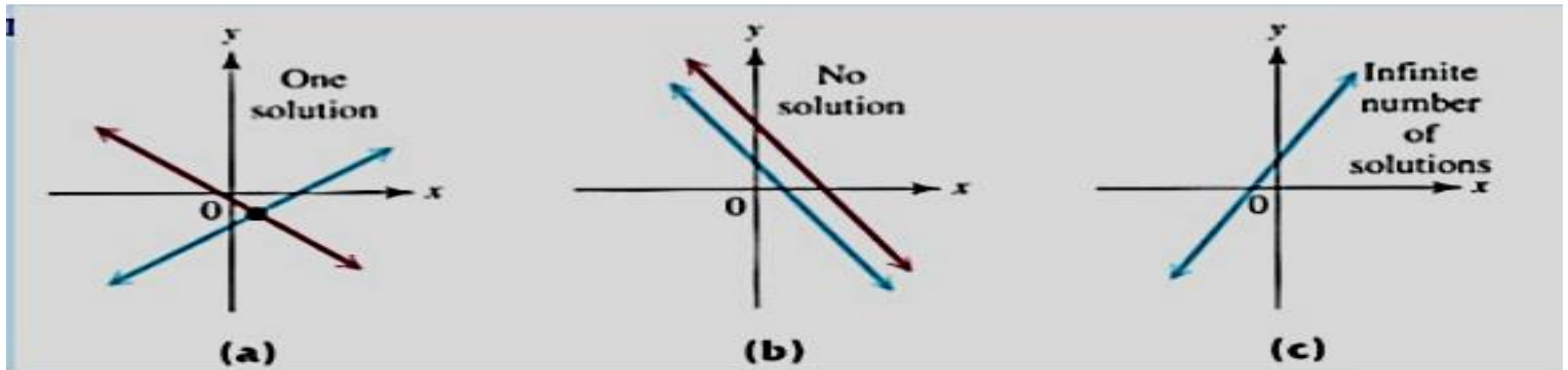
$$a_1x + b_1y + c_1 = 0 \text{ and}$$

$$a_2x + b_2y + c_2 = 0$$

In same Cartesian plane.

Step 2

Determine whether the lines intersect, are parallel, or are the same line (coincident)



Step 3

Identify the solution to the system.

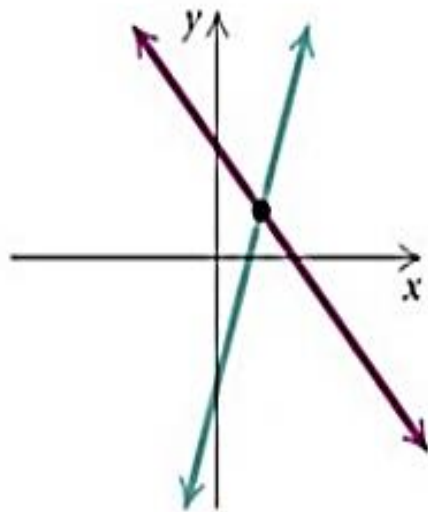
- If the lines intersect, identify the point of intersection. This is the solution to the system.
- If the lines are parallel, the system has no solution.
- If the lines are the same (identical) / (coincident) , the system has an infinite number of solutions.

RESULTS

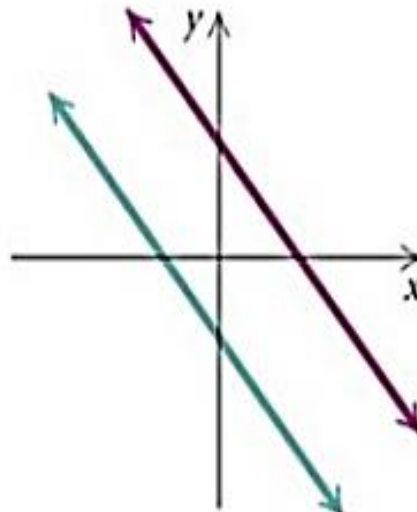
- (i) The lines may intersect in a single point. In this case, the pair of equations has a unique solution and it is called **consistent pair of equations**.
- (ii) The lines may be parallel. In this case, the equations have no solution and it is called **inconsistent pair of equations**.
- (iii) The lines may be coincident. In this case, the equations have infinitely many solutions and it is called **dependent (consistent) pair of equations**.

REMEMBER IT: ((A consistent system of equations is a system that has at least one solution or infinitely many solutions. An inconsistent system of equations is a system that has no solution))

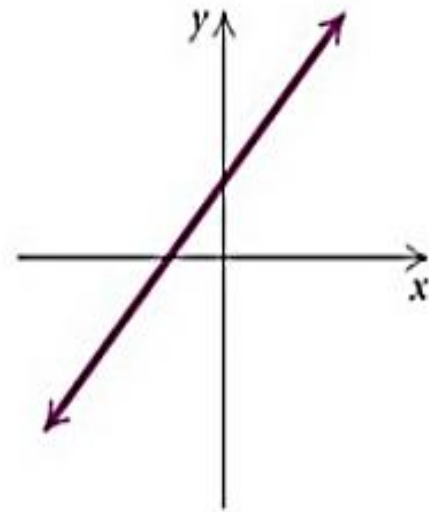
GRAPHICALLY



Exactly one common point
One solution



Parallel lines
No common points
No solution



Lines are identical.
Infinitely many common points
Infinitely many solutions

TOPIC 4 (V.IMP)

CONDITIONS OF SOLVABILITY OF PAIR OF LINEAR EQUATIONS

COMPARING THE RATIOS OF COEFFICIENTS OF A PAIR OF LINEAR EQUATIONS IN TWO VARIABLES

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

CONDITION	TYPE OF SOLUTION	GRAPHICAL REPRESENTATION	TYPE OF SYSTEM
$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Unique solution	Intersecting lines	consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Infinitely many solutions	Coincident lines	consistent
$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	No solution	Parallel lines	inconsistent

EXAMPLES

TABLE 3.4

Sl No.	Pair of lines	$\frac{a_1}{a_2}$	$\frac{b_1}{b_2}$	$\frac{c_1}{c_2}$	Compare the ratios	Graphical representation	Algebraic interpretation
1.	$x - 2y = 0$ $3x + 4y - 20 = 0$	$\frac{1}{3}$	$\frac{-2}{4}$	$\frac{0}{-20}$	$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$	Intersecting lines	Exactly one solution (unique)
2.	$2x + 3y - 9 = 0$ $4x + 6y - 18 = 0$	$\frac{2}{4}$	$\frac{3}{6}$	$\frac{-9}{-18}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$	Coincident lines	Infinitely many solutions
3.	$x + 2y - 4 = 0$ $2x + 4y - 12 = 0$	$\frac{1}{2}$	$\frac{2}{4}$	$\frac{-4}{-12}$	$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$	Parallel lines	No solution

POINTS TO BE REMEMBER

- (i) if $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$, the pair of equations are said to be consistent . Graph of the two equations intersect at unique point. The pair of linear equations have exactly one solution.
- (ii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$, the equations are said to be dependent . One equation can be obtained by multiplying the other equation with a non zero constant.. In this case graph of both the equations will coincide . Dependent equations are consistent. This pair of linear equations have infinitely many solutions.
- (iii) If $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$, the equations are said to be inconsistent. The graph of the equations are parallel to each other. The pair of equations have no solution

Ex: Check graphically whether the pair of equations

$$x + 3y = 6 \quad (1)$$

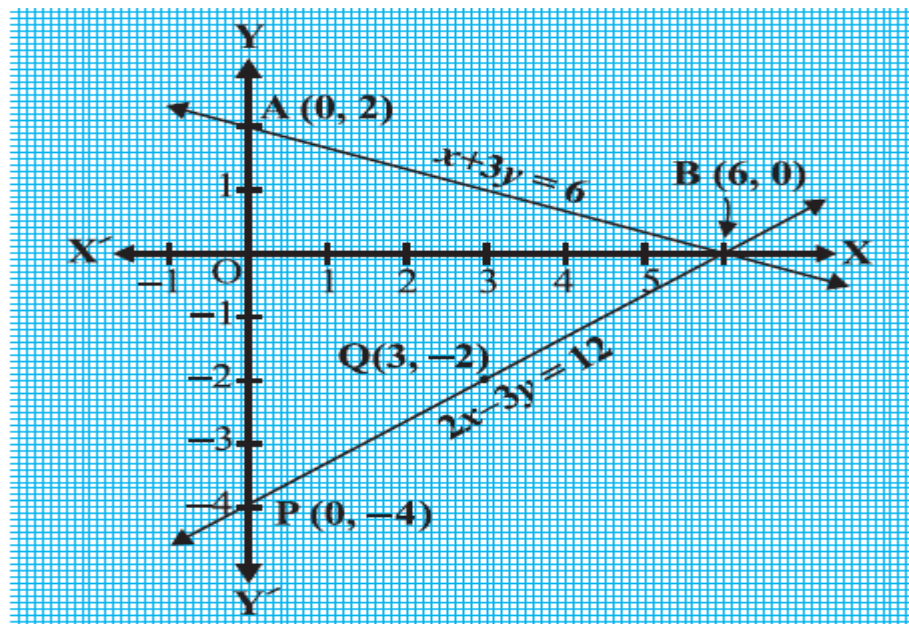
$$2x - 3y = 12 \quad (2)$$

is consistent. If so, solve them graphically.

Solution : Let us draw the graphs of the Equations (1) and (2). For this, we find two solutions of each of the equations, which are given .We observe that there is a point B (6, 0) common to both the lines AB and PQ. So, the solution of the pair of linear equations is $x = 6$ and $y = 0$, i.e., the given pair of equations is consistent.

x	0	6
$y = \frac{6-x}{3}$	2	0

x	0	3
$y = \frac{2x-12}{3}$	-4	-2



Example 5 : Graphically, find whether the following pair of equations has no solution, unique solution or infinitely many solutions:

$$5x - 8y + 1 = 0 \quad \dots\dots\dots (1)$$

$$3x - \frac{24}{5}y + \frac{3}{5} = 0 \quad \dots\dots\dots (2)$$

Solution : Multiplying Equation (2) by $\frac{5}{3}$

we get $5x - 8y + 1 = 0$ But, this is the same as Equation (1). Hence the lines represented by Equations (1) and (2) are coincident. Therefore, Equations (1) and (2) have infinitely many solutions. Plot few points on the graph and verify it yourself.

Example 6 : Champa went to a ‘Sale’ to purchase some pants and skirts. When her friends asked her how many of each she had bought, she answered, “The number of skirts is two less than twice the number of pants purchased. Also, the number of skirts is four less than four times the number of pants purchased”. Help her friends to find how many pants and skirts Champa bought.

Solution: Let us denote the number of pants by x and the number of skirts by y. Then the equations formed are :

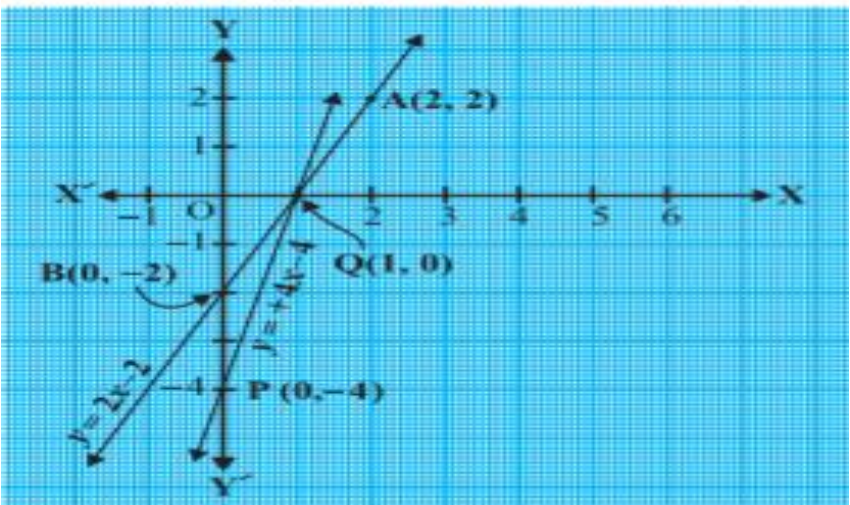
$y = 2x - 2$ (1)

$y = 4x - 4$ (2)

Let us draw the graphs of Equations (1) and (2) by finding two solutions for each of the equations. They are given in Table

x	2	0
$y = 2x - 2$	2	-2

x	0	1
$y = 4x - 4$	-4	0



The two lines intersect at the point (1, 0). So, $x = 1$, $y = 0$ is the required solution of the pair of linear equations, i.e., the number of pants she purchased is 1 and she did not buy any skirt.

- Verify the answer by checking whether it satisfies the conditions of the given problem.

$$y = 2x - 2$$

$$\text{put } x = 1, y = 0$$

$$0 = 2(1) - 2$$

$$0 = 2 - 2$$

$$0 = 0$$

$$\text{LHS} = \text{RHS}$$

Hence verified.

$$y = 4x - 4$$

$$\text{put } x = 1, y = 0$$

$$0 = 4(1) - 4$$

$$0 = 4 - 4$$

$$0 = 0$$

$$\text{LHS} = \text{RHS}$$

EXERCISE 3.2

1. Form the pair of linear equations in the following problems, and find their solutions graphically.

(i) 10 students of Class X took part in a Mathematics quiz. If the number of girls is 4 more than the number of boys, find the number of boys and girls who took part in the quiz.

(ii) 5 pencils and 7 pens together cost ₹ 50, whereas 7 pencils and 5 pens together cost ₹ 46. Find the cost of one pencil and that of one pen.

2. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the lines representing the

following pairs of linear equations intersect at a point, are parallel or coincident:

(i) $5x - 4y + 8 = 0$

$$7x + 6y - 9 = 0$$

(ii) $9x + 3y + 12 = 0$

$$18x + 6y + 24 = 0$$

(iii) $6x - 3y + 10 = 0$

$$2x - y + 9 = 0$$

3. On comparing the ratios $\frac{a_1}{a_2}$, $\frac{b_1}{b_2}$ and $\frac{c_1}{c_2}$, find out whether the following pair of linear equations are consistent, or inconsistent.

(i) $3x + 2y = 5$; $2x - 3y = 7$

(ii) $2x - 3y = 8$; $4x - 6y = 9$

(iii) $\frac{3}{2}x + \frac{5}{3}y = 7$; $9x - 10y = 14$

(iv) $5x - 3y = 11$; $-10x + 6y = -22$

(v) $\frac{4}{3}x + 2y = 8$; $2x + 3y = 12$

Solutions of previous assignment(Ex-3.1)

Question 1.

Aftab tells his daughter, “Seven years ago, I was seven times as old as you were then. Also, three years from now, I shall be three times as old as you will be”. Isn’t this interesting? Represent this situation algebraically and graphically.

Solution:

Let the present age of Aftab’s daughter be x years and that of Aftab be y years.

Then, seven years ago:

Aftab’s age was $(y - 7)$ years

and his daughter’s age was $(x - 7)$ years.

- According to the first condition:

$$y - 7 = 7(x - 7)$$

$$\Rightarrow y - 7 = 7x - 49$$

$$\Rightarrow 7x - y = 42 \quad \dots(i)$$

Five years later:

Aftab's age will be $(y + 5)$ years,

and his daughter's age will be $(x + 5)$ years

According to the second condition:

$$y + 5 = 3(x + 5)$$

$$\Rightarrow y + 5 = 3x + 15$$

$$\Rightarrow 3x - y = -10 \quad \dots(ii) \text{ For graphical representation:}$$

From equation (i), we have:

$$y = 7x - 42$$

$$\text{When } x = 11, \text{ then } y = 7 \times 11 - 42 = 35$$

$$\text{When } x = 12, \text{ then } y = 7 \times 12 - 42 = 42$$

$$\text{When } x = 13, \text{ then } y = 7 \times 13 - 42 = 49$$

- Thus, we have the following table of points

x	11	12	13
y	35	42	49

- From equation (ii), we have:

$$y = 3x + 10$$

$$\text{When } x = 9, \text{ then } y = 3 \times 9 + 10 = 37$$

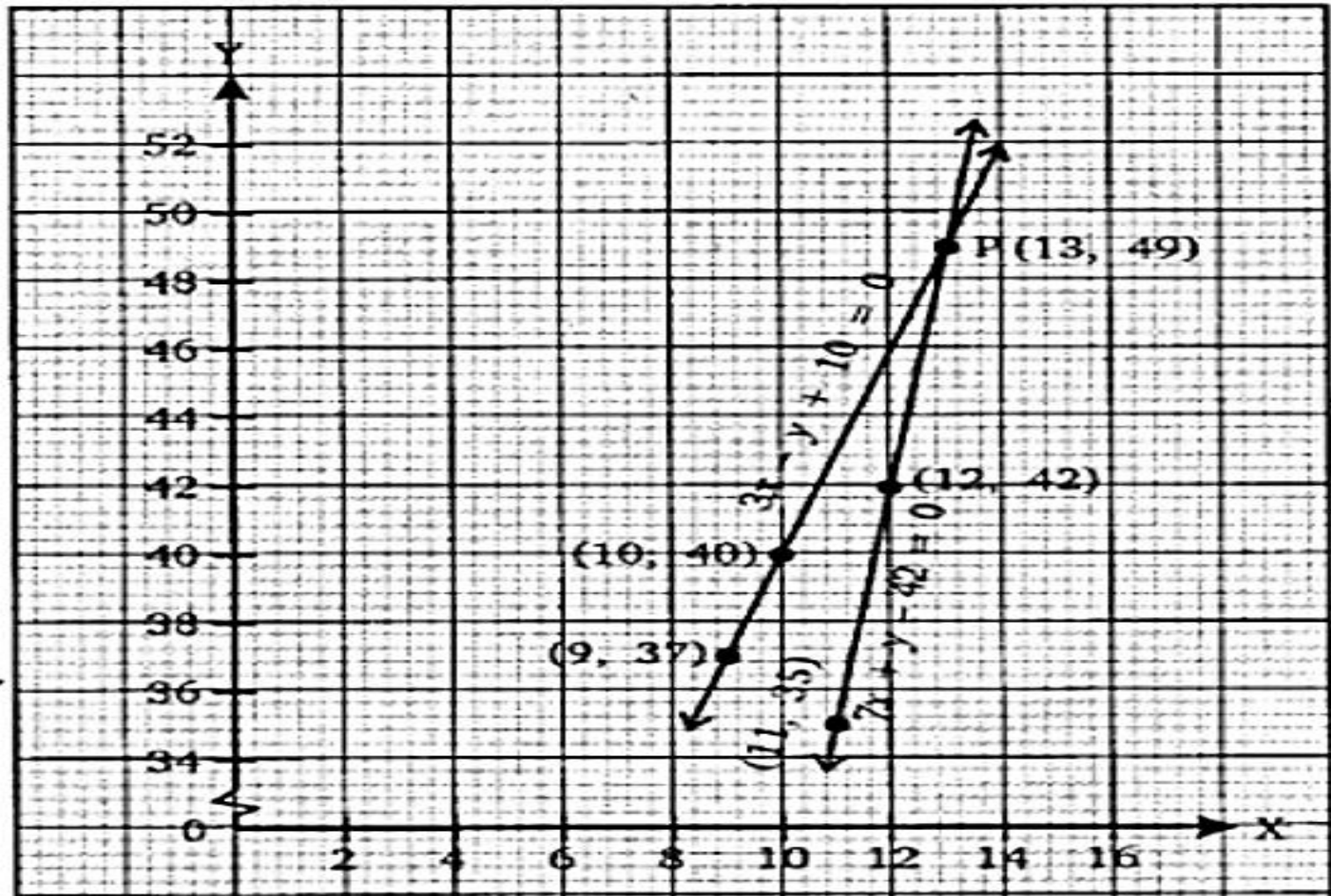
$$\text{When } x = 10, \text{ then } y = 3 \times 10 + 10 = 40$$

$$\text{When } x = 13, \text{ then } y = 3 \times 13 + 10 = 49$$

Thus, we have the following table of points:

x	9	10	13
y	37	40	49

Plotting the points of each table, we obtain the required graph of two intersecting lines.



Question 2.

The coach of a cricket team buys 3 bats and 6 balls for ₹ 3900. Later, she buys another bat and 3 more balls of the same kind for ₹1300. Represent this situation algebraically and geometrically.

• Solution:

Let the cost of 1 bat be ₹ x and the cost of 1 ball be ₹ y .

Then according to the question, we have:

$$3x + 6y = 3900 \quad \dots(i)$$

$$x + 3y = 1300 \quad \dots(ii)$$

For geometrical representation:

From equation (i), we have: $y = \frac{3900 - 3x}{6}$

When $x = 100$, then $y = \frac{3900 - 300}{6} = 600$

When $x = 300$, then $y = \frac{3900 - 900}{6} = 500$

When $x = 700$, then $y = \frac{3900 - 2100}{6} = 300$

Thus, we have the following table of points.

x	100	300	700
y	600	500	300

From equation (ii), we have: $y = \frac{1300 - x}{3}$

When $x = 100$, then $y = \frac{1300 - 100}{3} = 400$

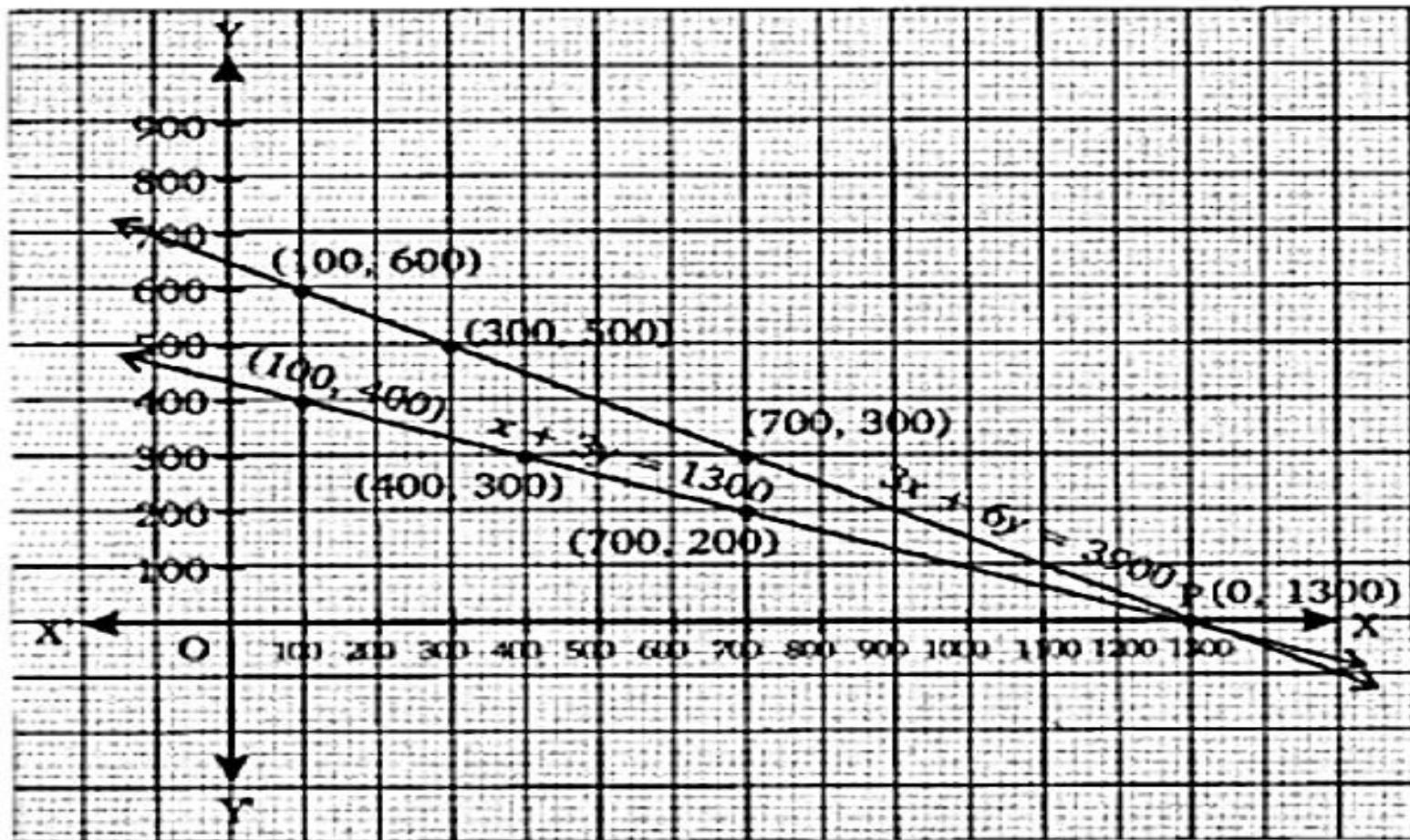
When $x = 400$, then $y = \frac{1300 - 400}{3} = 300$

When $x = 700$, then $y = \frac{1300 - 700}{3} = 200$

Thus, we have the following table of points:

x	100	400	700
y	400	300	200

Plotting the points of each table, we obtain the required graph of two intersecting lines.



Question 3.

The cost of 2 kg of apples and 1 kg of grapes on a day was found to be ₹160. After a month, the cost of 4 kg of apples and 2 kg of grapes is ₹300. Represent the situation algebraically and geometrically.

- **Solution:**

Let cost of 1 kg apples be ₹ x and the cost of 1 kg grapes be ₹ y .

Then according to the question, we have:

$$2x + y = 160 \quad \dots(i)$$

$$4x + 2y = 300 \quad \dots(ii)$$

For geometrical representation:

From equation (i), we have: $y = -2x + 160$

When $x = 50$, then $y = -2 \times 50 + 160 = 60$

When $x = 20$, then $y = -2 \times 20 + 160 = 120$

When $x = 45$, then $y = -2 \times 45 + 160 = 70$

Thus, we have the following table of points:

x	50	20	45
y	60	120	70

From equation (ii), we have: $y = \frac{300 - 4x}{2}$

When $x = 50$, then $y = \frac{300 - 200}{2} = \frac{100}{2} = 50$

When $x = 30$, then $y = \frac{300 - 120}{2} = \frac{180}{2} = 90$

When $x = 70$, then $y = \frac{300 - 280}{2} = \frac{20}{2} = 10$

Thus, we have the following table of points:

x	50	30	70
y	50	90	10

Plotting the points of each table on a graph paper, we obtain the required graph of two parallel lines.

