

CHAPTER- 1

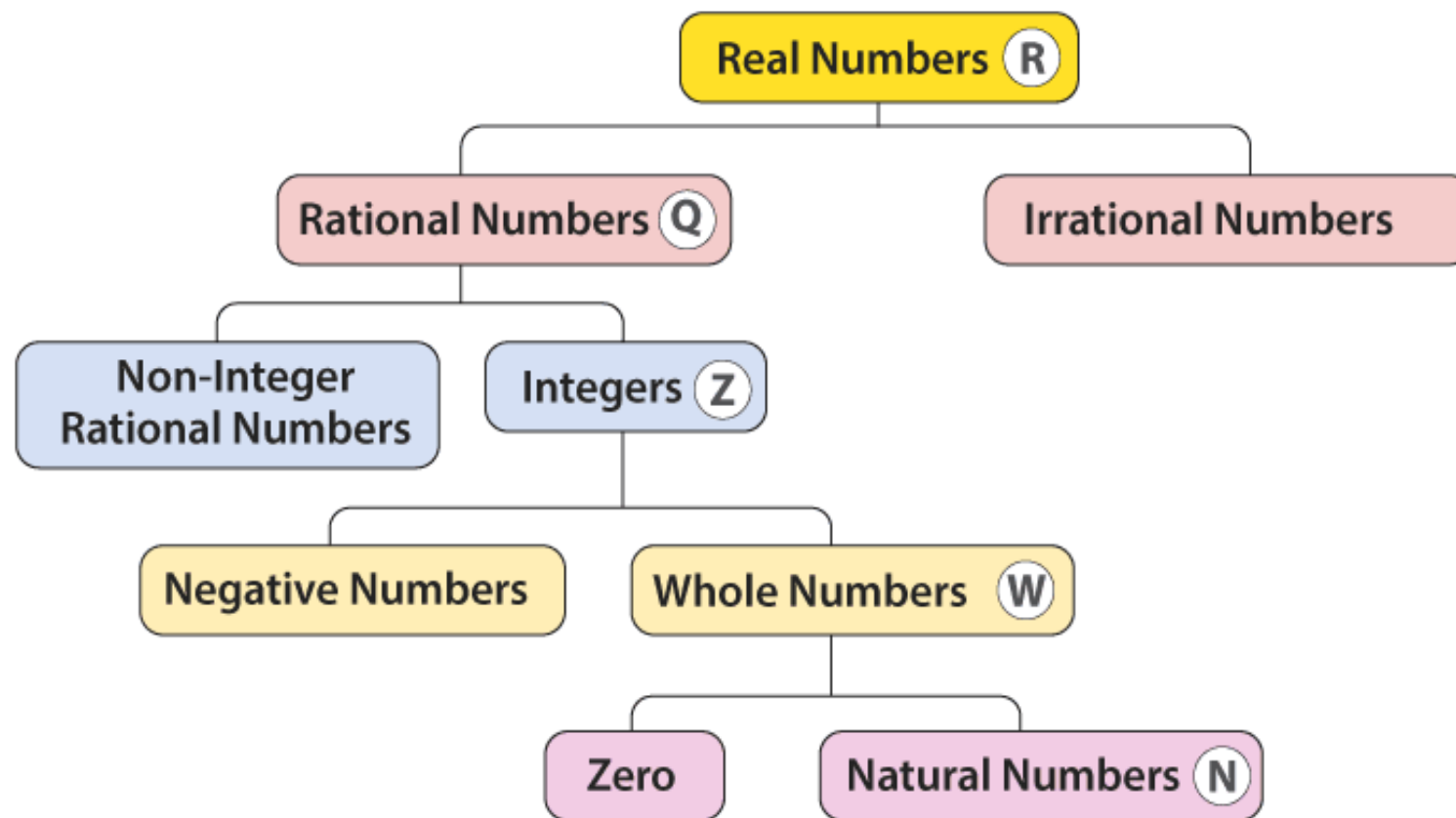
REAL NUMBERS

CLASS- X
MATHEMATICS



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REAL NUMBERS CHART



PRIME AND COMPOSITE NUMBERS

- › **Prime Number:** A prime number is a natural number greater than 1 that has no positive divisors other than 1 and the number itself.
 - › E.g. 3, 7, 11 etc
- › **Composite Number:** A natural number greater than 1 that is not a prime number is called a composite number.
 - › E.g. 4, 6, 9 etc

RATIONAL AND IRRATIONAL NUMBERS

- › **Rational Numbers:** Numbers that can be represented in the form of p/q where p & q are Integers & $q \neq 0$ are called Rational Numbers.
 - › It is denoted by Q .
 - › E.g.: $\frac{1}{2}$, $\frac{9}{5}$ etc.
- › **Irrational Numbers :** Numbers that can't be written in the form of p/q where p & q are integers & $q \neq 0$ are called irrational Numbers.
 - › E.g. : $\sqrt{2}$, $\sqrt{3}$, π etc.

REAL NUMBERS

- › **Real number:** Both Rational & Irrational Numbers together form Real number.
- › It is denoted by R.
- › NOTE- Every point on the number line is a Real number.
- › E.g.: $\sqrt{2}$, -7, $\frac{4}{9}$, 0, 5 etc.

Euclid's Division Lemma:

For any given positive integers a and b , there exists unique integers q and r such that $a = bq + r$ where $0 \leq r < b$

Note: If b divides a , then $r = 0$

Example: For $a = 15$, $b = 3$, it can be observed that

$$15 = 3 \times 5 + 0$$

Here, $q = 5$ and $r = 0$

If b does not divide a , then $0 < r < b$

Example: For $a = 20$, $b = 6$, it can be observed that $20 = 6 \times 3 + 2$

Here, $q = 3$, $r = 2$, $0 < 2 < 6$

EUCLID'S DIVISION ALGORITHM

- › Euclid's division algorithm deals with divisibility of integers.
- › It says any positive integer 'a' can be divided by another positive integer 'b' in such a way that it leaves a remainder r that is smaller than b.
- › It is mainly used to compute the HCF of two positive integers.

Example : Find HCF of 455 and 42 using Euclid's Division Algorithm

- › We start with the larger integer, that is, 455. Then we use Euclid's lemma to get
- › $455 = 42 \times 10 + 35$
- › Now consider the divisor 42 and the remainder 35, and apply the division lemma to get
- › $42 = 35 \times 1 + 7$
- › Now consider the divisor 35 and the remainder 7, and apply the division lemma to get
- › $35 = 7 \times 5 + 0$
- › Notice that the remainder has become zero, and we cannot proceed any further.
- › **HCF of 455 and 42 is 7.**

Example: Show that every positive even integer is of the form $2q$, and that every positive odd integer is of the form $2q + 1$, where q is some integer.

- › **Solution:** Let a be any positive integer and $b = 2$. Then, by Euclid's algorithm, $a = 2q + r$, for some integer $q \geq 0$, and $r = 0$ or $r = 1$, because $0 \leq r < 2$. So, $a = 2q$ or $2q + 1$.
- › If a is of the form $2q$, then a is an even integer. Also, a positive integer can be either even or odd. Therefore, any positive odd integer is of the form $2q + 1$.

FUNDAMENTAL THEOREM OF ARITHMETIC

- › The Fundamental Theorem of Arithmetic deals with multiplication of positive integers.
- › It states that ‘Every composite number can be expressed as a product of primes in a unique way.’
- › It is used to prove the irrationality of many of the numbers & to explore when exactly the decimal expansion of a rational number.

Example: Find the LCM and HCF of 6 and 20 by the prime factorisation method.

› **Solution:** We have :

$$6 = 2^1 \times 3^1 \text{ and } 20 = 2 \times 2 \times 5 = 2^2 \times 5^1.$$

› $\text{HCF}(6, 20) = 2^1 = 2$

› HCF is Product of the smallest power of each common prime factor in the numbers.

› $\text{LCM}(6, 20) = 2^2 \times 3^1 \times 5^1 = 60$

› LCM is Product of the greatest power of each prime factor, involved in the numbers.

THEOREM

Let p be a prime number. If p divides a^2 , then p divides a , where a is a positive integer.

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PROVING $\sqrt{2}$ IS IRRATIONAL

Suppose $\sqrt{2}$ is **rational**. That means it can be written as the ratio of two integers p and q

$$\sqrt{2} = \frac{p}{q} \tag{1}$$

where we may assume that p and q **have no common factors**. (If there are any common factors we cancel them in the numerator and denominator.) Squaring in (1) on both sides gives

$$2 = \frac{p^2}{q^2} \tag{2}$$

which implies

$$p^2 = 2q^2 \tag{3}$$

Thus p^2 is even. The only way this can be true is that p itself is even. But then p^2 is actually divisible by 4. Hence q^2 and therefore q must be even. So p and q are both even which is a contradiction to our assumption that they have no common factors. The square root of 2 cannot be rational!

Rational Numbers & Decimal Expression

- › Rational numbers have either a terminating decimal expansion or a non-terminating repeating decimal expansion.
- › **Theorem:** Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form $\frac{p}{q}$ where p and q are coprime, and the prime factorisation of q is of the form $2^n 5^m$, where n, m are non-negative integers
- › **Theorem :** Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is of the form $2^n 5^m$, where n, m are non-negative integers. Then x has a decimal expansion which terminates
- › **Theorem :** Let $x = \frac{p}{q}$ be a rational number, such that the prime factorization of q is not of the form $2^n 5^m$, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).