

Chapter-1

Exercise-1.4

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Class: X

Subject: Mathematics

The decimal expansion of every rational number is either terminating or non-terminating repeating.

Theorem: Let x be a rational number whose decimal expansion terminates. Then x can be expressed in the form p/q where p and q are coprime, and the prime factorization of q is of the form $2^m \times 5^n$, where n, m are non-negative integers.

Theorem: Let $x = p/q$, where p and q are co-primes, be a rational number, such that the prime factorization of q is not of the form $2^m \times 5^n$, where n, m are non-negative integers. Then, x has a decimal expansion which is non-terminating repeating (recurring).

1. Without actually performing the long division, state whether the following rational numbers will have a terminating decimal expansion or a non-terminating decimal expansion.

(a) $\frac{13}{3125}$

Solution: According to Theorem, any given rational number of the form $\frac{p}{q}$ where p and q are co-prime, has a terminating decimal expansion if q is of the form $2^m \times 5^n$, where m and n are non-negative integers.

$$q = 3125 = 5 \times 5 \times 5 \times 5 \times 5 = 2^0 \times 5^5$$

Here, denominator is of the form $2^m \times 5^n$, where $m = 0$ and $n = 5$. It means rational number has a **terminating** decimal expansion.

Q1. (b) $\frac{17}{8}$

Solution: According to Theorem, any given rational number of the form $\frac{p}{q}$ where p and q are co-prime, has a terminating decimal expansion if q is of the form $2^m \times 5^n$, where m and n are non-negative integers.

$$q = 8 = 2 \times 2 \times 2 = 2^3 \times 5^0$$

Here, denominator is of the form $2^m \times 5^n$, where $m = 3$ and $n = 0$

It means rational number has a **terminating** decimal expansion.

Q1. (c) $\frac{64}{455}$

Solution:, where m and n are non-negative integers.

$$q = 455 = 2^0 \times 5 \times 7 \times 13$$

Here, denominator is not of the form $2^m \times 5^n$ rational number has a **non-terminating repeating** decimal expansion,

It means that this rational number has a non- **terminating** decimal expansion.

H.

Q1. (d) $\frac{15}{1600}$

Solution: on simplifying $\frac{15}{1600} = \frac{3}{320}$

Now, $q = 320 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 = 2^6 \times 5$

here, denominator is of the form $2^m \times 5^n$, where $m = 6$ and $n = 1$

It means rational number has a **terminating** decimal expansion.

Q1. (e) $\frac{29}{343}$

Solution: $q = 343 = 7 \times 7 \times 7$

Here, denominator is not of the form $2^m \times 5^n$, where m and n are non-negative integers.

It means rational number has $\frac{29}{343}$ **non-terminating repeating** decimal expansion.

Q1. (f) $\frac{23}{2^3 \times 5^2}$

Solution: Here, $q = 2^3 \times 5^2$

denominator is of the form $2^m \times 5^n$, where $m = 3$ and $n = 2$

It means rational number has a **terminating** decimal expansion.

$$\text{Q1. (g)} \quad \frac{129}{2^2 \times 5^7 \times 7^5}$$

Solution: $q = 2^2 \times 5^7 \times 7^5$

Here, denominator is not of the form $2^m \times 5^n$, where m and n are non-negative integers.

It means rational number has **non-terminating repeating** decimal expansion.

Q1. (h) $\frac{6}{15}$

Solution: on simplifying, $\frac{6}{15} = \frac{2}{5}$
here $q = 5$

denominator is of the form $2^m \times 5^n$, where $m = 0$ and $n = 1$

It means rational number has a **terminating** decimal expansion.

Q1. (i) $\frac{35}{70}$

Solution: on simplifying, $\frac{35}{70} = \frac{7}{10}$

here $q = 2 \times 5$

denominator is of the form $2^m \times 5^n$, where $m = 1$ and $n = 1$

It means rational number has a **terminating** decimal expansion.

Q1. (j) $\frac{77}{210}$

Solution: on simplifying, $\frac{77}{210} = \frac{11}{30}$

$$q = 2 \times 3 \times 5$$

Here, denominator is not of the form $2^m \times 5^n$, where m and n are non-negative integers.

It means rational number has **non-terminating repeating** decimal expansion.

Q2. Write down the decimal expansions of those rational numbers in Question 1 which have terminating decimal expansions.

Solution :

$$(a) \quad \frac{13}{3125} = \frac{13}{5^5} = \frac{13 \times 2^5}{5^5 \times 2^5} = \frac{13 \times 2^5}{10^5} = \frac{416}{10^5} = 0.00416$$

$$(b) \quad \frac{17}{8} = \frac{17}{2^3} = \frac{17 \times 5^3}{2^3 \times 5^3} = \frac{17 \times 5^3}{10^3} = \frac{2125}{10^3} = 2.215$$

$$(d) \quad \frac{15}{1600} = \frac{15}{2^6 \times 5^2} = \frac{15 \times 5^4}{2^6 \times 5^2 \times 5^4} = \frac{15 \times 5^4}{10^6} = \frac{9375}{10^6} = 0.009375$$

$$(f) \quad \frac{23}{2^3 \times 5^2} = \frac{23 \times 5^1}{2^3 \times 5^2 \times 5^1} = \frac{23 \times 5^1}{10^3} = \frac{115}{10^3} = 0.115$$

$$(g) \quad \frac{6}{15} = \frac{2}{5} = \frac{2 \times 2}{5 \times 2} = \frac{4}{10} = 0.4$$

$$(h) \quad \frac{35}{70} = \frac{7}{10} = 0.7$$

3. The following real numbers have decimal expansions as given below. In each case, decide whether they are rational or not. If, they are rational, and of the form p/q , what can you say about the prime factors of q ?

(i) 43.123456789

Answer: It is rational because decimal expansion is terminating. Therefore, it can be expressed in p/q form where factors of q are of the form $2^m \times 5^n$.

(ii) 0.1201120012000120000...

Answer: It is irrational because decimal expansion is neither terminating nor non-terminating repeating.

Q.3

(iii) $43.\overline{123456789}$

Answer: It is rational because decimal expansion is non-terminating repeating. Therefore, it can be expressed in p/q form where factors of q are of the form $2^m \times 5^n$.