

Chapter-1

Exercise-1.1

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Class: X
Subject: Mathematics

1. Use Euclid's division algorithm to find the HCF of: (i) 135 and 225

Solution: We have $225 > 135$,

According to Euclid's division Lemma, $a = bq + r$ (where, q is quotient and r is remainder)

So, we apply the division lemma to 225 and 135 to obtain

$$225 = 135 \times 1 + 90$$

Here remainder $90 \neq 0$, we apply the division lemma again to 135 and 90 to obtain

$$135 = 90 \times 1 + 45$$

We consider the new divisor 90 and new remainder $45 \neq 0$, and apply the division lemma

$$90 = 45 \times 2 + 0$$

Since at this time the remainder is zero, the process is stopped.

The divisor at this stage is 45

Answer: HCF(135 and 225) = 45.

1. Use Euclid's division algorithm to find the HCF of: (ii) 196 and 38220

Solution: We have $38220 > 196$,

So, we apply the division lemma to 38220 and 196 to obtain

$$38220 = 196 \times 195 + 0$$

As the remainder is zero, the process stops.

The divisor at this stage is 196,

Answer: Therefore, $\text{HCF}(196 \text{ and } 38220) = 196$.

1. Use Euclid's division algorithm to find the HCF of: (iii) 867 and 255

Solution: We have $867 > 255$,

So, we apply the division lemma to 867 and 255 to obtain

$$867 = 255 \times 3 + 102$$

Here remainder $102 \neq 0$, we apply the division lemma again to 255 and 102

$$255 = 102 \times 2 + 51$$

Here remainder $51 \neq 0$, we apply the division lemma again to 102 and 51

$$102 = 51 \times 2 + 0$$

As the remainder is zero, the process stops.

The divisor at this stage is 51,

Answer: Therefore, HCF of 867 and 255 is 51.

2. Show that any positive odd integer is of the form $6q + 1$, or $6q + 3$, or $6q + 5$, where q is some integer.

Solution: Let a be any positive integer and $b = 6$. Then, by Euclid's algorithm,

$a = 6q + r$ for some integer $q \geq 0$, and $r = 0, 1, 2, 3, 4, 5$ because $0 \leq r < 6$.

Therefore, $a = 6q$ or $6q + 1$ or $6q + 2$ or $6q + 3$ or $6q + 4$ or $6q + 5$

When $a = 6q = 2 \times 3q$ (Even)

When $a = 6q + 1 = 2 \times 3q + 1 = 2k + 1$ (Odd, as it is not exactly divisible by 2)

When $a = 6q + 2 = 2(3q) + 2$ (Even)

When $a = 6q + 3 = 2 \times 3q + 3$ (Odd, as it is not exactly divisible by 2)

When $a = 6q + 4 = 2 \times 3q + 4$ (Even)

When $a = 6q + 5 = 2 \times 3q + 5$ (Odd, as it is not exactly divisible by 2)

Clearly, $6q + 1$, $6q + 3$, $6q + 5$ are odd expressions, as they are not exactly divisible by 2.

And therefore, any odd integer can be expressed in the form $6q + 1$, or $6q + 3$ or $6q + 5$

3. An army contingent of 616 members is to march behind an army band of 32 members in a parade. The two groups are to march in the same number of columns. What is the maximum number of columns in which they can march?

Solution: We have to find the HCF (616, 32) to find the maximum number of columns in which they can march.

To find the HCF, we can use Euclid's algorithm.

$$616 = 32 \times 19 + 8$$

$$32 = 8 \times 4 + 0$$

Since, the last divisor is 8.

Therefore, the HCF (616, 32) is 8.

Answer: Army contingent can march in 8 columns each.

4. Use Euclid's division lemma to show that the square of any positive integer is either of form $3m$ or $3m + 1$ for some integer m . [Hint: Let x be any positive integer then it is of the form $3q$, $3q + 1$ or $3q + 2$. Now square each of these and show that they can be rewritten in the form $3m$ or $3m + 1$.]

Solution: Let a be any positive integer and $b = 3$.

Then $a = 3q + r$ for some integer $q \geq 0$

and $r = 0, 1, 2$ because $0 \leq r < 3$

Therefore $a = 3q$ or $3q + 1$ or $3q + 2$

When $a = 3q$

$$a^2 = (3q)^2 = 9q^2 = 3 \times (\underline{3q^2}) = 3k_1 \quad \text{where, } k_1 = 3q^2$$

When, $a = 3q + 1$

$$a^2 = (3q + 1)^2 = 9q^2 + 6q + 1 = 3 \times (\underline{3q^2 + 2}) + 1 = 3k_2 + 1 \quad \text{where } k_2 = \underline{3q^2 + 2}$$

when $a = 3q + 2$

$$a^2 = (3q + 2)^2 = 9q^2 + 12q + 4 = 3 \times (\underline{3q^2 + 4}) + 3 + 1 = 3 \times (\underline{3q^2 + 4 + 3}) + 1 = 3k_3 + 1$$

Where, $k_3 = \underline{3q^2 + 4 + 3}$

Hence, it can be said that the square of any positive integer is either of the form $3m$ or $3m + 1$.

5. Use Euclid's division lemma to show that the cube of any positive integer is of the form $9m$, $9m + 1$ or $9m + 8$.

Solution: Let a be any positive integer and $b = 3$

$a = 3q + r$, where $q \geq 0$ and $r=0,1,2$ because $0 \leq r < 3$

Values of $a = 3q$ or $3q + 1$ or $3q + 2$

Therefore, we have three cases.

Case 1: When $a = 3q$,

Taking cube, $a^3 = (3q)^3 = 9q^3 = 3 \times (\underline{3q^3}) = 3m$ where, $m = 3q^3$

Case 2: When $a = 3q + 1$,

Taking cube $a^3 = (3q + 1)^3 = 27q^3 + 27q^2 + 9q + 1 = 9 \times (\underline{3q^3 + 3q^2 + q}) + 1 = 9m + 1$,

where $m = \underline{3q^3 + 3q^2 + q}$

Case 3: When $a = 3q + 2$,

$a^3 = (3q + 2)^3 = 27q^3 + 54q^2 + 36q + 8 = 9 \times (3q^3 + 6q^2 + 4q) + 8 = 9m + 8$,

where $m = 3q^3 + 6q^2 + 4q$

Therefore, the cube of any positive integer is of the form $9m$, $9m + 1$, or $9m + 8$.

Try These..

- Example 1 : Use Euclid's algorithm to find the HCF of 4052 and 12576
- Example 2 : Show that every positive even integer is of the form $2q$, and that every positive odd integer is of the form $2q + 1$, where q is some integer
- Example 3 : Show that any positive odd integer is of the form $4q + 1$ or $4q + 3$, where q is some integer.
- Example 4 : A sweetseller has 420 kaju barfis and 130 badam barfis. She wants to stack them in such a way that each stack has the same number, and they take up the least area of the tray. What is the number of that can be placed in each stack for this purpose? Example 1

Next Exercise..
To be continued..