

Chapter-1

Exercise-1.3

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Class: X
Subject: Mathematics

Q.1) Prove that $\sqrt{5}$ is an irrational number.

Solution: Let's assume, to the contrary that $\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\sqrt{5} = \frac{a}{b}$, Let a and b have a common factor 1 assuming that a and b are co-prime.

$\Rightarrow \sqrt{5} = \frac{a}{b}$, On squaring both the sides, we get

$$\Rightarrow (\sqrt{5})^2 = \left(\frac{a}{b}\right)^2$$

$$\Rightarrow 5b^2 = a^2$$

Therefore, a is divisible by 5 and it can be said that a is divisible by 5.

Let $a = 5c$, where c is an integer

$$\Rightarrow 5b^2 = (5c)^2$$

$$\Rightarrow 5b^2 = 25c^2$$

This means that b^2 is divisible by 5 and hence, b is divisible by 5.

This implies that a and b have 5 as a common factor.

And this is a contradiction to the fact that a and b are co-prime. Hence, $\sqrt{5}$ is irrational.

Q.2) Prove that $3 + 2\sqrt{5}$ is an irrational

Solution: Let's assume, to the contrary that $3+2\sqrt{5}$ is a rational number.

Therefore, we can find two integers a, b ($b \neq 0$) such that $3+2\sqrt{5} = \frac{a}{b}$, Let a and b have a common factor 1 assuming that a and b are co-prime.

$$\Rightarrow 3+2\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow 2\sqrt{5} = \frac{a}{b} - 3$$

$$\Rightarrow \sqrt{5} = \frac{1}{2}\left(\frac{a}{b} - 3\right)$$

Since a and b are integers, $\frac{1}{2}\left(\frac{a}{b} - 3\right)$ will also be rational and therefore, $\sqrt{5}$ is rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Hence, our assumption that $3 + 2\sqrt{5}$ is rational is false.

Therefore, $3 + 2\sqrt{5}$ is irrational.

Q.3) Prove that the following are irrationals:

(i) $\frac{1}{\sqrt{2}}$

Solution: Let's assume that $\frac{1}{\sqrt{2}}$ is rational.

Therefore, we can find two integers a, b ($b \neq 0$) such that $\frac{1}{\sqrt{2}} = \frac{a}{b}$, Let a and b have a common factor 1 assuming that a and b are co-prime.

$$\Rightarrow \frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\Rightarrow \sqrt{2} = \frac{b}{a}$$

It can be observed that $\frac{b}{a}$ is rational as a and b are integers. Therefore, $\sqrt{2}$ is rational which contradicts to the fact that $\sqrt{2}$ is irrational.

Hence, our assumption is false and $\frac{1}{\sqrt{2}}$ is irrational.

Q.3) (ii) $7\sqrt{5}$

Solution:

Let's assume, to the contrary that $7\sqrt{5}$ is rational. Therefore, we can find two co-prime integers a, b ($b \neq 0$) such that

$$\Rightarrow 7\sqrt{5} = \frac{a}{b}, \text{ on rearranging, we get}$$

$$\Rightarrow \sqrt{5} = \frac{a}{7b}$$

Therefore $\frac{a}{7b}$ is rational as a and b are integers. Therefore, $\sqrt{5}$ should be rational.

This contradicts the fact that $\sqrt{5}$ is irrational. Therefore, our assumption that $7\sqrt{5}$ is rational is false. Hence, $7\sqrt{5}$ is irrational.

Q.3) (iii) $6 + \sqrt{2}$

Solution: Let's assume to the contrary that $6 + \sqrt{2}$ be rational.

Therefore, we can find two co-prime integers a, b ($b \neq 0$) such that

$$\Rightarrow 6 + \sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$$

Since a and b are integers, $\frac{a}{b} - 6$ is also rational and hence, $\sqrt{2}$ should be rational. This contradicts the fact that $\sqrt{2}$ is irrational.

Therefore, our assumption is false and hence, $6 + \sqrt{2}$ is irrational.

Try These

- Prove that $\sqrt{2}$ is irrational.
- Prove that $\sqrt{3}$ is irrational.
- Show that $5 - \sqrt{3}$ is irrational.
- Show that $3\sqrt{2}$ is irrational.

Next exercise..
To be continued..