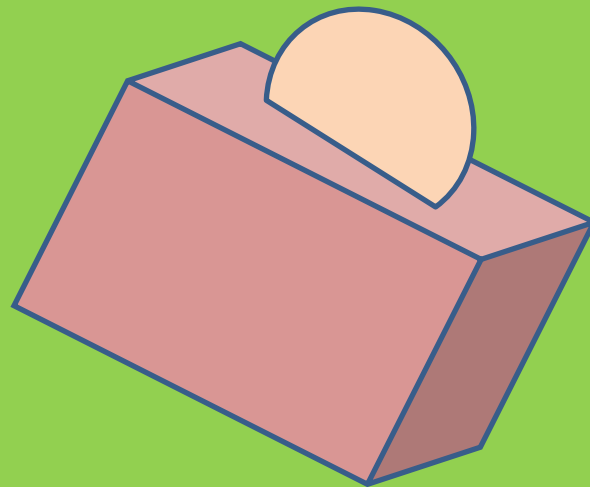
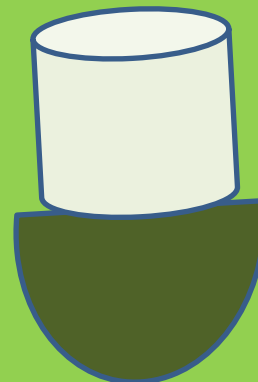


CHAPTER 13

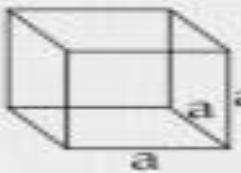
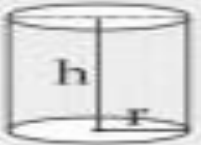
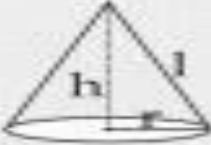
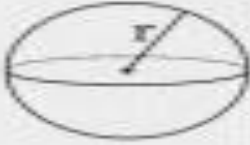


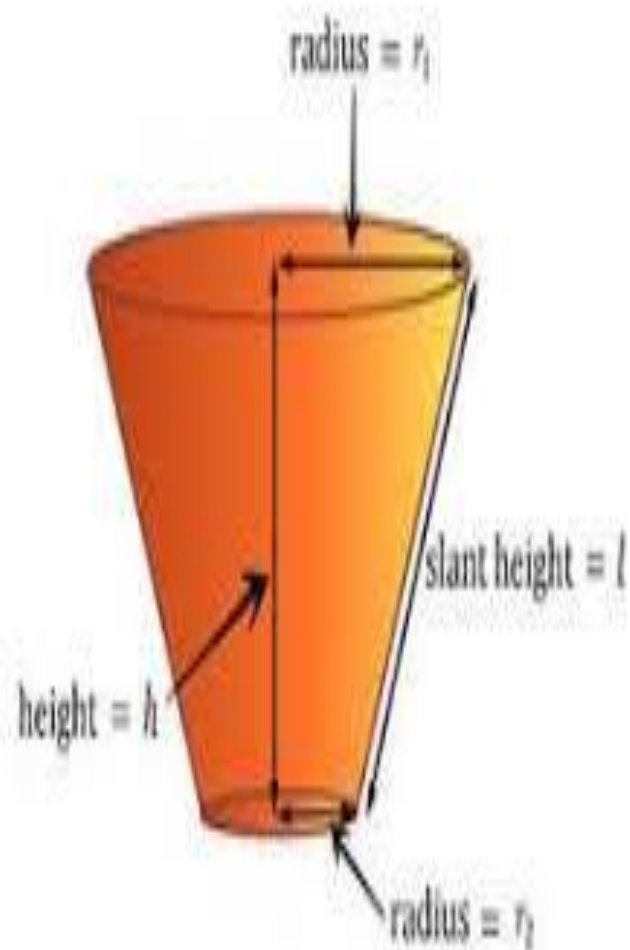
SURFACE AREAS AND VOLUMES OF COMBINED SHAPES



CONTENTS

- ❖ FORMULAE
- ❖ SURFACE AREAS
- ❖ VOLUMES
- ❖ CONVERSIONS OF SOLIDS
- ❖ FRUSTUM

Name of the solid	Figure	Volume	Lateral/Curved Surface Area	Total Surface Area
Cuboid		lbh	$2lh + 2bh$ or $2h(l+b)$	$2lh+2bh+2lb$ or $2(lh+bh+lb)$
Cube		a^3	$4a^2$	$4a^2+2a^2$ or $6a^2$
Right circular cylinder		$\pi r^2 h$	$2\pi r h$	$2\pi r h + 2\pi r^2$ or $2\pi r(h+r)$
Right circular cone		$\frac{1}{3} \pi r^2 h$	$\pi r l$	$\pi r l + \pi r^2$ or $\pi r(l+r)$
Sphere		$\frac{4}{3} \pi r^3$	$4\pi r^2$	$4\pi r^2$
Hemisphere		$\frac{2}{3} \pi r^3$	$2\pi r^2$	$2\pi r^2 + \pi r^2$ or $3\pi r^2$



FRUSTUM OF CONE

- Volume of the frustum of the cone =

$$\frac{1}{3}\pi h(r_1^2 + r_2^2 + r_1 r_2)$$

- Curved surface area of the frustum of the cone =

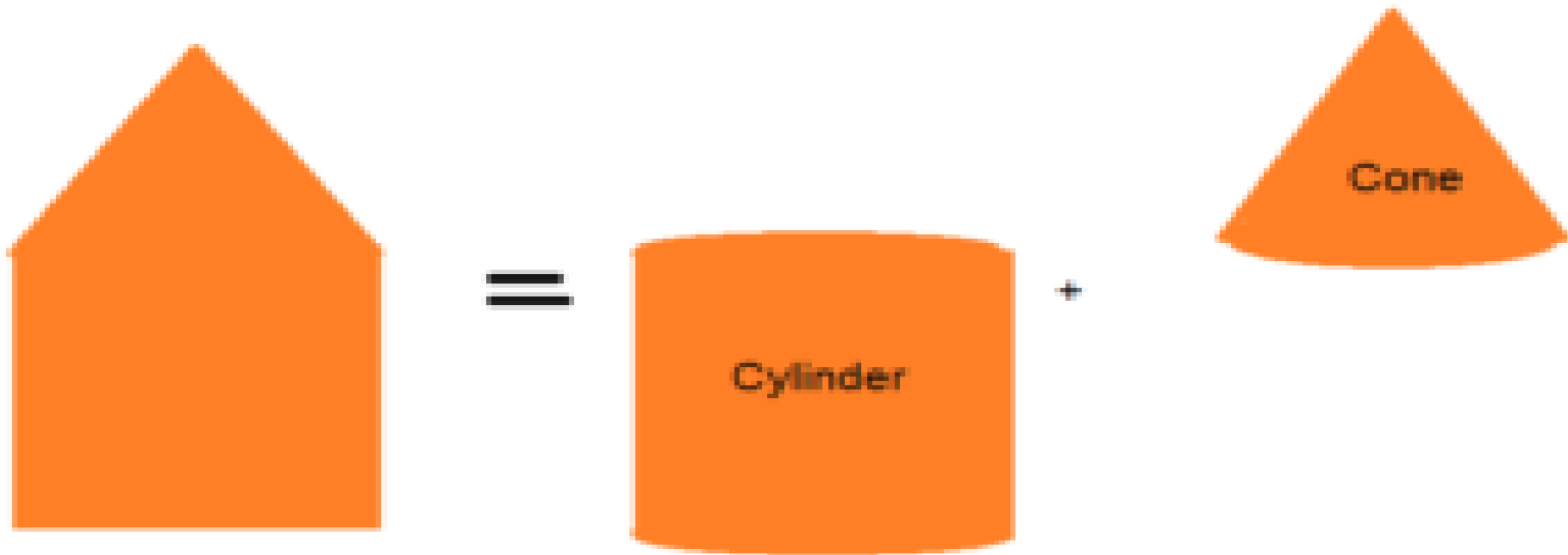
$$\pi(r_1 + r_2)l$$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$

- Total surface area of the frustum of the cone =

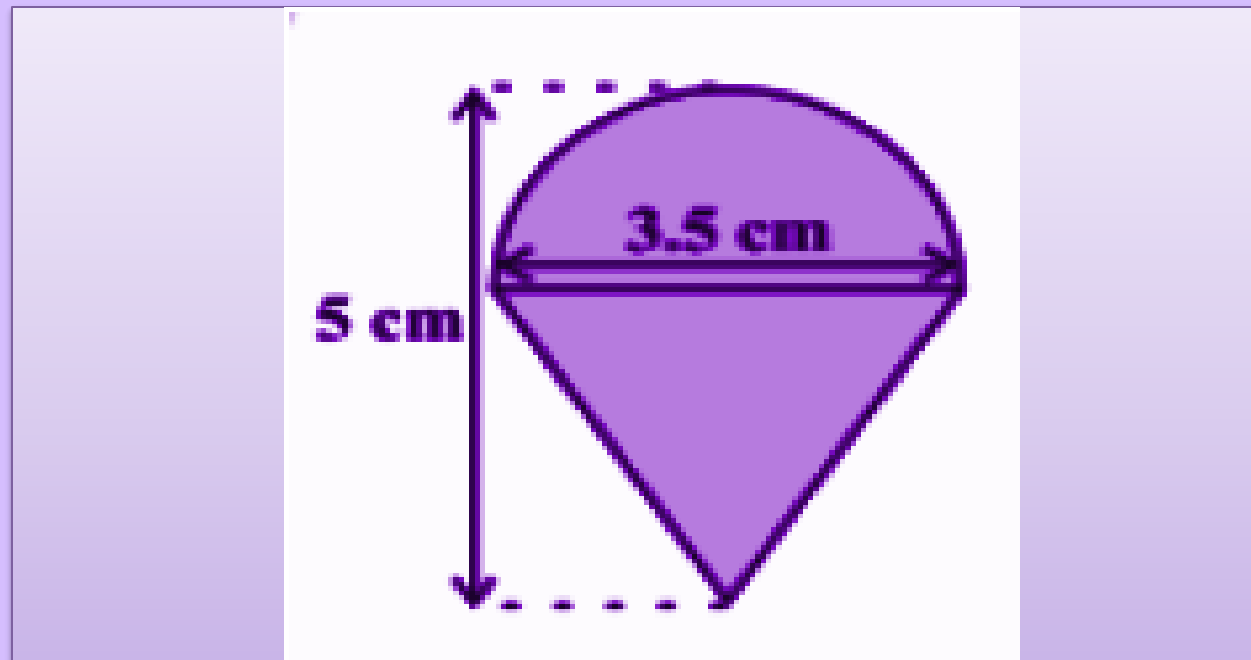
$$\pi l(r_1 + r_2) + \pi r_1^2 + \pi r_2^2$$

where $l = \sqrt{h^2 + (r_1 - r_2)^2}$



EXAMPLE OF
SURFACE AREAS OF
COMBINED FIGURES

EXAMPLE :--Ram got a playing top (lattu) as his birthday present, which surprisingly had no color on it. He wanted to color it with his crayons. The top is shaped like a cone surmounted by a hemisphere . The entire top is 5 cm in height and the diameter of the top is 3.5 cm. Find the area he has to color. (Take $\pi = 22/7$)



SOLUTION

TSA of the toy = CSA of hemisphere + CSA of cone

Curved surface area of the hemisphere = $\frac{1}{2} (4\pi r^2) = 2\pi r^2 = 2\left(\frac{22}{7}\right) \times \left(\frac{3.5}{2}\right) \times \left(\frac{3.5}{2}\right) \text{ cm}^2$

Height of the cone = Height of the top – Radius of the hemispherical part
= $(5 - 3.5/2) \text{ cm} = 3.25 \text{ cm}$

Slant height of the cone (l) = $\sqrt{r^2 + h^2} = \sqrt{\left(\frac{3.5}{2}\right)^2 + (3.25)^2} = 3.7 \text{ cm (approx.)}$

Therefore, CSA of cone = $\pi rl = \left(\frac{22}{7}\right) \times \left(\frac{3.5}{2}\right) \times 3.7 \text{ cm}^2$

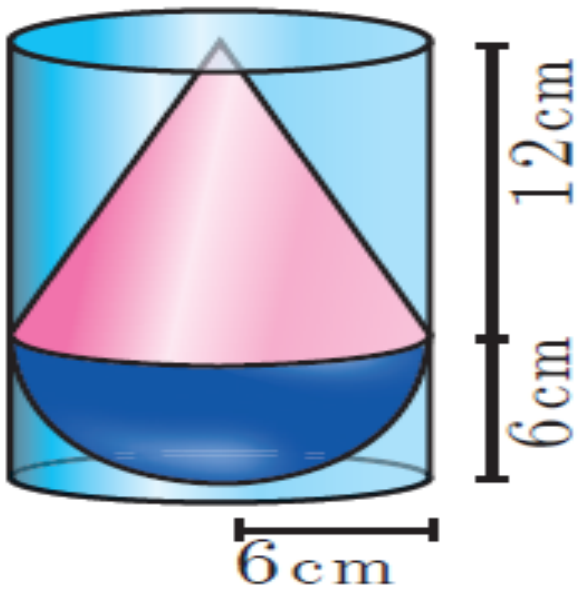
Hence, the surface area of the top

$$= \left[2\left(\frac{22}{7}\right) \times \left(\frac{3.5}{2}\right) \times \left(\frac{3.5}{2}\right) + \left(\frac{22}{7}\right) \times \left(\frac{3.5}{2}\right) \times 3.7 \right] \text{ cm}^2$$

$$= \left(\frac{22}{7}\right) \times \left(\frac{3.5}{2}\right) (3.5 + 3.7) \text{ cm}^2$$

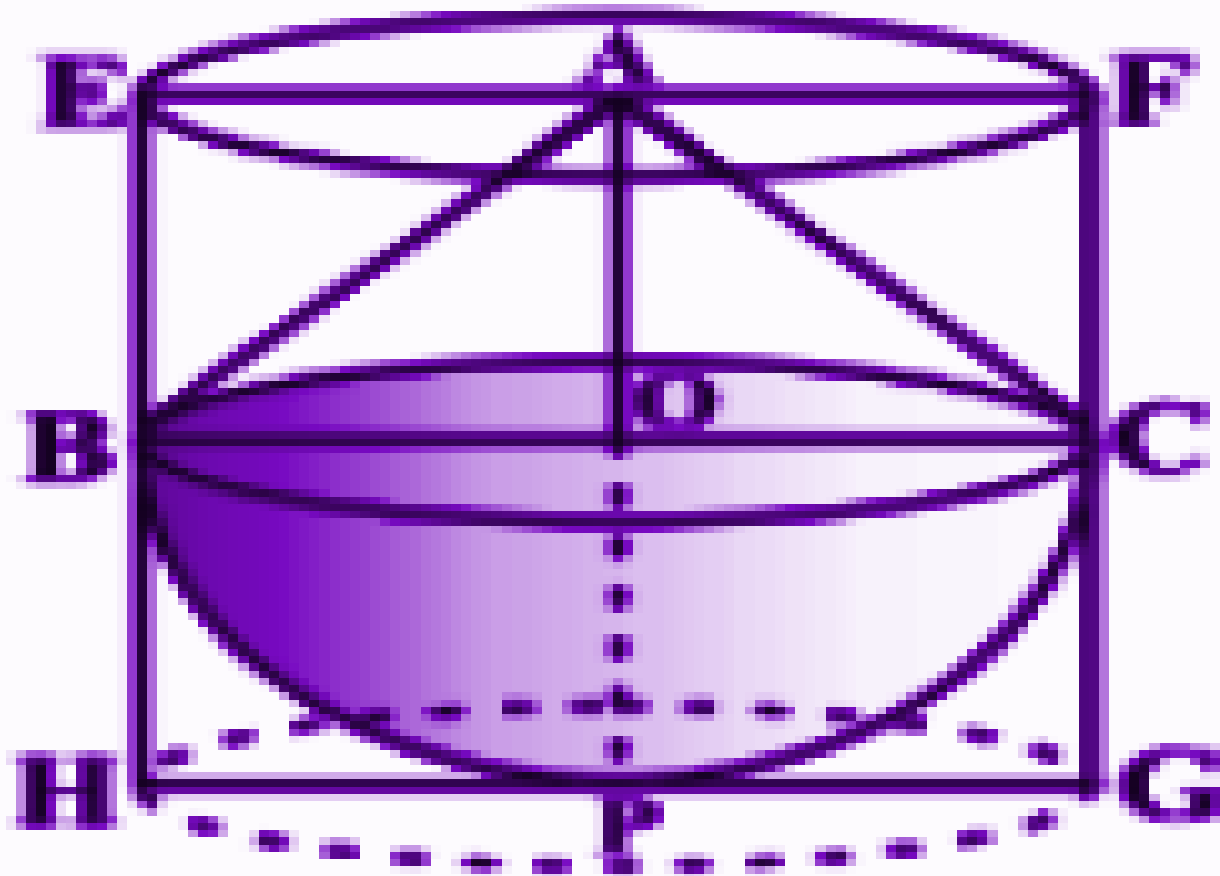
$$= \left(\frac{11}{2}\right) \times (3.5 + 3.7) \text{ cm}^2$$

$$= 39.6 \text{ cm}^2 \text{ (approx.)}$$



**EXAMPLE OF
VOLUME OF
COMBINED SOLIDS**

EXAMPLE :--A solid toy is in the form of a hemisphere surmounted by a right circular cone. The height of the cone is 2 cm and the diameter of the base is 4 cm. Determine the volume of the toy. If a right circular cylinder circumscribes the toy, find the difference between the volumes of the cylinder and the toy. (Take $\pi = 3.14$)



SOLUTION

Let BPC be the hemisphere and ABC be the cone standing on the base of the hemisphere as shown in the above figure.

The radius BO of the hemisphere (as well as of the cone) $= (\frac{1}{2}) \times 4 \text{ cm} = 2 \text{ cm}$.

So, volume of the toy $= (\frac{2}{3}) \pi r^3 + (\frac{1}{3}) \pi r^2 h$

$$= (\frac{2}{3}) \times 3.14 \times 2^3 + (\frac{1}{3}) \times 3.14 \times 2^2 \times 2$$

$$= 25.12 \text{ cm}^3$$

Now, let the right circular cylinder EFGH circumscribe the given solid.

The radius of the base of the right circular cylinder $= \text{HP} = \text{BO} = 2 \text{ cm}$, and its height is

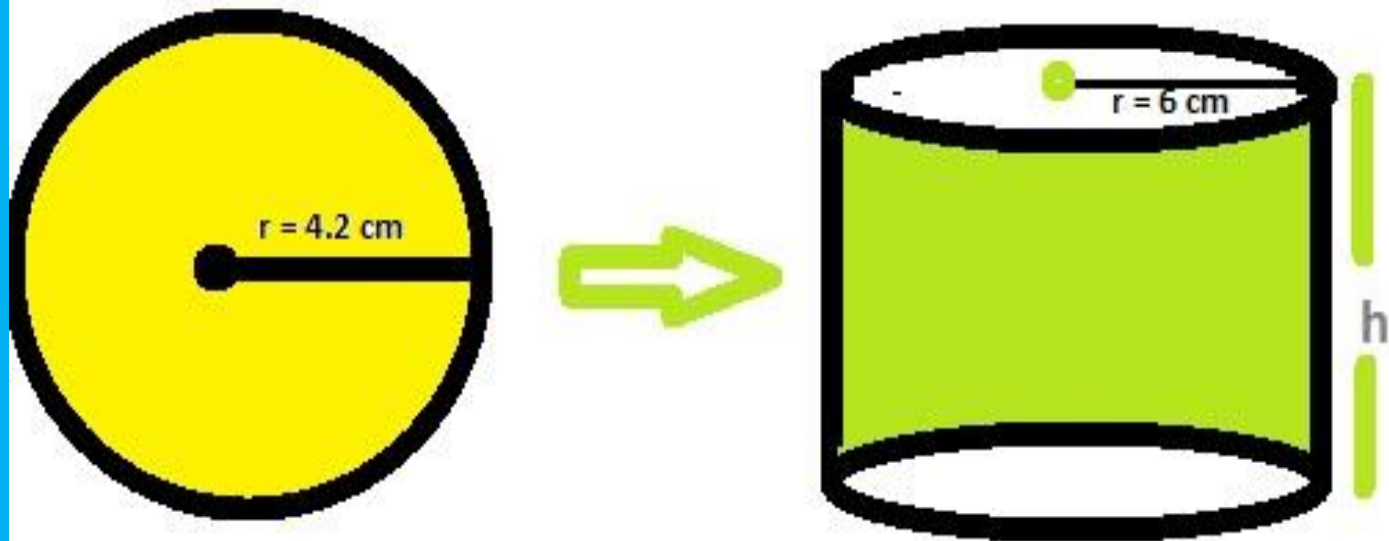
$$\text{EH} = \text{AO} + \text{OP} = (2 + 2) \text{ cm} = 4 \text{ cm}$$

So, the volume required $= \text{volume of the right circular cylinder} - \text{volume of the toy}$

$$= (3.14 \times 2^2 \times 4 - 25.12) \text{ cm}^3$$

$$= 25.12 \text{ cm}^3$$

Hence, the required difference of the two volumes $= 25.12 \text{ cm}^3$



EXAMPLE OF CONVERSION
OF SOLIDS

EXAMPLE :--A cone of height 24 cm and radius of base 6 cm is made up of modelling clay. A child reshapes it in the form of a sphere. Find the radius of the sphere.



SOLUTION

The volume of cone = $(\frac{1}{3}) \times \pi \times 6 \times 6 \times 24 \text{ cm}^3$

If r is the radius of the sphere, then its volume is $(\frac{4}{3}) \pi r^3$.

Since the volume of clay in the form of the cone and the sphere remains the same, we

$$\text{Have } (\frac{4}{3}) \pi r^3 = (\frac{1}{3}) \times \pi \times 6 \times 6 \times 24 \text{ cm}^3$$

$$r^3 = 3 \times 3 \times 24 = 3^3 \times 2^3$$

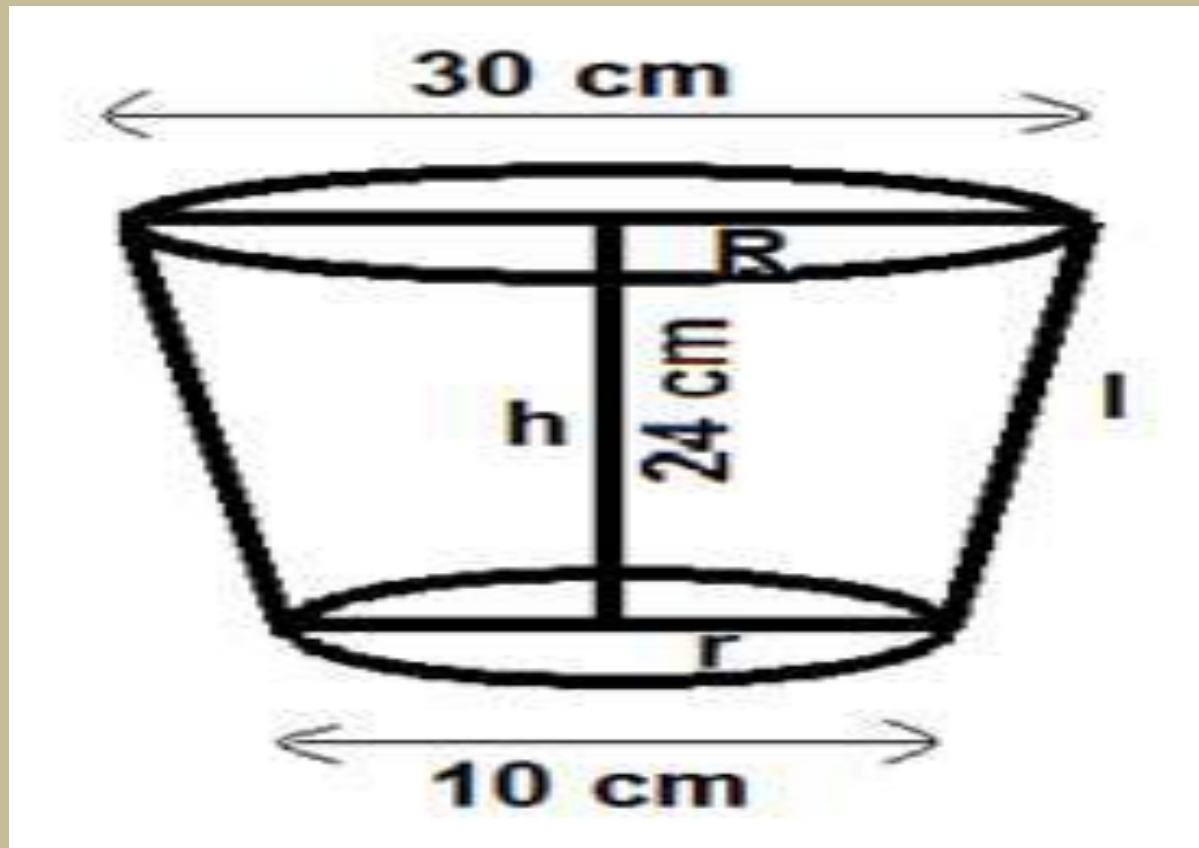
$$\text{So, } r = 3 \times 2 = 6$$

Therefore, the radius of the sphere is 6 cm.

EXAMPLE OF
FRUSTUM OF A
EXAMPLE OF
FRUSTUM OF A CONE



EXAMPLE :--A bucket made up of metal sheet is in the form of frustum of a cone. Its depth is 24 cm and the diameters of the top and bottom are 30 cm and 10 cm respectively. Find the cost of milk which will completely fill the bucket at the rate Rs. 20 per litre and cost of metal sheet used if it costs Rs. 10 per 100 cm².



SOLUTION

$$h = 24 \text{ cm}, r_1 = \frac{30}{2} = 15 \text{ cm}, r_2 = \frac{10}{2} = 5 \text{ cm}$$



$$l = \sqrt{h^2 + (r_1 - r_2)^2}$$

$$l = \sqrt{h^2 + (r_1 - r_2)^2} = \sqrt{24^2 + (15 - 5)^2}$$
$$= \sqrt{576 + 100} = \sqrt{676} = 26 \text{ cm}$$

VOLUME OF BUCKET $= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 r_2)$

$$= \frac{1}{3} \times 3.14 \times 24 (15^2 + 5^2 + 15 \times 5)$$
$$= 3.14 \times 8 (225 + 25 + 75) = 8164 \text{ cm}^3$$

QUANTITY OF MILK $= \frac{8164}{1000} = 8.164 \text{ LITRES}$

Cost of 1 litre of milk = Rs.20

Cost of 8.164 litres milk = Rs. 20 × 8.14
=Rs. 163.28

T.S.A. of bucket (excluding the upper end)

$$\begin{aligned} &= \pi l (r_1 + r_2) + \pi r_2^2 = 3.14 \times 26(15 + 5) + 3.14 \times 5^2 \\ &= 1632.8 + 78.5 = 1711.3 \text{ cm}^2 \end{aligned}$$

Cost of 100 cm² metal sheet = Rs.10

$$\text{Cost of 1711.3 cm}^2 \text{ metal sheet} = \frac{1711.3 \times 10}{100} = \text{Rs.171.13}$$

THANK YOU