

RELATIONS AND FUNCTIONS

1. If the functions $f: \mathbb{R} \rightarrow \mathbb{R}$, defined by $f(x) = 3x - 4$ is invertible, find f^{-1} . (Ans: $\frac{x+4}{3}$).
2. If $f(x) = |x|$ and $g(x) = [x]$. Evaluate : $\text{fog}(-5/3) - \text{gof}(-5/3)$. (Ans 1)
3. Write fog, if $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ are given by $f(x) = 8x^3$ and $g(x) = x^{\frac{1}{3}}$. (Ans: $8x$)
4. Show that the function f in $A = \mathbb{R} - \{\frac{2}{3}\}$ defined as $f(x) = \frac{4x+3}{6x-4}$ is one-one and onto. Hence find f^{-1} .
5. Consider $f: \mathbb{R} \rightarrow [4, \infty)$ given by $f(x) = x^2 + 4$. Show that f is invertible and find its inverse?
6. Let $A = \mathbb{R} - \{3\}$ and $B = \mathbb{R} - \{1\}$. Consider the function $f: A \rightarrow B$ defined by $f(x) = \frac{x-2}{x-3}$.
Show that f is one-one and onto hence find f^{-1} .
7. Show that $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(x) = \begin{cases} x+1 & \text{if } x \text{ is odd} \\ x-1 & \text{if } x \text{ is even} \end{cases}$ is bijective (both one-one and onto).
8. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = 10x + 7$. Find the function $g: \mathbb{R} \rightarrow \mathbb{R}$, such that $\text{gof} = \text{fog} = \text{Id}_{\mathbb{R}}$.
9. If $f: \mathbb{R} \rightarrow \mathbb{R}$ be the function defined by $f(x) = 4x^3 + 7$, Show that f is a bijection.
10. Let Z be the set of all integers and R is the relation on Z defined by as
 $R = \{(a, b) : a, b \in Z \text{ and } a-b \text{ is divisible by } 5\}$. Prove that R is equivalence relation.
11. Show that the relation S in the set $\mathbb{R}$ of real numbers defined as $S = \{(a, b) : a, b \in \mathbb{R} \text{ and } a \leq b^3\}$ is neither reflexive nor symmetric nor transitive.
12. Show that the relation R in the set $A = \{x : x \in \mathbb{Z}, 0 \leq x \leq 12\}$ given by $R = \{(a, b) : |a - b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.
13. Show that the relation S defined on the set $\mathbb{N} \times \mathbb{N}$ by $(a, b) S (c, d) \rightarrow a + d = b + c$ is an equivalence relation.
14. Consider $f: \mathbb{R}_+ \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$, Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{y+6}-1}{3}$.
15. Consider $f: \mathbb{R}_+ \rightarrow [-9, \infty)$ given by $f(x) = 5x^2 + 6x - 9$, Show that f is invertible with $f^{-1}(y) = \frac{\sqrt{54+5y}-3}{5}$.
16. Let $f: X \rightarrow Y$ be a function. Define a relation R on X given by $R = \{(a, b) : f(a) = f(b)\}$.
Show that R is an equivalence relation on X .
17. Show that a function $f: \mathbb{R} \rightarrow \mathbb{R}$ given by $f(x) = ax + b, a, b \in \mathbb{R}, a \neq 0$ is a bijection.
18. Let $f: \mathbb{N} \rightarrow \mathbb{N}$ be defined by $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ for all $n \in \mathbb{N}$. Find whether the function f is bijective.
19. Show that the relation R in $\mathbb{R}$ defined as $R = \{(a, b) : a \leq b^2\}$ is neither reflexive nor transitive nor symmetric.
20. If $f: \mathbb{R} \rightarrow \mathbb{R}$ and $g: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = x^2 + 3x + 1$ and $g(x) = 2x - 3$, find fog and gof. (Ans: $4x^2 - 6x + 1$ and $2x^2 + 6x - 1$)
21. Define a binary operation on the set $\{0, 1, 2, 3, 4, 5\}$ as $a * b = \begin{cases} a + b & \text{if } a + b < 6 \\ a + b - 6 & \text{if } a + b \geq 6 \end{cases}$.
Show that zero is the identity for this operation and each element a of the set is invertible with $6-a$ being the inverse of a .
22. If the operation $*$ defined on the set Q of all rational numbers by $a * b = \frac{3ab}{5}$ for all $a, b \in Q$.
Show that $*$ is commutative as well as associative on Q .
23. Let $A = \mathbb{N} \times \mathbb{N}$ and $*$ be the binary operation on A defined by $(a, b) * (c, d) = (a + c, b + d)$. Show that $*$ is commutative and associative. Also, find the identity element for $*$ on A , if any.
24. Consider the binary operation $*$ on the set $\{1, 2, 3, 4, 5\}$ defined by $a * b = \min\{a, b\}$.
Write operation table of operation $*$.

25. Let N denotes the set of all natural numbers and R be the relation on $N \times N$ defined by $(a,b)R(c,d)$, if $ad(b+c)=bc(a+d)$. Show that R is an equivalence relation.
26. If $f, g : R \rightarrow R$ are two functions defined as $f(x) = |x| + x$ and $g(x) = |x| - x$, for all $x \in R$. Then find $f \circ g(-3)$, $g \circ f(-2)$ and $f \circ g(5)$. (Ans: 12, 0 and 0)
27. If the operation $*$ defined on the set Q of all rational numbers by $a*b = a + b - ab$ for all $a, b \in Q$. show that $*$ is commutative as well as associative on Q .
28. Let $A = Q \times Q$ and let $*$ be a binary operation on A defined by $(a, b) * (c, d) = (ac, b + ad)$ for $(a, b), (c, d) \in A$. Determine whether $*$ is commutative and associative. Then, with respect to $*$ on A
 - (i) Find the identity element in A
 - (ii) Find the invertible elements of A .