

Relations and Functions

Types of Functions

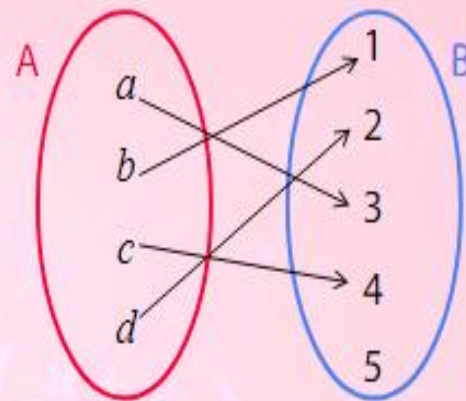
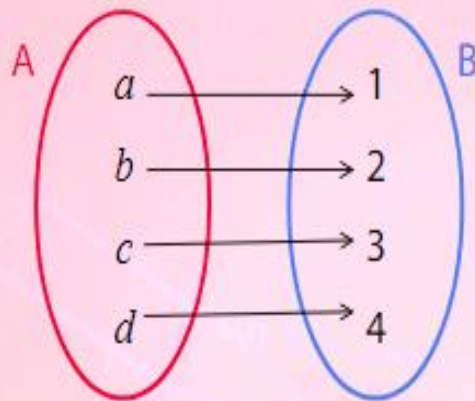
Introduction

This topic covers:

- One-to-one function
- Many-to-one function
- Onto function
- Into function

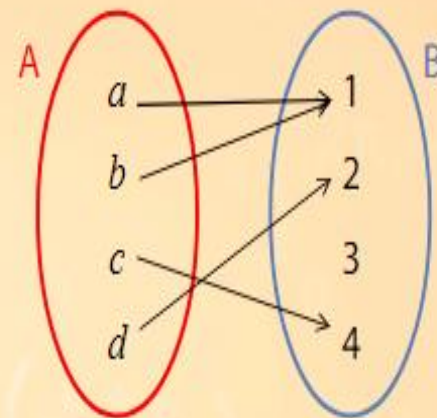
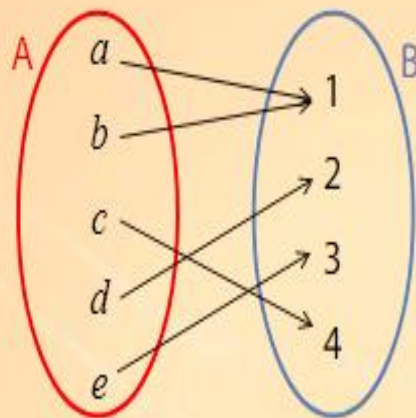
One-to-One Function

A function $f: A \rightarrow B$ is said to be one-to-one (or injective), if each element of the set A is uniquely related to an element of set B , that is, for every $a_1, a_2 \in A$, $f(a_1) = f(a_2)$ implies $a_1 = a_2$.



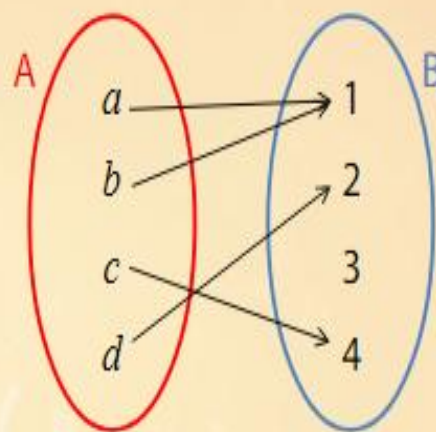
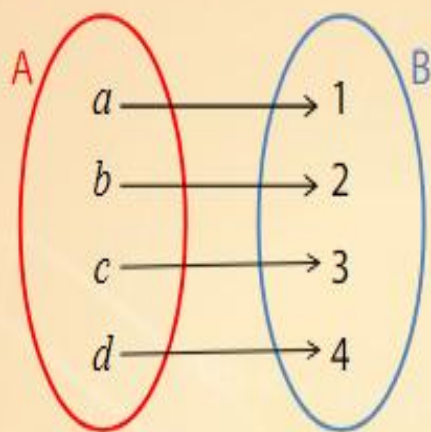
Many-to-One Function

A function $f: A \rightarrow B$ is said to be many-to-one, if two or more elements of the set A are related to the same element of set B , that is, for some $a_1, a_2 \in A$, $f(a_1) = f(a_2)$ implies $a_1 \neq a_2$.



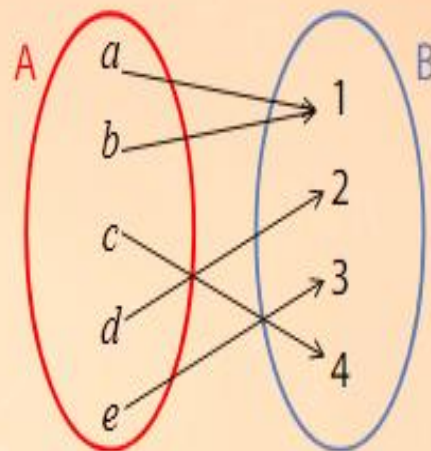
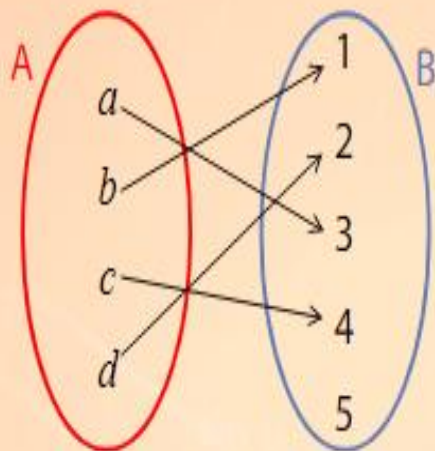
Onto Function

A function $f: A \rightarrow B$ is said to be onto (or surjective), if every element of set B has a pre-image in set A , that is, for every $b \in B$, there exist an element $a \in A$ such that $f(a) = b$.



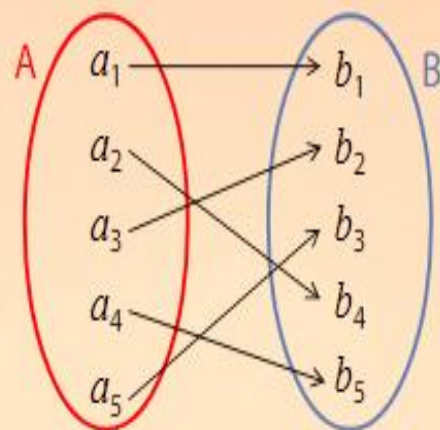
Into Function

A function $f: A \rightarrow B$ is said to be into, if there exists an element in set B such that it doesn't have pre-image in set A.



Bijjective Function

A function $f: A \rightarrow B$ is said to be bijective, if f is both one-to-one and onto.

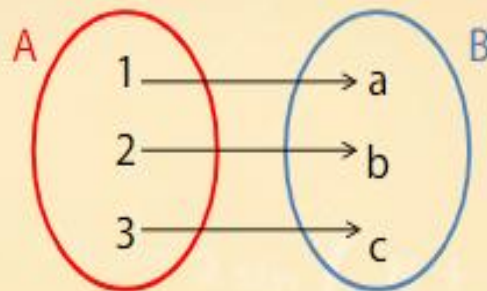


Example 1

Let $A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$ and let $f = \{(1, a), (2, b), (3, c)\}$ be a function from A to B . Show that f is one-to-one.

Solution:

$A = \{1, 2, 3\}$, $B = \{a, b, c, d\}$ and $f = \{(1, a), (2, b), (3, c)\}$



Since all the elements of A are mapped to a unique element of B , the function f is one-to-one.

Example 2

Prove that the function $f: \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = 5x + 3$ is bijective function.

Solution:

For $x_1, x_2 \in \mathbb{R}$; let $f(x_1) = f(x_2)$

$$\text{i.e. } 5x_1 + 3 = 5x_2 + 3$$

$$\Rightarrow x_1 = x_2$$

Since for all $x_1, x_2 \in \mathbb{R}$, $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

The given function is one-to-one.

Example 2 (Contd.)

Prove that the function $f: \mathbb{R} \rightarrow \mathbb{R}$, given by $f(x) = 5x + 3$, is bijective function.

Solution:

For any real number $y \in \mathbb{R}$,

there exist $\frac{y-3}{5} \in \mathbb{R}$ such that

$$f\left(\frac{y-3}{5}\right) = 5\left(\frac{y-3}{5}\right) + 3 = y$$

$\therefore f$ is onto.

Hence f is bijective function.

Example 2 (Contd.)

Show that the function $f : \mathbb{R} \rightarrow \mathbb{R}$, defined as $f(x) = x^2 + 3$, is neither one-to-one nor onto.

Solution :

Now, consider the element -1 in the co-domain $\mathbb{R}$. Let's see if we can find pre-image of this element.

$$x^2 + 3 = -1$$

$$\Rightarrow x^2 = -4$$

$$\Rightarrow x = \sqrt{-4}$$

$$\Rightarrow x \notin \mathbb{R}$$

$\therefore f$ is not onto.

Hence, f is neither one-to-one nor onto.

A Quick Recap

Here are the main points covered in this topic:

Let f be the function such that $f: A \rightarrow B$. Then the function f can be of the following types:

Types	Conditions
One-to-One (Injective)	If each element of the set A is uniquely related to an element of set B
Many-to-One	If two or more elements of the set A are related to the same element of set B
Onto (Surjective)	If every element of set B has a pre-image in set A
Into	If there exists an element in set B such that it doesn't have pre-image in set A
One-to-One and Onto (Bijective)	If f is both one-to-one and onto