

INVERSE TRIGONOMETRIC FUNCTIONS

Sin⁻¹x: For $x \in [-1, 1]$, $\sin^{-1} x$ is defined as an angle θ whose sin is x . i. e. $\sin^{-1} x = \theta \Rightarrow \sin \theta = x$. If $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ then θ is uniquely defined and it is called principle value of $\sin^{-1} x$ e.g. $\sin^{-1}(1/2) = \frac{\pi}{6}$.

Cos⁻¹x: For $x \in [-1, 1]$, $\cos^{-1} x$ is defined as an angle θ whose cos is x . i. e. $\cos^{-1} x = \theta \Rightarrow \cos \theta = x$. If $\theta \in [0, \pi]$ then θ is uniquely defined and it is called principle value of $\cos^{-1} x$ e.g. $\cos^{-1}(1/2) = \frac{\pi}{3}$.

tan⁻¹x: For $x \in \mathbb{R}$, $\tan^{-1} x$ is defined as an angle θ whose tan is x . i. e. $\tan^{-1} x = \theta \Rightarrow \tan \theta = x$. If $\theta \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then θ is uniquely defined and it is called principle value of $\tan^{-1} x$ e.g. $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.

FUNCTION	DOMAIN	RANGE (P. Value)
$\sin^{-1} x$	$[-1, 1]$	$[-\pi/2, \pi/2]$
$\cos^{-1} x$	$[-1, 1]$	$[0, \pi]$
$\tan^{-1} x$	$\mathbb{R}$	$(-\pi/2, \pi/2)$
$\operatorname{Cosec}^{-1} x$	$\mathbb{R} - (-1, 1)$	$[-\pi/2, \pi/2] - \{0\}$
$\sec^{-1} x$	$\mathbb{R} - (-1, 1)$	$[0, \pi] - \pi/2$
$\operatorname{Cot}^{-1} x$	$\mathbb{R}$	$(0, \pi)$

Note: $\sin^{-1} x \neq 1 / \sin x$

PROPERTIES: (I) (i) $\sin^{-1}(\sin x) = x$ (ii) $\cos^{-1}(\cos x) = x$ (iii) $\tan^{-1}(\tan x) = x$

(II) (i) $\sin(\sin^{-1} x) = x$ (ii) $\cos(\cos^{-1} x) = x$ (iii) $\tan(\tan^{-1} x) = x$

(III) (i) $\sin^{-1}(-x) = -\sin^{-1}(x)$ (ii) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$ (iii) $\tan^{-1}(-x) = -\tan^{-1}(x)$

(iv) $\cos^{-1}(-x) = \pi - \cos^{-1}(x)$ (v) $\sec^{-1}(-x) = \pi - \sec^{-1}(x)$ (vi) $\cot^{-1}(-x) = \pi - \cot^{-1} x$

(IV) (i) $\operatorname{cosec}^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$ (ii) $\sec^{-1}(x) = \cos^{-1}\left(\frac{1}{x}\right)$ (iii) $\cot^{-1}(x) = \tan^{-1}\left(\frac{1}{x}\right)$

(iv) $\sin^{-1}(x) = \operatorname{cosec}^{-1}\left(\frac{1}{x}\right)$ (ii) $\cos^{-1}(x) = \sec^{-1}\left(\frac{1}{x}\right)$ (iii) $\tan^{-1}(x) = \cot^{-1}\left(\frac{1}{x}\right)$

(V) (i) $\sin^{-1} x + \cos^{-1} x = \pi/2$ (ii) $\operatorname{cosec}^{-1}(x) + \sec^{-1}(x) = \pi/2$ (iii) $\tan^{-1}(x) + \cot^{-1}(x) = \pi/2$.

(VI) $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \frac{x+y}{1-xy}$; $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$;

(VII) (i) $\sin^{-1} x \pm \sin^{-1} y = \sin^{-1}(x\sqrt{1-y^2} \pm y\sqrt{1-x^2})$ (ii)

$\cos^{-1} x \pm \cos^{-1} y = \cos^{-1}(xy \mp \sqrt{1-x^2}\sqrt{1-y^2})$

(VIII) $2 \tan^{-1} x = \sin^{-1} \frac{2x}{1+x^2} = \cos^{-1} \frac{1-x^2}{1+x^2} = \tan^{-1} \frac{2x}{1-x^2}$

(IX) $2 \sin^{-1} x = \sin^{-1}(2x\sqrt{1-x^2})$, $2 \cos^{-1} x = \cos^{-1}(2x^2 - 1)$

(X) $3 \sin^{-1} x = \sin^{-1}(3x - 4x^3)$, $3 \cos^{-1} x = \cos^{-1}(4x^3 - 3x)$, $3 \tan^{-1} x = \tan^{-1} \frac{3x - x^3}{1 - 3x^2}$

(XI) $\sin^{-1} x = \cos^{-1} \sqrt{1-x^2}$, $\cos^{-1} x = \sin^{-1} \sqrt{1-x^2}$,

$\sin^{-1} x = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$, $\cos^{-1} x = \tan^{-1} \frac{\sqrt{1-x^2}}{x}$

Important Substitutions:

1. For $\sqrt{a^2 - x^2}$ Put $x = a \sin \theta$,

$\sqrt{1-x^2}$ Put $x = \sin \theta$

2. For $\sqrt{a^2 + x^2}$, Put $x = a \tan \theta$,

$\sqrt{1+x^2}$, Put $x = \tan \theta$

3. For $\sqrt{x^2 - a^2}$ Put $x = a \sec \theta$,

$\sqrt{x^2 - 1}$ Put $x = \sec \theta$

4. For $\sqrt{a-x}/\sqrt{a+x}$ Put $x = a \cos \theta$,

$\sqrt{1-x}/\sqrt{1+x}$ Put $x = \cos \theta$ or $x = \sin \theta$

Important Trigonometric Formulas:

(i) $\sin x = 2 \sin \frac{x}{2} \cdot \cos \frac{x}{2} = \frac{2 \tan \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

(ii) $\tan 2x = \frac{2 \tan \frac{x}{2}}{1 - \tan^2 \frac{x}{2}}$

(iii) $\cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} = 2 \cos^2 \frac{x}{2} - 1 = 1 - 2 \sin^2 \frac{x}{2} = \frac{1 - \tan^2 \frac{x}{2}}{1 + \tan^2 \frac{x}{2}}$

(iv) $\sin 3x = 3 \sin x - 4 \sin^3 x$

(v) $\cos 3x = 4 \cos^3 x - 3 \cos x$

(vi) $1 + \cos x = 2 \cos^2 \frac{x}{2}$

(vii) $1 - \cos x = 2 \sin^2 \frac{x}{2}$

(viii) $\sqrt{1 + \sin x} = \sqrt{\left(\cos \frac{x}{2} + \sin \frac{x}{2}\right)^2}$

(ix) $\sqrt{1 - \sin x} = \sqrt{\left(\cos \frac{x}{2} - \sin \frac{x}{2}\right)^2}$