

INVERSE TRIGONOMETRIC FUNCTIONS

1. Write the principal value of $\tan^{-1}(1) + \cos^{-1}\left(\frac{-1}{2}\right)$ (Ans: $\frac{11\pi}{12}$)
2. Write the principal value of $\cos^{-1}\left(\frac{-1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$. (Ans: π)
3. Write the value of $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ (Ans: $\frac{-\pi}{4}$)
4. Write the value of $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$ (Ans: $\frac{5\pi}{6}$).
5. Find the value of the following: $\tan^{-1}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$
(Ans: $\frac{x+y}{1-xy}$).
6. Show that $\tan^{-1}\left(\frac{1}{2}\right) + \tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right) = \frac{\pi}{4}$
7. Show that $\tan\left(\frac{1}{2}\sin^{-1}\frac{3}{4}\right) = \frac{4-\sqrt{7}}{3}$
8. Solve the equation $\cos(\tan^{-1}x) = \sin(\cot^{-1}\frac{3}{4})$.
9. Prove that $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \cos^{-1}\frac{36}{85}$.
10. Prove that $\tan^{-1}\left(\frac{\cos x}{1+\sin x}\right) = \frac{\pi}{4} - \frac{x}{2}$.
11. Prove that $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$.
12. Solve for x : $2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x)$. (Ans: $\frac{\pi}{4}$).
13. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$
14. Find the value of $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\left(\frac{x-y}{x+y}\right)$.
15. Prove that $\tan^{-1}\left[\frac{\sqrt{1+x} + \sqrt{1-x}}{\sqrt{1+x} - \sqrt{1-x}}\right] = \frac{\pi}{4} + \frac{1}{2}\cos^{-1}x$.
16. Prove that $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$.
17. Prove that $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\left(\frac{1}{3}\right) = \frac{9}{4}\sin^{-1}\left(\frac{2\sqrt{2}}{3}\right)$.
18. Prove that $\tan^{-1}\frac{1}{4} + \tan^{-1}\frac{2}{9} = \frac{1}{2}\tan^{-1}\frac{4}{3}$.
19. Solve for x , $\cos(2\sin^{-1}x) = \frac{1}{9}, x > 0$.
20. Prove that $2\tan^{-1}\frac{3}{4} - \tan^{-1}\frac{17}{31} = \frac{\pi}{4}$.
21. Solve the following : $\tan^{-1}\left(\frac{2x}{1-x^2}\right) + \cot^{-1}\left(\frac{1-x^2}{2x}\right) = \frac{\pi}{3}$. (Ans: $-\sqrt{3} \pm 2$).
22. Prove that $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$.
23. $\tan^{-1}x + \tan^{-1}\left(\frac{2x}{1-x^2}\right) = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$.

24. Prove that $\cos[\tan^{-1}\{\sin(\cot^{-1}x)\}] = \sqrt{\frac{1+x^2}{2+x^2}}$.
25. Solve for x, $\cos^{-1}x + \sin^{-1}(\frac{x}{2}) = \frac{\pi}{6}$. (Ans: ± 1).
26. Solve for x, $\tan^{-1}(x+1) + \tan^{-1}(x-1) = \tan^{-1}(\frac{8}{31})$. (Ans: $\frac{1}{4}$)
27. Solve for x : $2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$. (Ans: $\frac{\pi}{4}$).
28. Solve for x, $\tan^{-1}2x + \tan^{-1}3x = \frac{\pi}{4}$. (Ans: $-1, \frac{1}{6}$)
29. Prove that $\tan^{-1}(\frac{1}{3}) + \tan^{-1}(\frac{1}{5}) + \tan^{-1}(\frac{1}{7}) + \tan^{-1}(\frac{1}{8}) = \frac{\pi}{4}$.
30. If $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, then find the value of x. (Ans: $\pm \frac{1}{\sqrt{2}}$).
31. Solve for x, $\tan^{-1}(\frac{1+x}{1-x}) = \frac{\pi}{4} + \tan^{-1}x$, $0 < x < 1$. (Ans: 1)
32. Prove that: $\tan^{-1} 1 + \tan^{-1} 2 + \tan^{-1} 3 = \pi$
33. Prove that $\tan\left\{\frac{\pi}{4} + \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} + \tan\left\{\frac{\pi}{4} - \frac{1}{2}\cos^{-1}\frac{a}{b}\right\} = \frac{2b}{a}$.
34. If $\tan^{-1}\frac{x-3}{x-4} + \tan^{-1}\frac{x+3}{x+4} = \frac{\pi}{4}$, then find the value of x.
35. If $\tan^{-1}\left[\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right] = \alpha$, then prove that $x^2 = \sin 2\alpha$.
36. Prove that $\cos^{-1}\left(\frac{\cos\alpha + \cos\beta}{1 + \cos\alpha\cos\beta}\right) = 2\tan^{-1}\left(\tan\frac{\alpha}{2}\tan\frac{\beta}{2}\right)$.
37. Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$. (Ans: 0, $\frac{1}{2}$)