

CONTINUITY

Left hand limit: For function $f(x)$ on an open interval $(a - h, a + h)$, a real number ℓ is said to be left hand limit of f at a , if for all x sufficiently close to a on the left of a the value of $f(x)$ can be made as close as desired to the number ℓ and is expressed as $\lim_{x \rightarrow a^-} f(x) = \ell$

Right hand limit: For function $f(x)$ on an open interval $(a - h, a + h)$, a real number ℓ is said to be right hand limit of f at a , if for all x sufficiently close to a on the right of a the value of $f(x)$ can be made as close as desired to the number ℓ and is expressed as $\lim_{x \rightarrow a^+} f(x) = \ell$

Important formulas: (i) $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a} = na^{n-1}$ (ii) $\lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$ (iii) $\lim_{x \rightarrow 0} \frac{x}{\tan x} = 1$ (iv)

$\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$ (v) $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$ (vi) $\lim_{x \rightarrow 0} \cos x = 1$

NOTE: For problems having trigonometric functions and $x \rightarrow a$ with $a \neq 0$, put $x = a + h$

Procedure to find R H L $\lim_{x \rightarrow a^+} f(x)$ put $x = a + h$ with $x \rightarrow a^+, h \rightarrow 0$

Procedure to find L H L $\lim_{x \rightarrow a^-} f(x)$ put $x = a - h$ with $x \rightarrow a^-, h \rightarrow 0$

Continuity

A function $y = f(x)$ is said to be continuous at $x = a$ if $\lim_{x \rightarrow a} f(x) = f(a)$.

Procedure to check the continuity of the function at a point

(i) If function consists of $[x], |x|$ or $\leq >$ Calculate $\lim_{x \rightarrow a^+} f(x)$, $\lim_{x \rightarrow a^-} f(x)$ and $f(a)$ and if all the three have same value, function is continuous at $x = a$.

(ii) If function does not have any of the above calculate $\lim_{x \rightarrow a} f(x)$ and $f(a)$ and if both of these are equal, $f(x)$ is continuous at $x = a$

Algebra of continuous functions : For f and g be two real functions continuous at a real number c , then (1) $f + g$ is continuous at $x = c$. (2) $f - g$ is continuous at $x = c$. (3) $f \cdot g$ is continuous at $x = c$. (4) f/g is continuous at $x = c$ provided $g \neq 0$ at $x = c$.

Theorem: Suppose f and g are real valued functions such that $(f \circ g)$ is defined at c . If g is continuous at c and if f is continuous at $g(c)$, then $(f \circ g)$ is continuous at c .

Standard Continuous Functions:

- (i) Identity Function $f(x) = x$
- (ii) Constant Function $f(x) = k$
- (iii) Polynomial Function : $f(x) = 2x^2 + 3$
- (iv) Rational Function $f(x) = g(x)/h(x)$, $h(x) \neq 0$
- (v) Modulus Function : $f(x) = |x|$
- (vi) Log Function $f(x) = \log x$, $x > 0$
- (vii) Sine function $f(x) = \sin x$
- (viii) Cos function $f(x) = \cos x$
- (ix) Reciprocal Function $f(x) = 1/x$, $x \in \mathbb{R} - \{0\}$