

LINEAR PROGRAMMING PROBLEMS

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LINEAR PROGRAMMING PROBLEM

DEFINITION:

- ▶ A linear programming problem is one that is concerned with finding the optimal value (maximum or minimum value) of a linear function of two variables, subject to the constraints that the variables are non negative and satisfy a set of linear inequalities.

IMPORTANT TERMS RELATED TO LPP:

1. OBJECTIVE FUNCTION :

A linear function of two or more variables which has to be maximised or minimised under the given restrictions is called an objective function. The variables used in the objective functions are called decision variables.

2. LINEAR CONSTRAINTS :

The linear inequations or the restrictions on the variable are called linear constraints. The conditions $x \geq 0, y \geq 0$ are called non - negative constraints.

3. OPTIMISATION PROBLEM :

A problem which seeks to maximise or minimise a linear function subject to certain constraints is called an optimisation problem. Linear programming problems are special type of optimisation problems.

4. OPTIMAL VALUE :

The maximum or minimum value of the objective function is known as the optimal value of LPP.

5. FEASIBLE AND INFEASIBLE REGION :

The common region determined by all the constraints including non - negative constraints $x \geq 0, y \geq 0$ is called the feasible region or solution region. The region other than the feasible region is called an infeasible region.

6. FEASIBLE AND INFEASIBLE SOLUTION :

Points within and on the boundary of the feasible region, represents feasible solution. Any point outside the feasible region is called an infeasible solution.

7. OPTIMAL FEASIBLE SOLUTION :

A feasible solution at which the objective function has optimal (maximum or minimum value) is called the optimal solution or optimal feasible solution.

MATHEMATICAL FORM OF LPP :

The general mathematical form of a linear programming problem may be written as follows :

Maximise or Minimise, $Z = c_1x + c_2y$

subject to the linear constraints;

$$a_1x + b_1y \leq 0$$

$$a_2x + b_2y \leq 0 \text{ etc}$$

and non - negative constraints;

$$x \geq 0, y \geq 0$$

TYPES OF LPP :

- ▶ **MANUFACTURING PROBLEM** : These problems involve optimizing the production rate or the net profits of the manufactured products, which could be a function of the available workspace, the number of labourers, machine hours, packaging material used, raw materials required, the market value of the product etc.
- ▶ **DIET PROBLEM** : As the name suggests in itself, such problems involve optimizing the intake of certain types of foods rich in certain nutrients that could help one follow a particular diet plan. More precisely, the goal of a diet problem is to select a set of foods that will satisfy a set of a daily nutritional requirement at a minimum cost.

- **TRANSPORTATION PROBLEMS** : These problems are related to the study of the efficient transportation routes i.e. how efficiently the product from different sources of production is transported to the different markets, such as the total transportation cost is minimum. Analysis of such problems is very crucial for big companies with several production plants and a widespread area to cater to.
- **ALLOCATION PROBLEMS** : These problems involve the distribution of resources among competing alternatives in order to minimize total costs or maximize total return. Such problems have the following components: a set of resources available in given amounts; a set of jobs to be done, each consuming a specified amount of resources; and a set of costs or returns for each job and resource. The problem is to determine how much of each resource to allocate to each job.

SOLUTION OF LPP BY GRAPHICAL METHOD :

The major steps involved in this method are as follows:

- (i) State the problem mathematically
- (ii) Write all the constraints in the form of equations and draw the graph
- (iii) Find the feasible region
- (iv) Find the coordinates of each vertex (corner points) of the feasible region. The coordinates of the vertex can be obtained either by inspection or by solving the two equations of the lines intersecting at the point.
- (v) By substituting these corner points in the objective function we can get the values of the objective function
- (vi) If the problem is maximization then the maximum of the above values is the optimum value.
If the problem is minimization then the minimum of the above values is the optimum value

Example 1 : Solve the following LPP

Maximize $Z = 2x_1 + 5x_2$ subject to the conditions $x_1 + 4x_2 \leq 24$, $3x_1 + x_2 \leq 21$,
 $x_1 + x_2 \leq 9$ and $x_1, x_2 \geq 0$

Solution:

Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant.

Write all the inequalities of the constraints in the form of equations.

Therefore we have the lines $x_1 + 4x_2 = 24$; $3x_1 + x_2 = 21$; $x_1 + x_2 = 9$

$x_1 + 4x_2 = 24$ is a line passing through the points (0, 6) and (24, 0).

Any point lying on or below the line $x_1 + 4x_2 = 24$ satisfies the constraint $x_1 + 4x_2 \leq 24$.

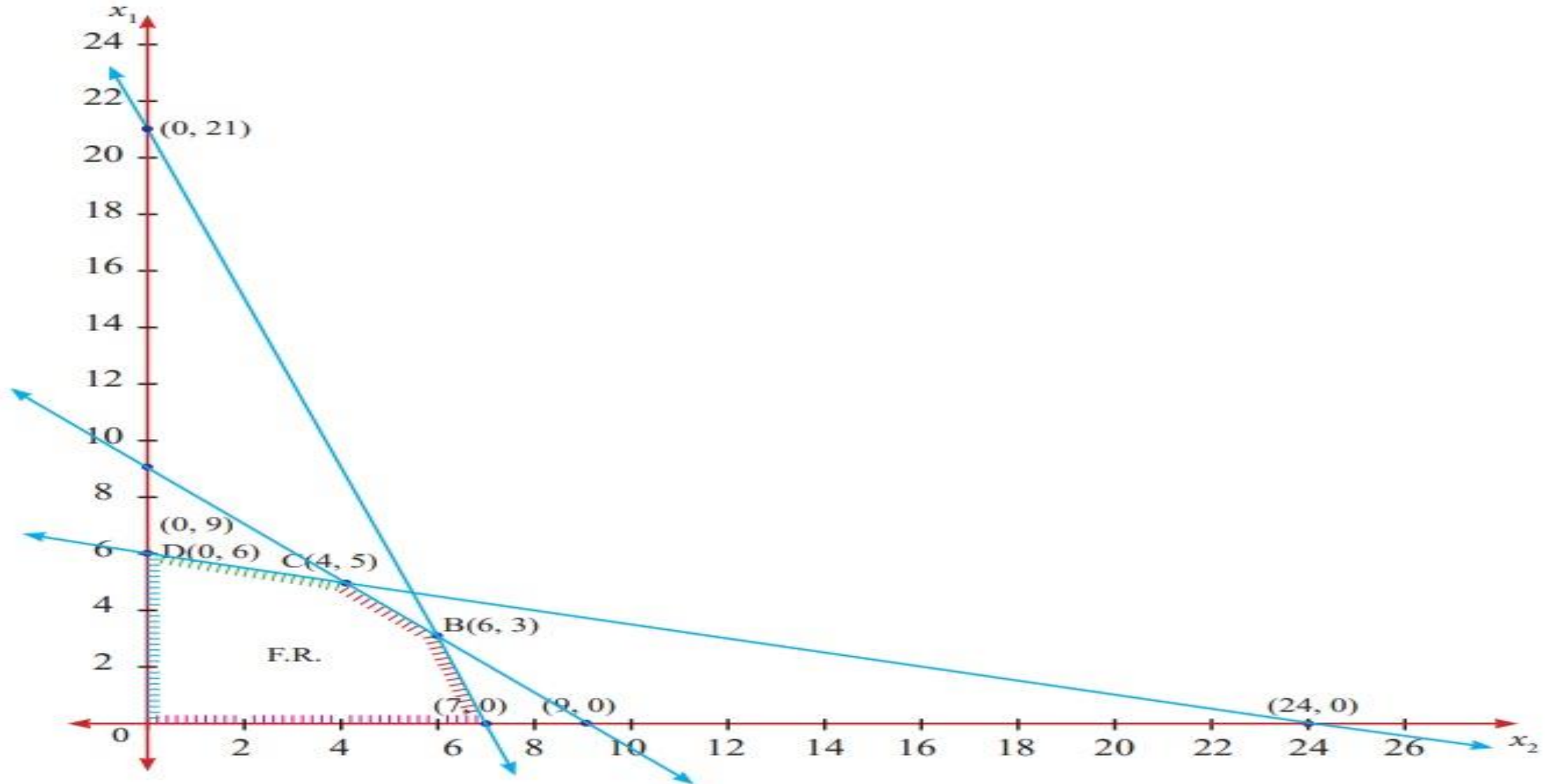
$3x_1 + x_2 = 21$ is a line passing through the points (0, 21) and (7, 0).

Any point lying on or below the line $3x_1 + x_2 = 21$ satisfies the constraint $3x_1 + x_2 \leq 21$.

$x_1 + x_2 = 9$ is a line passing through the points (0, 9) and (9, 0).

Any point lying on or below the line $x_1 + x_2 = 9$ satisfies the constraint $x_1 + x_2 \leq 9$.

Now we draw the graph



The feasible region satisfying all the conditions is OABCD. The co-ordinates of the points are O(0,0); A(7,0); B(6,3) ; C(4,5) and D(0,6).

Corner points	$Z = 2x_1 + 5x_2$
O(0,0)	0
A(7,0)	14
B(6,3)	27
C(4,5)	33
D(0,6)	30

- Maximum value of Z occurs at C. Therefore the solution is $x_1 = 4$, $x_2 = 5$,
 $Z_{\max} = 33$

Example 2

Solve the following LPP by graphical method Minimize $z = 5x_1 + 4x_2$ Subject to constraints $4x_1 + x_2 \geq 40$; $2x_1 + 3x_2 \geq 90$ and $x_1, x_2 \geq 0$

Solution:

Since both the decision variables x_1 and x_2 are non-negative, the solution lies in the first quadrant of the plane.

Consider the equations $4x_1 + x_2 = 40$ and $2x_1 + 3x_2 = 90$

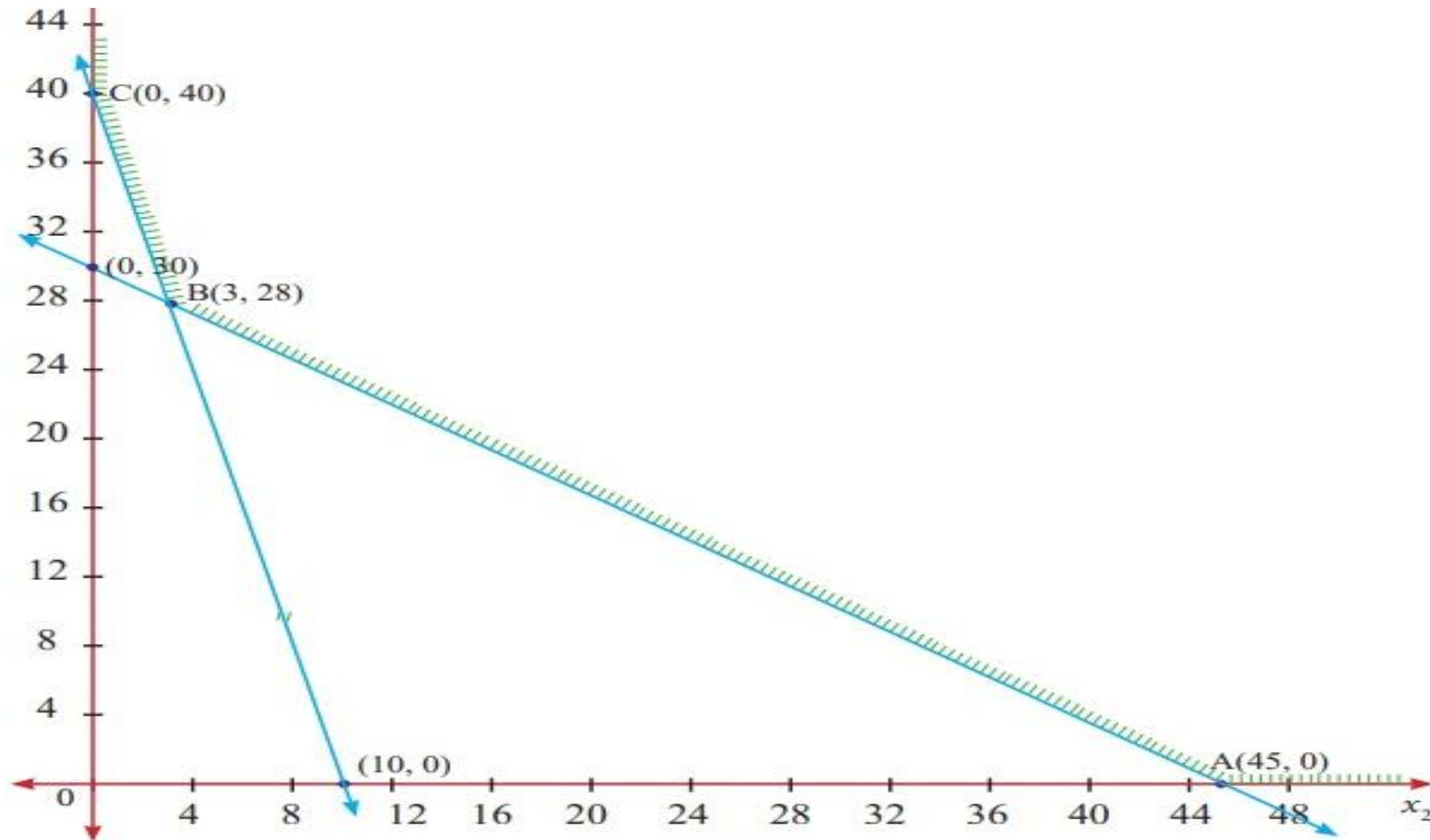
$4x_1 + x_2 = 40$ is a line passing through the points (0,40) and (10,0).

Any point lying on or above the line $4x_1 + x_2 = 40$ satisfies the constraint $4x_1 + x_2 \geq 40$.

$2x_1 + 3x_2 = 90$ is a line passing through the points (0,30) and (45,0).

Any point lying on or above the line $2x_1 + 3x_2 = 90$ satisfies the constraint $2x_1 + 3x_2 \geq 90$.

Draw the graph using the given constraints.



The feasible region is ABC which is unbounded.

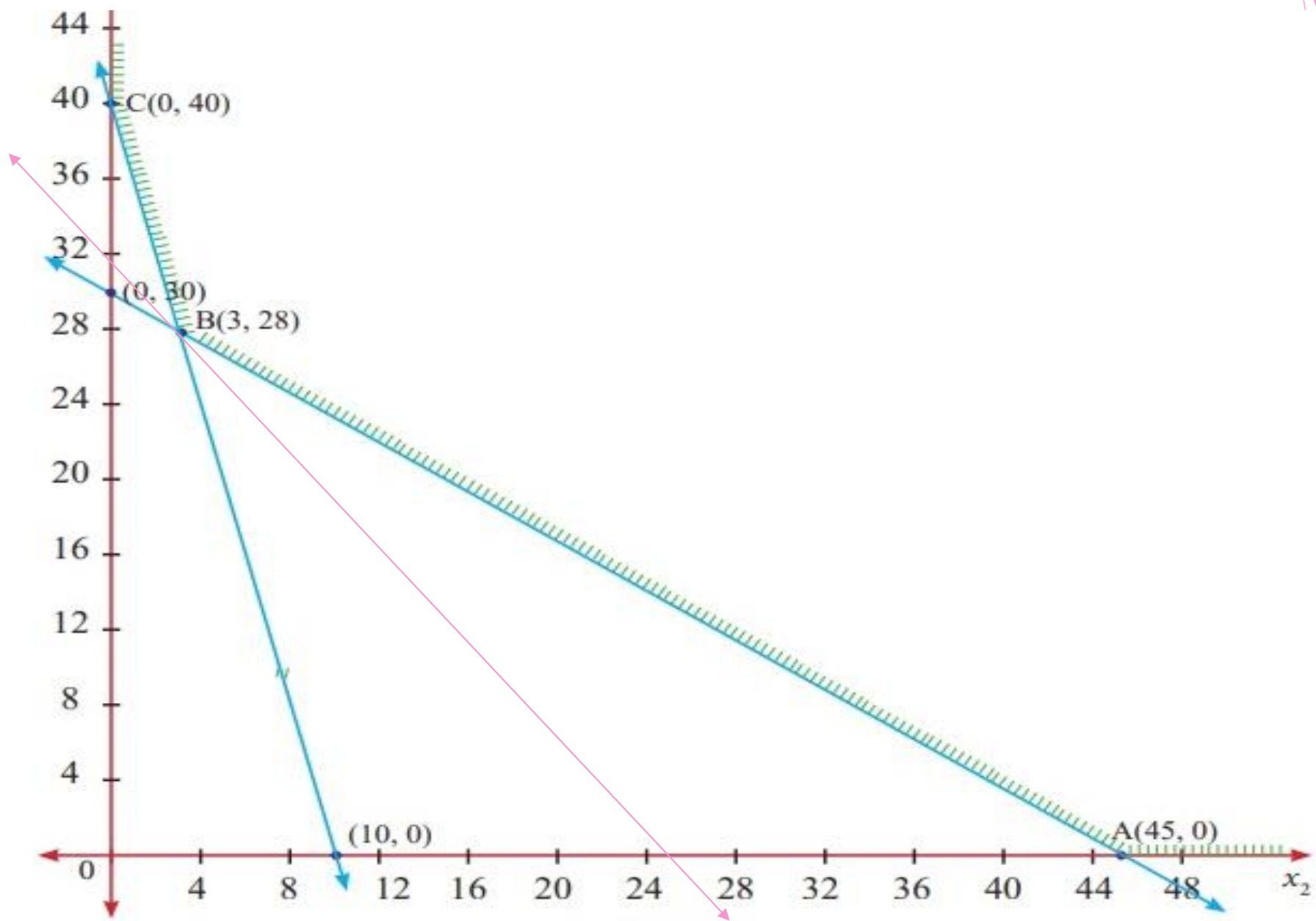
Corner points	$z = 5x_1 + 4x_2$
A(45,0)	225
B(3,28)	127
C(0,40)	160

The minimum value of Z is 127 which occurs at B(3,28).

Since the feasible region is unbounded.

Hence, 127 may or may not be the min. value of Z.

So, we need to plot the inequality $5x_1 + 4x_2 < 127$



As there are no points in common between the feasible region and the inequality $5x_1 + 4x_2 < 127$.

Therefore, minimum value of Z is 127 at $x_1 = 3$, $x_2 = 28$.