

CHAPTER-7

ALTERNATING CURRENT

► Average or Mean Value of Alternating Current:

Average or Mean Value of Alternating Current:

Average or Mean value of alternating current over half cycle is that steady current which will send the same amount of charge in a circuit in the time of half cycle as is sent by the given alternating current in the same circuit in the same time.

$$dq = I dt = I_0 \sin \omega t dt$$

$$q = \int_0^{T/2} I_0 \sin \omega t dt$$

$$q = 2 I_0 / \omega = 2 I_0 T / 2\pi = I_0 T / \pi$$

$$\text{Mean Value of AC, } I_m = I_{av} = q / (T/2)$$

$$I_m = I_{av} = 2 I_0 / \pi = 0.637 I_0 = 63.7 \% I_0$$

Average or Mean Value of Alternating emf:

$$E_m = E_{av} = 2 E_0 / \pi = 0.637 E_0 = 63.7 \% E_0$$

Note: Average or Mean value of alternating current or emf is zero over a cycle as the + ve and – ve values get cancelled.

Root Mean Square or Virtual or Effective Value of Alternating Current:

Root Mean Square (rms) value of alternating current is that steady current which would produce the same heat in a given resistance in a given time as is produced by the given alternating current in the same resistance in the same time.

$$dH = I^2 R dt = I_0^2 R \sin^2 \omega t dt$$

$$H = \int_0^T I_0^2 R \sin^2 \omega t dt$$

$$H = I_0^2 R T / 2 \quad (\text{After integration, } \omega \text{ is replaced with } 2\pi / T)$$

If I_v be the virtual value of AC, then

$$H = I_v^2 R T \quad \therefore I_v = I_{rms} = I_{eff} = I_0 / \sqrt{2} = 0.707 I_0 = 70.7 \% I_0$$

Root Mean Square or Virtual or Effective Value of Alternating emf:

$$E_v = E_{rms} = E_{eff} = E_0 / \sqrt{2} = 0.707 E_0 = 70.7 \% E_0$$

Note:

1. Root Mean Square value of alternating current or emf can be calculated over any period of the cycle since it is based on the heat energy produced.
2. Do not use the above formulae if the time interval under the consideration is less than one period.

AC Circuit Containing Resistance only

All a.c. instruments measure virtual values of a.c. The behavior of an ohmic resistance R in a.c. circuit is the same as in d.c. circuit. Through alternating EMF and alternating current are in same phase.

AC Circuit with a Pure Resistor:

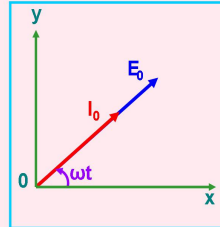
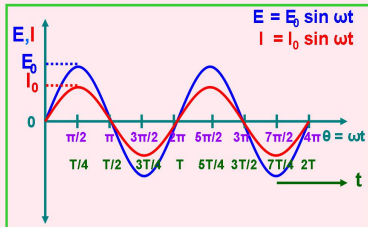
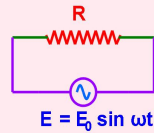
$$E = E_0 \sin \omega t$$

$$I = E / R$$

$$= (E_0 / R) \sin \omega t$$

$$I = I_0 \sin \omega t \quad (\text{where } I_0 = E_0 / R \quad \text{and} \quad R = E_0 / I_0)$$

Emf and current are in same phase.



AC Circuit Containing Pure Inductance only

Inductive reactance (X_L) is the resistance offered by an inductor and is given by

$$X_L = \omega L = 2\pi\nu L$$

Through a pure inductor, alternating current lags behind the alternating EMF by phase angle of 90°

AC Circuit with a Pure Inductor:

$$E = E_0 \sin \omega t$$

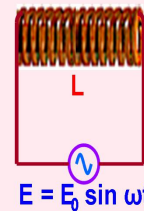
Induced emf in the inductor is $-L (dI / dt)$

In order to maintain the flow of current, the applied emf must be equal and opposite to the induced emf.

$$\begin{aligned} \therefore E &= L (dI / dt) \\ E_0 \sin \omega t &= L (dI / dt) \\ dI &= (E_0 / L) \sin \omega t \, dt \end{aligned}$$

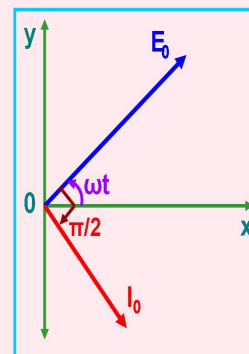
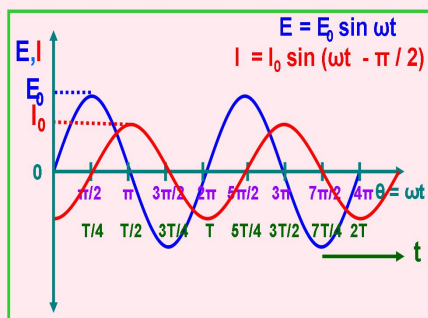


$$\begin{aligned} I &= \int (E_0 / L) \sin \omega t \, dt \\ I &= (E_0 / \omega L) (-\cos \omega t) \\ I &= I_0 \sin (\omega t - \pi / 2) \end{aligned}$$



(where $I_0 = E_0 / \omega L$ and $X_L = \omega L = E_0 / I_0$) Current lags behind emf by $\pi/2$ rad.

X_L is Inductive Reactance. Its SI unit is ohm.



AC Circuit with Pure Capacitance only -Through a pure capacitor, alternating current leads the alternating

EMF by a phase angle of 90° . X_L and X_C both are measured in ohms.

AC Circuit with a Capacitor:

$$E = E_0 \sin \omega t$$

$$q = CE = CE_0 \sin \omega t$$

$$I = dq / dt$$

$$= (d / dt) [CE_0 \sin \omega t]$$

$$I = [E_0 / (1 / \omega C)] (\cos \omega t)$$

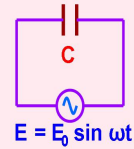
$$I = I_0 \sin (\omega t + \pi / 2)$$

(where $I_0 = E_0 / (1 / \omega C)$ and

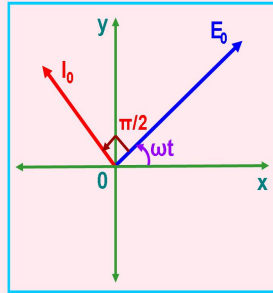
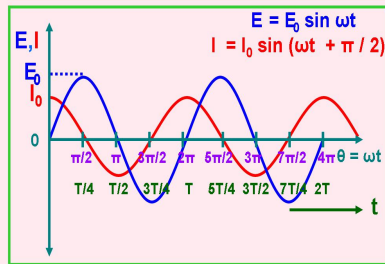
$$X_C = 1 / \omega C = E_0 / I_0)$$

X_C is Capacitive Reactance.

Its SI unit is ohm.



Current leads the emf by $\pi/2$ radians.

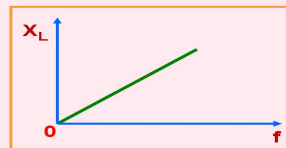


Variation of X_L with Frequency:

$$I_0 = E_0 / \omega L \text{ and } X_L = \omega L$$

X_L is Inductive Reactance and $\omega = 2\pi f$

$$X_L = 2\pi f L \quad \text{i.e.} \quad X_L \propto f$$



Variation of X_C with Frequency:

$$I_0 = E_0 / (1/\omega C) \text{ and } X_C = 1 / \omega C$$

X_C is Inductive Reactance and $\omega = 2\pi f$

$$X_C = 1 / 2\pi f C \quad \text{i.e.} \quad X_C \propto 1 / f$$



TIPS:

- 1) Inductance (L) can not decrease Direct Current. It can only decrease Alternating Current.
- 2) Capacitance (C) allows AC to flow through it but blocks DC.

AC Circuit with L, C, R in Series Combination:

The applied emf appears as Voltage drops V_R , V_L and V_C across R, L and C respectively.

- 1) In R, current and voltage are in phase.

- 2) In L, current lags behind voltage by $\pi/2$

- 3) In C, current leads the voltage by $\pi/2$

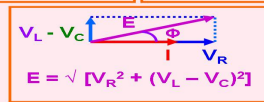
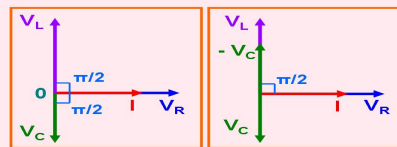
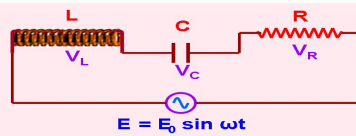
$$E = \sqrt{V_R^2 + (V_L - V_C)^2}$$

$$I = \frac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$$

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

$$Z = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

$$\tan \Phi = \frac{X_L - X_C}{R} \quad \text{or} \quad \tan \Phi = \frac{\omega L - 1/\omega C}{R}$$



Resonance in AC Circuit with L, C, R:

When $X_L = X_C$ i.e. $\omega L = 1/\omega C$, $\tan \Phi = 0$ or Φ is 0° and

$$Z = \sqrt{[R^2 + (\omega L - 1/\omega C)^2]} \text{ becomes } Z_{\min} = R \text{ and } I_{0\max} = E/R$$

i.e. The impedance offered by the circuit is minimum and the current is maximum. This condition is called resonant condition of LCR circuit and the frequency is called resonant frequency.

At resonant angular frequency ω_r ,

$$\omega_r L = 1/\omega_r C \text{ or } \omega_r = 1/\sqrt{LC} \text{ or } f_r = 1/(2\pi\sqrt{LC})$$

Resonant Curve & Q – Factor (sharpness of resonance)

$$\text{Band width} = 2 \Delta \omega$$

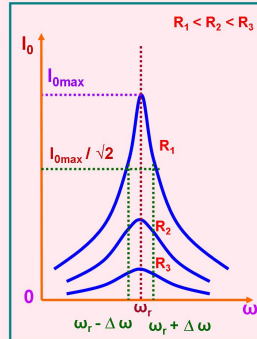
Quality factor (Q – factor) is defined as the ratio of resonant frequency to band width.

$$Q = \omega_r / 2 \Delta \omega$$

It can also be defined as the ratio of potential drop across either the inductance or the capacitance to the potential drop across the resistance.

$$Q = V_L / V_R \text{ or } Q = V_C / V_R$$

$$\text{or } Q = \omega_r L / R \text{ or } Q = 1 / \omega_r C R$$



POWER IN AC CIRCUIT: THE POWER FACTOR

$$I_m = \frac{V_m}{Z} \text{ and } \phi = \tan^{-1} \left(\frac{X_C - X_L}{R} \right)$$

$$p = v i = (v_m \sin \omega t) \times [i_m \sin(\omega t + \phi)]$$

$$= \frac{v_m i_m}{2} [\cos \phi - \cos(2\omega t + \phi)]$$

$$P = \frac{v_m i_m}{2} \cos \phi = \frac{v_m}{\sqrt{2}} \frac{i_m}{\sqrt{2}} \cos \phi$$

$$P = V I \cos \phi$$

$$P_{av} = E_v I_v \cos \Phi$$

Power in AC Circuit with R:

In R, current and emf are in phase.

$$\Phi = 0^\circ$$

$$P_{av} = E_v I_v \cos \Phi = E_v I_v \cos 0^\circ = E_v I_v$$

Power in AC Circuit with L:

In L, current lags behind emf by $\pi/2$.

$$\Phi = -\pi/2$$

$$P_{av} = E_v I_v \cos(-\pi/2) = E_v I_v (0) = 0$$

Power in AC Circuit with C:

In C, current leads emf by $\pi/2$.

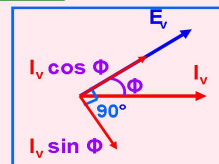
$$\Phi = +\pi/2$$

$$P_{av} = E_v I_v \cos(\pi/2) = E_v I_v (0) = 0$$

Note:

Power (Energy) is not dissipated in Inductor and Capacitor and hence they find a lot of practical applications and in devices using alternating current.

Wattless Current or Idle Current:

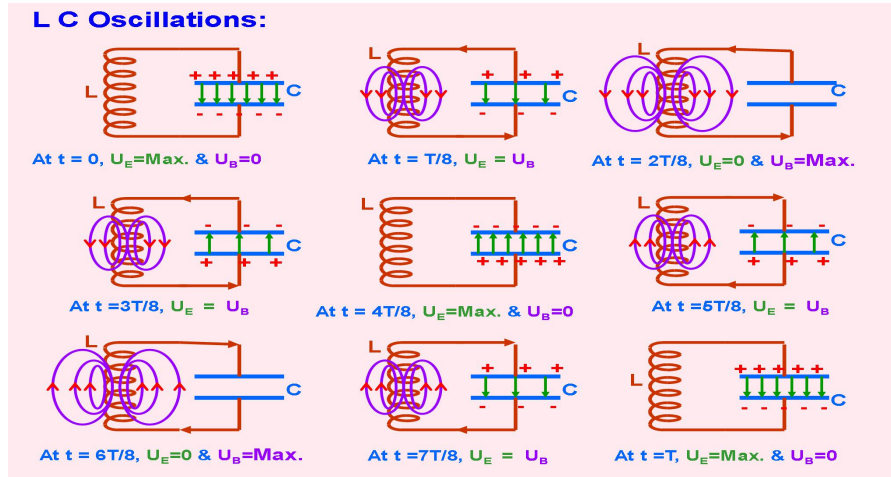


The component $I_v \cos \Phi$ generates power with E_v .

However, the component $I_v \sin \Phi$ does not contribute to power along E_v and hence power generated is zero. This component of current is called wattless or idle current.

$$P = E_v I_v \sin \Phi \cos 90^\circ = 0$$

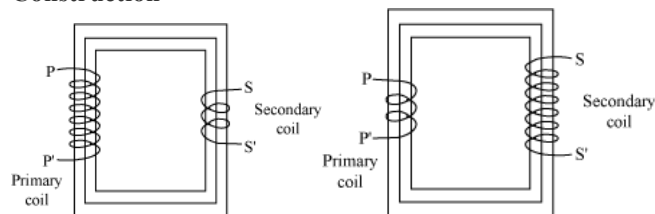
LC Oscillations:



TRANSFORMER

Principle – It works on the principle of mutual induction. It converts low voltage high current into high voltage at low current or vice versa.

Construction



Step-down transformer

Step-up transformer

Working

Alternating *emf* is supplied to the primary coil PP'. The resulting current produces an induced current in secondary.

Magnetic flux linked with primary is also linked with the secondary. The induced *emf* in each turn of the secondary is equal to that induced in each turn of the primary.

Let,

E_p – Alternating *emf* applied to primary

n_p – Number of turns in the primary

$\frac{d\phi}{dt}$ – Rate of change of flux through each turn of primary coil

$$\therefore E_p = -n_p \frac{d\phi}{dt} \quad \dots(1)$$

E_s – Alternating *emf* of secondary

n_s – Number of turns in secondary

$$\therefore E_s = -n_s \frac{d\phi}{dt} \quad \dots(2)$$

Dividing equation (2) by (1),

$$\frac{E_s}{E_p} = \frac{n_s}{n_p} = k$$

- For step-up transformer, $K > 1$

$$\therefore E_s > E_p$$

- For step-down transformer, $K < 1$

$$\therefore E_s < E_p$$

- According to law of conservation of energy,

Input electrical power = Output electrical power

$$E_p I_p = E_s I_s$$

$$\therefore \frac{E_s}{E_p} = \frac{I_p}{I_s}$$

Transformers are used in telegraph, telephone, power stations, etc.

Losses in transformer:

Copper loss – Heat in copper wire is generated by working of a transformer. It can be diminished using thick copper wires.

Iron loss – Loss is in the bulk of iron core due to the induced eddy currents. It is minimized by using thin laminated core.

Hysteresis loss – Alternately magnetizing and demagnetizing, the iron core cause loss of energy. It is minimized

using a special alloy of iron core with silicon.

Magnetic loss – It is due to the leakage of magnetic flux.

RESOURCES:

NCERT TEXT BOOK- CLASS XII, TUTOR VISTA

PPT- MR.C. MANI, RETD DEPUTY COMMISSIONER, KVS RO, CHENNAI