

CHAPTER 8

MOTION

Motion

- Motion defines as the change in position with respect to time and surroundings. Objects changing their places from one point to another with respect to time.
- To describe the position of an object we need to specify a reference point called the origin. An object may seem to be moving to one observer and stationary to another.
- A frame of reference is another object or scene with respect to which we compare an object's position.

Types of Motion

There are three types of motion:

- Translatory motion
- Rotatory motion
- Vibratory motion

Translatory Motion

In translatory motion the particle moves from one point in space to another. This motion may be along a straight line or along a curved path.

- Motion along a straight line is called rectilinear motion.
- Motion along a curved path is called curvilinear motion.
- Example: A car moving on a straight road

Rotatory Motion

In rotatory motion, the particles of the body describe concentric circles about the axis of motion. For example rotation of needles of watch.

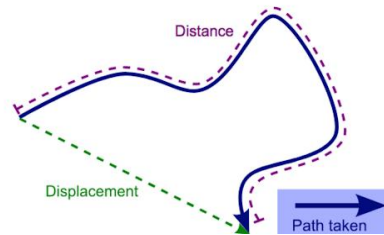
Vibratory Motion

In vibratory motion the particles move to and fro about a fixed point. For pendulum of watch.

Distance

The magnitude of the length covered by a moving object is called distance. It has no direction.

Displacement Displacement is the shortest distance between two points or the distance between the starting and final positions with respect to time. It has magnitude as well direction. Displacement can be zero, but distance cannot be zero.



Magnitude

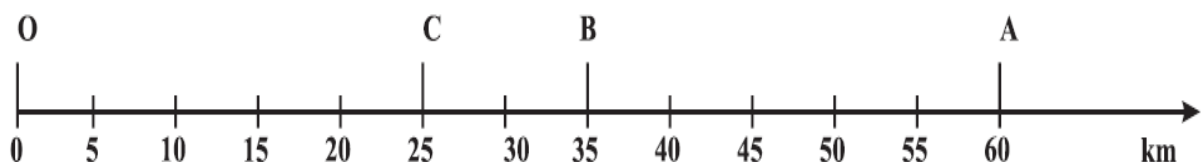
Magnitude is the size or extent of a physical quantity. In physics, we have scalar and vector quantities.

Scalar quantities are only expressed as magnitude. E.g: time, distance, mass, temperature, area, volume

Vector quantities are expressed in magnitude as well as the direction of the object. E.g: Velocity, displacement, weight, momentum, force, acceleration etc.

Motion Along A Straight Line

- The simplest type of motion is the motion along a straight line.
- To distance we need to specify only the numerical value and not the direction of motion.
- The shortest distance measured from the initial to the final position of an object is known as the displacement.
- The magnitude of displacement is less than or equal to the path length.
- The magnitude of the displacement for a course of motion may be zero but the corresponding distance covered or path length can not be zero.



Difference Between Distance And Displacement

Distance	Displacement
Distance provides the complete details of the path taken by the object	Displacement does not provide the complete details of the path taken by the object
Distance is always positive	Displacement can be positive, negative or zero
It is a scalar quantity	It is a vector quantity
The distance between two points may not be unique	Displacement between two points is always unique

Uniform Motion When an object travels equal distances in equal intervals of time the object is said to have a uniform motion.

Non Uniform Motion When an object travels unequal distances in equal intervals of time the object is said to have a non-uniform motion.

Speed It is defined as the distance traveled by the object per unit time. Speed is the rate of change of distance. If a body covers a certain distance in a certain amount of time, its speed is given by

$$\text{Speed} = \text{Distance} / \text{Time}$$

- The SI unit of speed is **metre per second**. This is represented by the symbol **m s^{-1} or m/s**.
- The other units of speed include **centimetre per second (cm s^{-1})** and **kilometre per hour (km h^{-1})**.
- To specify the speed of an object, we require only its magnitude.

Average Speed – If the motion of the object is non-uniform then we calculate the average speed to signify the rate of motion of that object

$$\text{Average Speed (v)} = \text{Total distance travelled (s)} / \text{Total time taken (t)}$$

$$v = s / t$$

Velocity

- Velocity is the speed of an object moving in a definite direction.

- The velocity of an object can be uniform or variable.
- It can be changed by changing the object's speed, direction of motion or both.

Average velocity

When an object is moving along a straight line at a variable speed, we can express the magnitude of its rate of motion in terms of average velocity.

- It is calculated in the same way as we calculate average speed.

$$\text{Average Velocity} = \text{Total Displacement} / \text{Total Time taken}$$

In case the velocity of the object is changing at a uniform rate, then average velocity is given by the arithmetic mean of initial velocity and final velocity for a given period of time. That is,

$$\text{Average velocity} = (\text{initial velocity} + \text{final velocity})/2$$

Mathematically, $v_{av} = (u + v) / 2$

where v_{av} is the average velocity, u is the initial velocity and v is the final velocity of the object.

Speed and velocity have the same units, that is, m s^{-1} or m/s.

Acceleration

- Acceleration, is a measure of the change in the velocity of an object per unit time. That is,

$$\text{Acceleration} = \text{Rate of change of velocity with time}$$

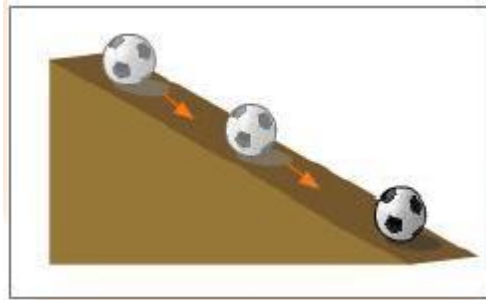
$$\text{Acceleration} = \text{change in velocity} / \text{time taken}$$

- If the velocity of an object changes from an initial value u to the final value v in time t , the acceleration a is,

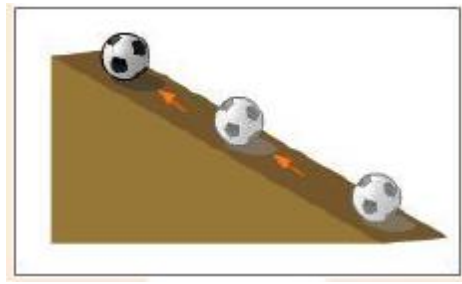
$$a = (v - u) / t$$

- The acceleration is taken to be positive if it is in the direction of velocity and negative when it is opposite to the direction of velocity.
- The SI unit of acceleration is m s^{-2} .
- Acceleration may be uniform and non uniform

Positive Acceleration If the velocity of an object increases, then the object is said to be moving with positive acceleration. Example: A ball rolling down on an inclined plane.



Negative Acceleration If the velocity of an object decreases, then the object is said to be moving with negative acceleration. Negative acceleration is also known as retardation or deceleration. Example: (1) A ball moving up an inclined plane



Zero Acceleration If the change in velocity is zero, i.e., either the object is at rest or moving with uniform velocity, then the object is said to have zero acceleration. Example: a parked car, a train moving with a constant speed of 90 km/hr

Uniform Acceleration If the change in velocity in equal intervals of time is always the same, then the object is said to be moving with uniform acceleration. Example: a body falling from a height towards the surface of the earth.

Non-uniform or Variable Acceleration If the change in velocity in equal intervals of time is not the same, then the object is said to be moving with variable acceleration.

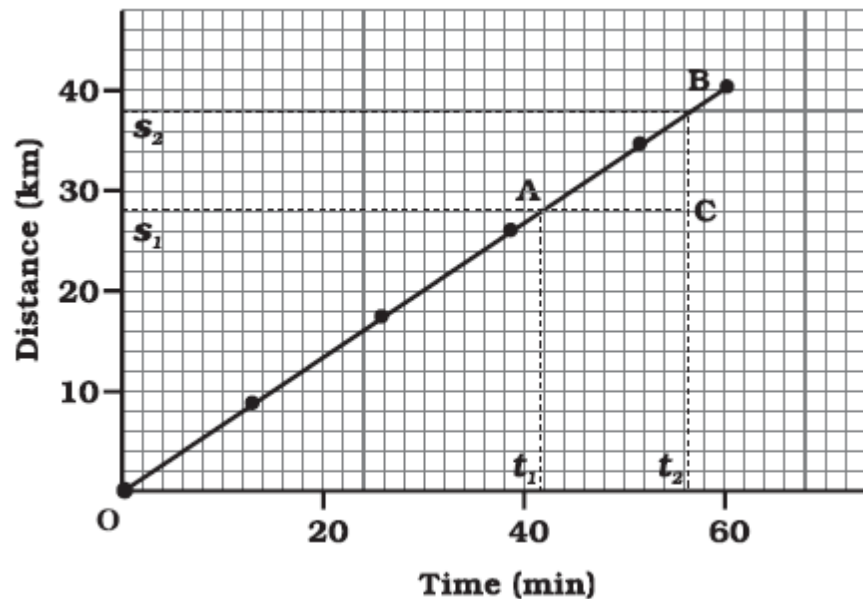
Graphical Representation of Motion

Graphs provide a convenient method to present basic information about a variety of events.

To describe the motion of an object, we can use line graphs. In this case, line graphs show dependence of one physical quantity, such as distance or velocity, on another quantity, such as time.

Distance–Time Graphs

The change in the position of an object with time can be represented on the distance-time graph adopting a convenient scale of choice.

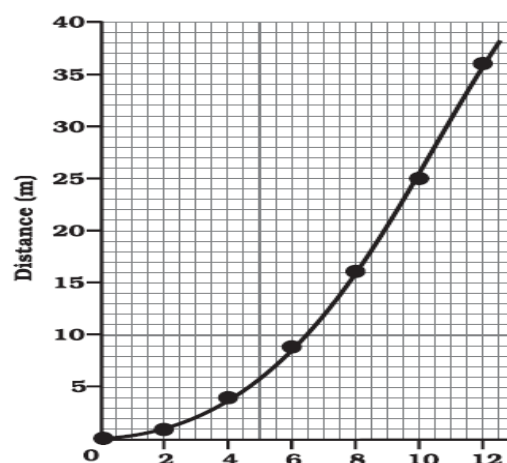


In this graph, time is taken along the x-axis and distance is taken along the y-axis. We know that when an object travels equal distances in equal intervals of time, it moves with uniform speed. This shows that the distance travelled by the object is directly proportional to time taken. Thus, for uniform speed, a graph of distance travelled against time is a straight line. The portion OB of the graph shows that the distance is increasing at a uniform rate.

We can use the distance-time graph to determine the speed of an object. The graph shows, AC denotes the time interval ($t_2 - t_1$) while BC corresponds to the distance ($s_2 - s_1$). We can see from the graph that as the object moves from the point A to B, it covers a distance ($s_2 - s_1$) in time ($t_2 - t_1$). The speed, v of the object, therefore can be represented as

$$v = \frac{s_2 - s_1}{t_2 - t_1}$$

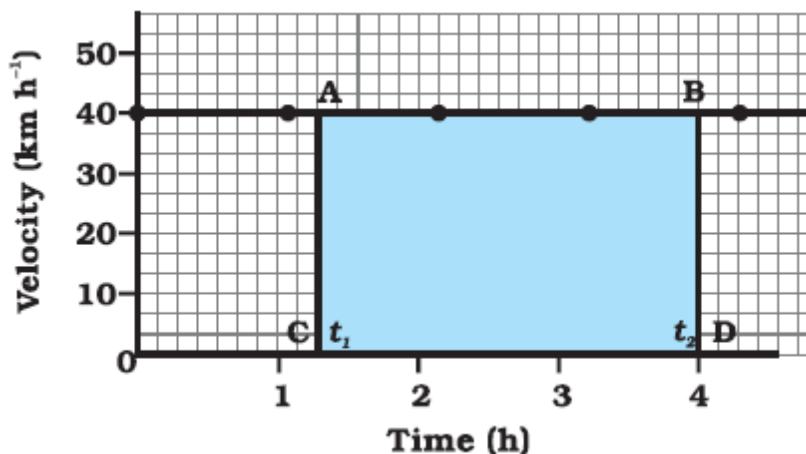
The nature of this graph shows nonlinear variation of the distance travelled by the car with time. Thus, this graph represents motion with non-uniform speed.



Velocity-Time Graphs

The variation in velocity with time for an object moving in a straight line can be represented by a velocity-time graph. In this graph, time is represented along the x-axis and the velocity is represented along the y-axis. If the object moves at uniform velocity, the height of its velocity-time graph will not change with time. It will be a straight line parallel to the x-axis.

We know that the product of velocity and time give displacement of an object moving with uniform velocity. The area enclosed by velocity-time graph and the time axis will be equal to the magnitude of the displacement.

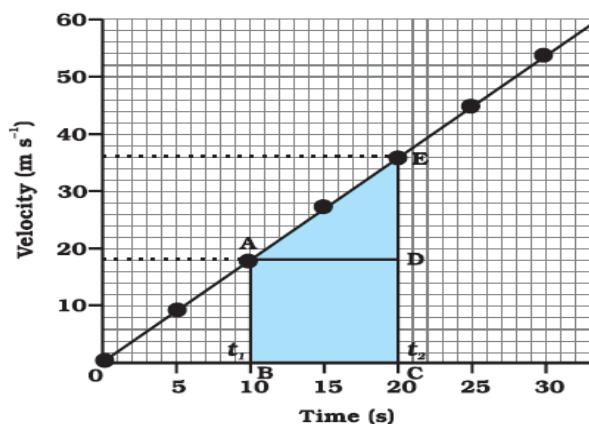


The velocity of 40 km h^{-1} is represented by the height AC or BD and the time $(t_2 - t_1)$ is represented by the length AB.

So, the distance s moved by the car in time $(t_2 - t_1)$ can be expressed as

$$\begin{aligned} s &= AC \times CD \\ &= [(40 \text{ km h}^{-1}) \times (t_2 - t_1) \text{ h}] \\ &= 40 (t_2 - t_1) \text{ km} \\ &= \text{area of the rectangle ABDC (shaded region)} \end{aligned}$$

This graph shows for all uniformly accelerated motion, the velocity-time graph is a straight line. The nature of the graph shows that velocity changes by equal amounts in equal intervals of time.



We can also determine the distance moved by the car from its velocity-time graph.

The area under the velocity-time graph gives the distance (magnitude of displacement) moved by the car in a given interval of time. If the car would have been moving with uniform velocity, the distance travelled by it would be represented by the area ABCD under the graph. Since the magnitude of the velocity of the car is changing due to acceleration, the distance s travelled by the car will be given by the area ABCDE under the velocity-time graph.

That is,

$$\begin{aligned} s &= \text{area ABCDE} \\ &= \text{area of the rectangle ABCD} + \text{area of the triangle ADE} \\ &= AB \times BC + \frac{1}{2} (AD \times DE) \end{aligned}$$

In the case of non-uniformly accelerated motion, velocity-time graphs can have any shape.

Equations of Motion by Graphical Method

When an object moves along a straight line with uniform acceleration, it is possible to relate its velocity, acceleration during motion and the distance covered by it in a certain time interval by a set of equations known as the equations of motion. There are three such equations. These are:

$$v = u + at \dots\dots\dots(1)$$

$$s = ut + \frac{1}{2} at^2 \dots\dots\dots(2)$$

$$2as = v^2 - u^2 \dots\dots\dots(3)$$

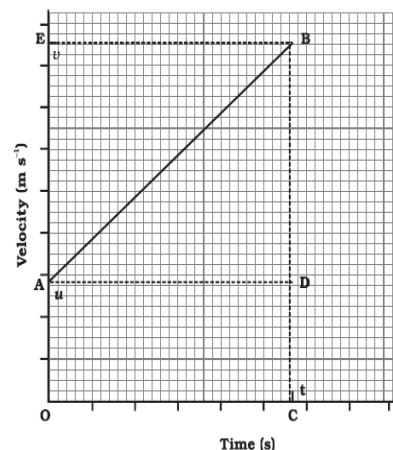
where u is the initial velocity of the object which moves with uniform acceleration a for time t , v is the final velocity, and s is the distance travelled by the object in time t .

Equation For Velocity-Time Relation

Consider the velocity-time graph of an object that moves under uniform acceleration as shown in graph.

From this graph, we can see that initial velocity of the object is u (at point A) and then it increases to v (at point B) in time t . The velocity changes at a uniform rate a .

In Fig. 8.8 graph, the perpendicular lines BC and BE are drawn from point B on the time and the velocity axes respectively, so that the initial velocity is represented by OA, the final velocity is represented by BC and the time interval t is represented by OC. $BD = BC - CD$, represents the change in velocity in time interval t .



Let us draw AD parallel to OC. From the graph, we observe that
 $BC = BD + DC = BD + OA$

Substituting $BC = v$ and $OA = u$,

we get $v = BD + u$

or $BD = v - u \dots\dots\dots(1)$

From the velocity-time graph the acceleration of the object is given by

$a = \text{Change in velocity} / \text{time taken} = BD / AD = BD / OC$

Substituting $OC = t$, we get

$a = BD / t$

$BD = at \dots\dots\dots(2)$

Using Eqs. (1) and (2) we get

$v = u + at$

Equation for position-time Relation

Let us consider that the object has travelled a distance s in time t under uniform acceleration a .

From graph, the distance travelled by the object is obtained by the area enclosed within OABC under the velocity-time graph AB.

Thus, the distance s travelled by the object is given by

$s = \text{area OABC (which is a trapezium)}$

$= \text{area of the rectangle OADC} + \text{area of the triangle ABD}$

$= OA \times OC + \frac{1}{2} (AD \times BD) \dots\dots\dots(3)$

Substituting $OA = u$, $OC = AD = t$ and

$BD = at$,

we get

$$s = u \times t + \frac{1}{2} (t \times at)$$

or $s = u t + \frac{1}{2} at^2$

$s = u t + \frac{1}{2} at^2$

Equation For Position–Velocity Relation

From the velocity-time graph shown, the distance s travelled by the object in time t , moving under uniform acceleration a is given by the area enclosed within the trapezium OABC under the graph. That is,

$s = \text{area of the trapezium OABC}$

$(OA + BC) \times OC / 2$

Substituting $OA = u$, $BC = v$ and $OC = t$,

we get

$s = (u + v) t / 2 \dots\dots\dots(5)$

From the velocity-time relation

$$v = u + at$$

we get

$$t = (v - u) / a \dots\dots\dots(6)$$

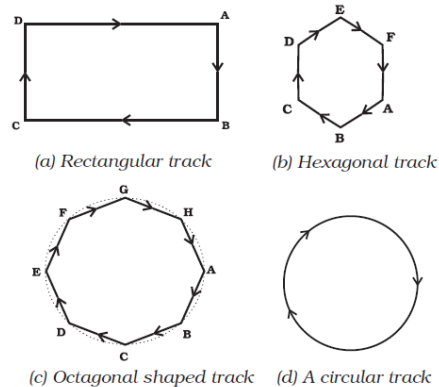
Using equations. (5) and (6) we have

$$s = (v + u) (v - u) / 2a$$

$$\text{or} \quad 2as = v^2 - u^2$$

Uniform Circular Motion

When the velocity of an object changes, we say that the object is accelerating. The change in the velocity could be due to change in its magnitude or the direction of the motion or both. Uniform circular motion is an example when an object does not change its magnitude of velocity but only its direction of motion.



Let us consider an example of the motion of a body along a closed path. Figure shows the path of an athlete along a rectangular track ABCD, hexagonal shaped path ABCDEF, octagon ABCDEFGH. Each of the track has fixed no. of sides. If the athlete runs at a uniform speed on the straight parts AB, BC, CD and DE, EF, FG, GH, and HA of the track. In order to keep himself on track, he quickly changes his speed at the corners. The circular shaped track defines infinite number of small sides, it mean it has infinite number of corners at which the direction of athlete changes. If the athlete moves with a velocity of constant magnitude along the circular path, the only change in his velocity is due to the change in the direction of motion. The motion of the athlete moving along a circular path is, therefore, an example of an accelerated motion.

We know that the circumference of a circle of radius r is given by $2\pi r$. If the athlete takes t seconds to go once around the circular path of radius r , the velocity v is given by

$$v = \frac{2\pi r}{t}$$

When an object moves in a circular path with uniform speed, its motion is called uniform circular motion.