

CHAPTER-4- MOVING CHARGES AND MAGNETISM

Lorentz Force

The force experienced by a charged particle moving in space where both electric and magnetic field exist is called Lorentz force. The Lorentz

force $\vec{F}$ is therefore the vector sum of force due to electric field and the force due to magnetic field. Mathematically

$$\vec{F} = \vec{F}_E + \vec{F}_M$$

$$\vec{F} = q\vec{E} + q\left(\vec{V} \times \vec{B}\right)$$

or

$$\vec{F} = q\left(\vec{E} + \left(\vec{V} \times \vec{B}\right)\right)$$

Definition of 1 Tesla

The magnetic field induction at a point is said to be one tesla if a charge of 1 coulomb while moving at right angles to a magnetic field with a velocity of 1m/sec experiences a force of 1N at that point.

Magnetic Force on a Moving Charge

The magnetic force $\vec{F}_m = q\left(\vec{V} \times \vec{B}\right)$ has some important features like,

- 1) The force is zero when charges are at rest, i.e., $\vec{V} = 0$,
- 2) When the charges move parallel or antiparallel to the magnetic field $\theta = 0^\circ$ or 180° F_m is zero.
- 3) The force always acts perpendicular to velocity and magnetic field.

Magnetic Force on a Current Carrying Conductor in an Uniform Magnetic Field

A straight rod carrying current is considered.

Let,

A – Cross-sectional area of the rod

l – Length of the rod

n – Number density of mobile charge carriers

I – Current in the rod

v_d – Average drift velocity of mobile charge carrier

B – External magnetic field

Force on the carriers,

$$F = (nAl) qv_d \times B$$

Since current density, $j = nqv_d$

$$\therefore F = [(nqv_d) Al] \times B$$

$$F = [jAl] \times B$$

$$F = I l \times B$$

Where,

l is the vector magnitude of length of the rod.

For a wire of arbitrary shape, $F = \sum Idl \times B$

The total force on conductor due to free electrons

$$\text{or } \vec{F} = I\vec{l} \times \vec{B} \sin \theta$$

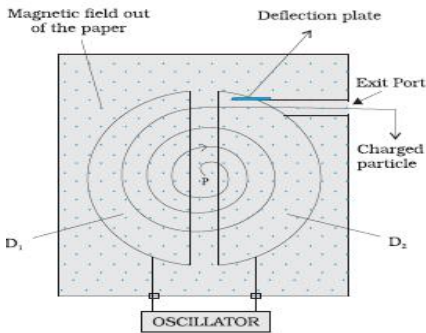
The direction of the force can be given by right hand screw rule or Fleming's left hand rule.

Note:

- 1) If $= 0^\circ$ or 180° then $F = 0$ (minimum)
- 2) If $= 90^\circ$ then $F = I \times l \times B$ (maximum)

CYCLOTRON

A cyclotron is device by which positively charged particle can be accelerated and the desired nuclear reaction can be brought about

**Principle**

A positively charged particle can be accelerated to high energy with the help of an oscillating electric field, by making it cross the same electric field time and again with the use of a strong magnetic field.

Theory and Working

The positive ion P to be accelerated is placed in between the two dees. If at any instant, D_1 is at negative potential and D_2 is at positive potential, then the ion gets accelerated towards D_1 but since its perpendicular to B , it describes a circular path of radius r and Lorentz force provides the centripetal force.

$$\text{Now, } Bqv = \frac{mv^2}{r}$$

$$r = \frac{mv}{Bq}$$

Time taken to describe a semicircle is

$$t = \frac{\pi r}{v} = \frac{\pi m}{Bq} = \frac{\pi}{B \left(\frac{q}{m} \right)} = \text{constant}$$

If this time is equal to the time during which D_1 and D_2 change their polarity, the ion gets accelerated when it arrives in between the gaps. The electric field accelerates the ion further. Once the ion is inside the dee D_2 , it now describes a greater semicircle due to the magnetic field. This process repeats and the ion goes on describing a circular path of greater radius and finally acquires a high energy ion is further removed from a window W. The maximum energy acquired by the ion source is

$$\frac{mv_0^2}{r_0} = Bqv_0$$

$$v_0 = \frac{Bqr_0}{m}$$

$$K.E_{\text{max}} = \frac{1}{2}mv_0^2 = \frac{1}{2}m \left(\frac{Bqr_0}{m} \right)^2$$

$$K.E_{\text{max}} = \frac{B^2 q^2 r_0^2}{2m}$$

The frequency of cyclotron is given by

$$f = \frac{1}{T} = \frac{1}{2t} = \frac{Bq}{2\pi m}$$

Limitations of Cyclotron

Only when the speed of the circulating ion is less than 'c' the speed of light, we find the frequency of revolution to be independent of its speed.

At higher speeds, the mass of the ion will increase and this changes the time period of the ion revolution. This results in the ion lagging behind the electric field and it eventually loses by collisions against the walls of the dees.

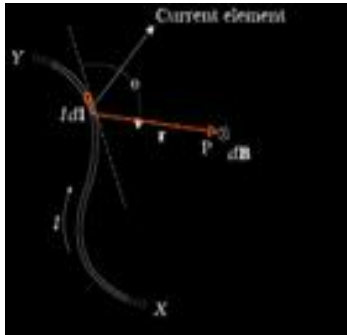
The cyclotron is suitable for accelerating heavy charged particles but not electrons.

Cyclotrons cannot accelerate in charged particles.

It is not suited for very high kinetic energy.

BIOT-SAVART LAW

If AB represents a current element of a conductor PQ carrying current I and $\vec{r}$ the position vector of P from the current element AB (i.e., of length $\vec{dl}$), then the law states that magnetic field (dB) at P due to current element depends on



$$dB \propto I$$

$$dB \propto dl$$

$$dB \propto \sin \theta (\text{angle between } \vec{dl} \text{ and } \vec{r})$$

$$db \propto \frac{1}{r^2}$$

Combining we get,

$$dB \propto \frac{I dl \sin \theta}{r^2}$$

$$dB = \frac{\mu_0}{4\pi} \frac{I dl \sin \theta}{r^2}$$

$$\text{where } \mu_0 = 4\pi \times 10^{-7} \frac{\text{Tm}}{\text{A}} \left(\text{i.e. Tesla metre (Ampere)}^{-1} \right)$$

and is called permeability of free space.

In the vector form,

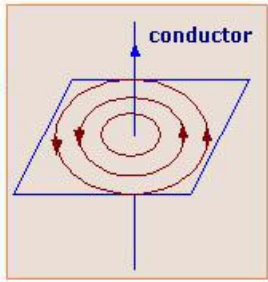
$$\vec{dB} = \frac{\mu_0 I (\vec{dl} \times \vec{r})}{4\pi r^3}$$

Direction of $\vec{dB} \rightarrow$ It is given by right handed screw rule

S.I. unit is 1 tesla or 1T.

Magnetic Field due to a Straight Conductor Carrying Current

The magnitude of the magnetic field at a distance R from a long, straight wire carrying a current i is given by:



$$B = \frac{\mu_0 I}{2\pi R}$$

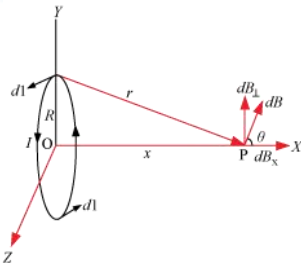
The field lines are circles concentric with the wire

MAGNETIC FIELD ON THE AXIS OF A CIRCULAR CURRENT LOOP

Consider a conducting element $d\mathbf{l}$ of the loop. This is shown in Fig. 4.11. The magnitude dB of the magnetic field due to $d\mathbf{l}$ is given by the Biot-Savart law [Eq. 4.11(a)],

$$dB = \frac{\mu_0}{4\pi} \frac{I |d\mathbf{l} \times \mathbf{r}|}{r^3} \quad (4.12)$$

$$\text{Now } r^2 = x^2 + R^2.$$



$$dB = \frac{\mu_0}{4\pi} \frac{I dl}{(x^2 + R^2)}$$

$$\cos \theta = \frac{R}{(x^2 + R^2)^{1/2}}$$

$$dB_x = dB \cos \theta$$

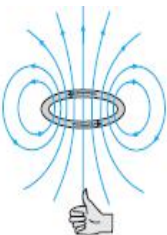
$$dB_x = \frac{\mu_0 I dl}{4\pi} \frac{R}{(x^2 + R^2)^{3/2}}$$

Summation of dl over the loop is given by $2\pi R$

$$\mathbf{B} = B_x \hat{\mathbf{i}} = \frac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}} \hat{\mathbf{i}}$$

When point P is at the centre, $x=0$

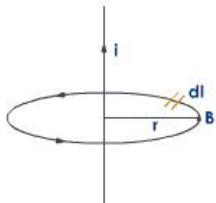
$$\mathbf{B}_0 = \frac{\mu_0 I}{2R} \hat{\mathbf{i}}$$



The magnetic field lines for a current loop. The direction of the field is given by the right-hand thumb rule described in the text. The upper side of the loop may be thought of as the north pole and the lower side as the south pole of a magnet.

Ampere's Circuital Law

The line integral of the magnetic field B around any closed path is equal to μ_0 times the net current across the area bounded by the path."

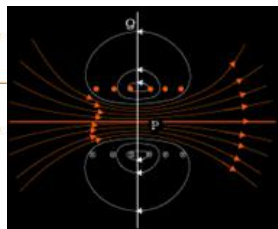
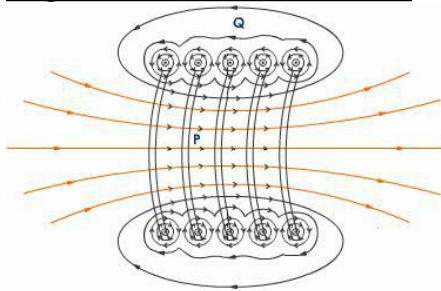


$$\oint B \cdot dl = \oint \frac{\mu_0 I}{2\pi r} dl = \frac{\mu_0 I}{2\pi r} \oint dl$$

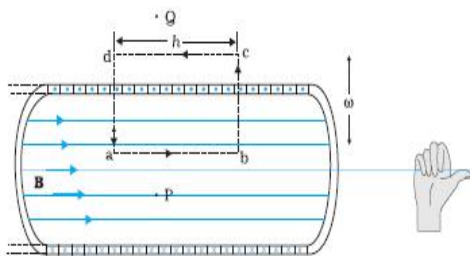
$$\oint dl = \text{circumference of the circle} = 2\pi r$$

$$\oint B \cdot dl = \mu_0 I$$

Magnetic Field due to Solenoid



MAGNETIC FIELD due TO FINITE SOLENOID



Solenoid is long wire wound in form of helix such that the length of solenoid is large compared to the radius of the closely spaced turns.

To find the magnetic field due to a solenoid consider the Amperian loop (imaginary closed path) as shown in the diagram.

The field along cd is zero as it is outside the solenoid. Along da and bc the transverse section the field is zero

outside the solenoid (also, B is perpendicular to dl so $\vec{B} \cdot d\vec{l} = 0$). Therefore the only contribution is from ab. Let the length ab be 'h'. If there are n turns per unit length, then the enclosed current i_e is

$i_e = i(nh)$, where i is the current in the solenoid.

$$\therefore \oint \vec{B} \cdot d\vec{l} = \mu_0 i_e$$

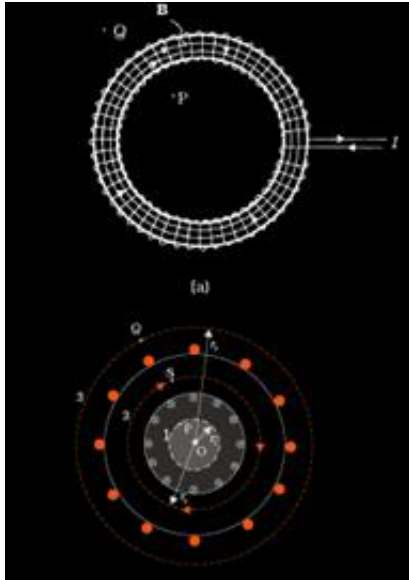
$$\int_{ab} B dl = \mu_0 i_e$$

$$Bh = \mu_0 n h i \quad \left(\because \int_{ab} dl = ab = h \right)$$

$$B = \mu_0 n i$$

Magnetic Field due to Toroidal Solenoids

Toroid is a hollow circular ring (like a medu vadai) on which a large number of turns of a wire are wound.



Using Ampere's law

$$2\pi r B = \mu_0 N i$$

$$B = \frac{\mu_0 N i}{2\pi r}$$

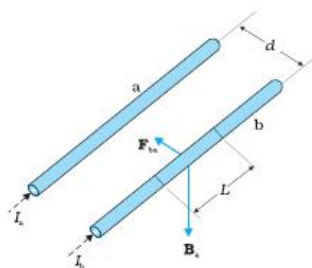
If the radial thickness of the core is small, field is almost constant across the section.

Here $2\pi r$ circumferential length of the toroid.

$$\frac{N}{2\pi r} - \text{number of turns per unit length 'n' then } B = \mu_0 n i$$

Field outside the toroid and inside the core of the toroid is zero and within the toroid $= \mu_0 n i$

Force between Two Parallel Conductors Carrying Currents



Magnetic field induction at a point on conductor b due to current I_a passing through a is

$$B_a = \frac{\mu_0 I_a}{2\pi d}$$

•

Unit length of b will experience a force as

$$F_{ba} = I_b L B_a$$

$$= \frac{\mu_0 I_a I_b}{2\pi d} L$$

Magnetic field induction at a point on conductor a due to current I_b passing through b is

$$B_b = \frac{\mu_0 I_b}{2\pi d}$$

$$F_{ab} = I_a L B_b$$

$$= \frac{\mu_0 I_a I_b}{2\pi d} L$$

Conductor a also experiences the same amount of force, directed towards b . Hence, a and b attract each other.

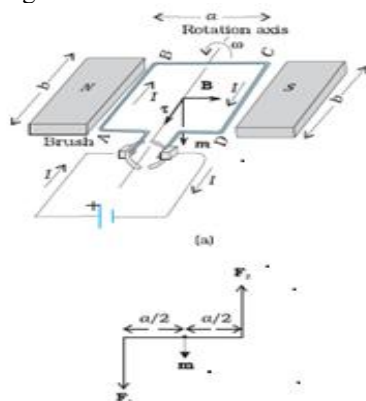
Two linear parallel conductors carrying currents in the same direction attract each other while in opposite direction repel each other.

One ampere is that much current which when flowing through each of the two parallel conductors of length 1m and separated by 1m in vacuum experiences an attractive or repulsive force of 2×10^{-7} N per metre of their length.

The Current Loop as a Magnetic Dipole

Torque on a Current Carrying Coil in a Magnetic Field

When an electric current flows in a closed loop of wire, placed in a uniform magnetic field, the magnetic forces produce a torque which tends to rotate the loop so that area of the loop is perpendicular to the direction of the magnetic field.



Case I - The rectangular loop is placed such that the uniform magnetic field B is in the plane of loop.

No force is exerted by the magnetic field on the arms AD and BC .

Magnetic field exerts a force F_1 on arm AB .

$$\therefore F_1 = IbB$$

Magnetic field exerts a force F_2 on arm CD .

$$\therefore F_2 = IbB = F_1$$

Net force on the loop is zero. The torque on the loop rotates the loop in anti-clockwise direction.

$$\text{Torque, } \tau = F_1 \frac{a}{2} + F_2 \frac{a}{2}$$

$$= IbB \frac{a}{2} + IbB \frac{a}{2}$$

$$= I(ab)B$$

$$\tau = IAB$$

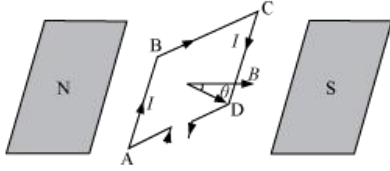
If there are 'n' such turns the torque will be $nIAB$

Where, $b \rightarrow$ Breadth of the rectangular coil

$a \rightarrow$ Length of the rectangular coil

$A = ab \rightarrow$ Area of the coil

Case II - Plane of the loop is not along the magnetic field, but makes angle with it.



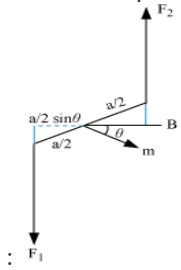
Angle between the field and the normal is θ .

Forces on BC and DA are equal and opposite and they cancel each other as they are collinear.

Force on AB is F_1 and force on CD is F_2 .

$$F_1 = F_2 = IbB$$

Magnitude of torque on the loop as in the figure



$$\therefore \tau = F_1 \frac{a}{2} \sin \theta + F_2 \frac{a}{2} \sin \theta$$

$$= Iab B \sin \theta$$

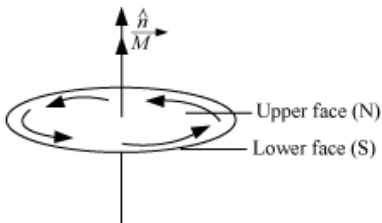
$$\tau = IAB \sin \theta$$

If there are 'n' such turns the torque will be $nIAB \sin \theta$

Magnetic moment of the current, $m = IA$

$$\therefore \vec{\tau} = \vec{m} \times \vec{B}$$

Current carrying wire can be considered as a magnetic dipole.



• Its upper face has current flowing in anti-clockwise direction.

$\therefore$ It has North polarity.

Its lower face has current flowing in clockwise direction.

$\therefore$ It has South polarity.

Magnetic dipole moment of current loop (M) is

directly proportional to strength of current (I)

$$\therefore M \propto I$$

directly proportional to the area of enclosed loop (A)

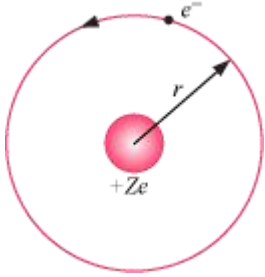
$$\therefore M \propto A$$

$$\Rightarrow M = kIA$$

Where, k is constant of proportionality

For $k = 1, M = AI$ For N such turns, $M = NIA$

Magnetic dipole moment of a revolving electron



An electron of charge e revolves in a path of radius r having a uniform angular velocity ω .

$$\text{Current, } i = \frac{\text{Charge}}{\text{Time}} = \frac{e}{t}$$

Where, t is period of revolution of electron

$$\therefore t = \frac{2\pi}{\omega}$$

$$\therefore i = \frac{e\omega}{2\pi}$$

Let r be the radius of the orbit.

$$\therefore \text{Area of the orbit, } A = \pi r^2$$

$$\text{Magnetic moment of the atom, } M = iA$$

$$M = \frac{\omega e}{2\pi} \pi r^2$$

$$\therefore M = \frac{1}{2} e\omega r^2 \quad \dots\dots\dots (1)$$

According to Bohr's theory, an electron in an atom can revolve only in a certain stationary orbit. Therefore, the

angular momentum of electron (mvr) is an integral multiple of $\left(\frac{h}{2\pi}\right)$.

$$\therefore mvr = \frac{nh}{2\pi}$$

$$\therefore v = r\omega$$

$$\therefore m(r\omega)r = \frac{nh}{2\pi}$$

$$\omega r^2 = \frac{nh}{2\pi m}$$

From equation (1),

$$M = \frac{1}{2} e \cdot \frac{nh}{2\pi m}$$

$$= n \frac{eh}{4\pi m} = n\mu_B$$

$$\mu_B = \frac{eh}{4\pi m}$$

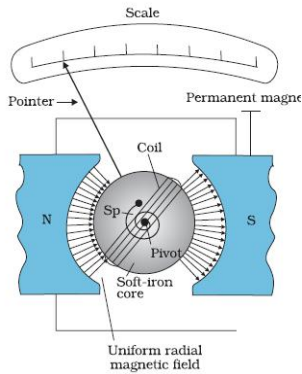
Where, μ_B is the least value of atomic dipole moment known as Bohr's magneton.

Moving Coil Galvanometer

Moving coil galvanometer is an instrument used for detection and measurement of small electric currents.

Principle

A current carrying conductor placed in a magnetic field experiences a torque.



Radial Field (Top view of the Galvanometer)

Working

When a current is passed through the coil in the direction, then the coil experiences a torque

$$\tau = nIBA \sin \alpha$$

Since the magnified field is radial the plane of the coil is parallel to the magnetic field $\vec{B}$ such that $\alpha = 90^\circ$ and hence

$$\tau = NIBA.$$

The coil rotates and the phosphor bronze strip gets twisted. As a result a restoring torque comes into play trying to restore the coil back to original position.

If Φ be the twist produced in the strip and C be the restoring torque per unit twist then the restoring torque = $C \Phi$

In equilibrium,

$$NIBA = C \Phi$$

$$I = \left(\frac{C}{NBA} \right) \Phi$$

$$I = G \Phi$$

where G is the galvanometer constant.

Hence $I \propto \Phi$

This shows that galvanometer has a linear scale and detects the presence of current.

Current sensitivity of galvanometer is the deflection produced for a unit current flowing through it. i.e.,

$$\frac{\Phi}{I} = \frac{NBC}{C} \text{ unit rad/Ampere}$$

A sensitive galvanometer should have a long deflection for small current through it.

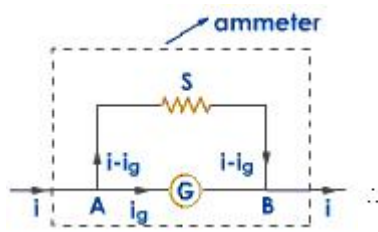
Voltage sensitivity is the deflection produced for a unit voltage applied across the two terminals of the

galvanometer

$$\frac{\phi}{IR} = \frac{NBA}{CR}$$

Conversion of Galvanometer into Ammeter

A galvanometer can be converted into an ammeter (device measuring the current flowing through a conductor) by connecting a low resistance (called shunt resistance) in parallel to the galvanometer as shown in the figure.



$$S = \left(\frac{i_g}{i - i_g} \right) R_g$$

$R_g = \text{Galvanometer Resistance}$

The effective resistance of shunt and galvanometer is

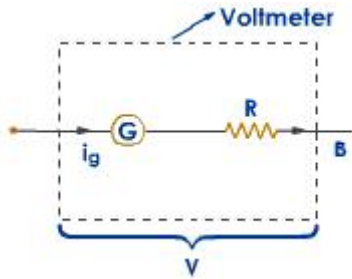
$$\frac{1}{R_{\text{effective}}} = \frac{1}{R_g} + \frac{1}{S}$$

It is to be noted that as the value of S is low, the parallel combination has a much lower resistance. It is for this reason that ammeter have very low resistance.

'Ideal Ammeters have zero resistance'.

Conversion of Galvanometer to Voltmeter

A Galvanometer can be converted into a voltmeter by connecting a high resistance in series with a galvanometer as shown.



The total resistance of voltmeter

$$= R_g + R$$

$$i_g = \frac{V}{R_g + R}$$

$$R = \frac{V}{i_g} - R_g$$

This works as a voltmeter of range 0 to V volt. Since the value of R is high, the effective resistance also has a higher value. Thus voltmeters have high resistance.

'Ideal Voltmeter has infinite resistance'.

$$\frac{\phi}{I} = \frac{NAB}{k} \quad \text{----- 1}$$

$$\frac{\phi}{V} = \left(\frac{NAB}{k} \right) \frac{I}{V} = \left(\frac{NAB}{k} \right) \frac{1}{R} \quad \text{----- 2}$$

An interesting point to note is that increasing the current sensitivity may not necessarily increase the voltage sensitivity. Let us take Eq₁ which provides a measure of current sensitivity. If $N \rightarrow 2N$, i.e., we double the number of turns, then

$$\frac{\phi}{I} \rightarrow 2 \frac{\phi}{I} \quad \text{g g}$$

Thus, the current sensitivity doubles. However, the resistance of the galvanometer is also likely to double, since it is proportional to the length of the wire. In the eqn 2 $N \rightarrow 2N$, and $R \rightarrow 2R$, thus the voltage sensitivity,

$$\frac{\phi}{V} \rightarrow \frac{\phi}{V} \quad \text{1 1}$$

remains unchanged. So in general, the modification needed for conversion of a galvanometer to an ammeter will be different from what is needed for converting it into a voltmeter.

RESOURCES:

NCERT TEXT BOOK- CLASS XII, TUTOR VISTA

PPT- MR.C. MANI, RETD DEPUTY COMMISSIONER, KVS RO, CHENNAI