

SUBJECT: MATHEMATICS
TOPIC: ARITHMETIC PROGRESSIONS
CLASS: X

Consider the following series:

(i) 2,4,6,8,..... (ii) 2,4,8,16,..... (iii) 2,4,5,10,.... (iv) 19,15,11,7....

Examples (i) and (iv) have a **common difference** but in examples (ii) and (iii) don't have a **common difference**.

An arithmetic progression is a list of numbers in which each term is obtained by adding a fixed number to the preceding term except the first term.

The common difference of the AP. Remember that it can be *positive, negative or zero*.

The general form of an AP thus can be represented as $a, a + d, a + 2d, a + 3d, \dots$

2,4,6,8,.....200 is an *finite series*

2,4,6,8,..... is an *infinite series*

Example (v): 5,9,13,.....

Here the *first term* is 5 while the *common difference* is $9-5=4$. As its Infinite AP, the *last term* is taken as a_n

Example (v): 5,9,13,.....121

Here the *first term* is 5 while the *common difference* is $9-5=4$. As its Infinite AP, the *last term* is taken as $a_n = 121$.

Thus a, d, n and a_n are important terms of an AP

Type I: To determine if the given series is AP or not.

Working: (i) Find the difference between the second term and first term.

$$d = a_2 - a_1$$

(ii) Find the difference between the third term and second term.

$$d = a_3 - a_2$$

(iii) If the difference in (i) and (ii) are same then it is an AP otherwise it's not.

Find out which of the following sequences are arithmetic progressions. For those which are arithmetic progressions, find out the common differences.

Q1. 3, 6, 12, 24, Q2. 0, 4, 8, 12, Q3. $1/2, 1/3, 1/4, 1/5, \dots$ Q4. 3, 3, 3, 3,

Q5. 11, 15, 17, 25,

Type I: To determine n^{th} term of an AP using the formula $a_n = a + (n - 1)d$

Working: (i) Write the data of the given AP

$a=?$, $d=?$, $n=?$, $a_n=?$

(ii) Write the formula $a_n = a + (n - 1)d$

(iii) Substitute the values of a, d, n, a_n and simplify.

Q1. Find the 10^{th} term of the AP 1, 4, 7, 10.

Q2. Find the 10^{th} term of the AP 40, 15, 10, 35,

Q3. Which term of the AP 3, 8, 13, is 248?

Q4. Which term of the AP 84, 80, 76, ... is 0?

Q5. Which term of the AP 84, 80, 76, ... is 0?

Q6 ★. Which term of the AP 121, 117, 113, ... is its first negative term?

Q7. Is 68 a term of the AP 7, 10, 13,?

Q8. Is 302 a term of the AP 3, 8, 13, ?

Q9. How many terms are there in the AP?

(i) 7, 10, 13, 43

(ii) 18, $15/2$, 13, 47

Q10. Find the of two digit numbers divisible by 7.

Q11. Find the of two digit numbers divisible by 4.

Q12. Find the of two digit numbers divisible by 3.

Type II: To n^{th} term from the end of given AP

Working: (i) Write the given AP

(ii) take a =last term

(ii) find d of the given AP and then change the sign of d

(iii) Write the formula $a_n = a + (n - 1)d$

Find a_n

Questions based on such condition:

Q1. Find the 12^{th} term from the end of the following arithmetic progressions:

(i) 3 5 7, 9, ... 201 (ii) 3, 8, 13, ... , 253 (iii) 1, 4, 7, 10, ..., 88

Type III: To determine Sum of n terms (S_n) of an AP

Using the formula $S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} [a + a_n]$

Working: (i) Write the data of the given AP

$a=?$, $d=?$, $n=?$, $a_n=?$

(ii) If n not given then use the formula $a_n = a + (n - 1)d$ and find n .

(iii) Substitute the values in $S_n = \frac{n}{2} [2a + (n - 1)d]$ and simplify.

Questions based on such condition:

Q1. Find the sum of the following arithmetic progressions:

(i) 50, 46, 42, ... to 10 terms (ii) $-26, -24, -22, \dots$ to 36 terms.

(iii) $-6, 0, 6, 12, \dots$ upto 13 terms

Q2. Find the sum to n term of the A.P. 5, 2, -1, -4, -7, ...

Q3. If the sum of a certain number of terms starting from first term of an A.P. is 25, 22, 19, ... is 116. Find the last term.

Q4. How many terms are there in the A.P. whose first and fifth terms are -14 and 2 respectively and the sum of the terms is 40 ?

Q5. How many terms of the A.P. 9, 17, 25, ... must be taken so that their sum is 636?

Q6. The first and the last terms of an A.P. are 17 and 350 respectively. If the common difference is 9, how many terms are there and what is their sum?

Q7. If 12th term of an A.P. is -13 and the sum of the first four terms is 24, what is the sum of first 10 terms?

Q8. Find the of two digit numbers divisible by 7. Hence, find their sum.

TypeIV*: Special Questions.

Q1. If $a_n = 5n - 7$ then find a , d and S_{15} .

Q2. If $a_n = -4n + 15$ then find a , d and S_{10} .

Q3. If $a_n = 4n + 3$ then find a , d and S_{50} .

Q4. If $a_n = 9 - 5n$ then find a , d and S_{20} .

Working: (i) Write the general term given in the Question.

(ii) Put $n=1$, and find the value of a .

(iii) Put $n=2$, and find the value of a_2 .

(iv) Find d , using the formula $d = a_2 - a_1$.

(v) Find S_n using the formula $S_n = \frac{n}{2} [2a + (n-1)d]$.

Type V*: Special Questions.

Q1. In an AP, if $S_n = \frac{3n^2}{2} + \frac{13n}{2}$ then find a_{20} .

Working: (i) Write the general term given in the Question.

(ii) Put $n=1$, and find the value of S_1 (Remember $S_1=a$).

(iii) Put $n=2$, and find the value of S_2 .

(iv) Find a_2 , using the formula $a_2 = S_2 - S_1$.

(v) Find d , using the formula $d = a_2 - a$.

(vi) Find a_{20} , using the formula $a_n = a + (n-1)d$.

You can solve similar questions using the above working.

Undertaking

I, Jagdish Gannavarapu mathematics teacher in Jain Heritage A Cambridge School, Nagpur hereby declare that the above teaching material(format) is purely drafted by me under the guidance of my Principal Mr.Praveen Sharma.