

CHAPTER 9

FORCE AND LAWS OF MOTION

Force

In our everyday life we observe that some effort is required to put a stationary object into motion or to stop a moving object. We ordinarily experience this as a muscular effort and say that we must push or hit or pull on an object to change its state of motion. The concept of force is based on this push, hit or pull. In fact, no one has seen, tasted or felt a force. We always see or feel the effect of a force. It can only be explained by describing what happens when a force is applied to an object. Pushing, hitting and pulling of objects are all ways of bringing objects in motion. They move because we make a force act on them.

- a) Force can be used to change the magnitude of velocity of an object (that is, to make the object move faster or slower) or to change its direction of motion.
- b) A force can change the shape and size of objects.
- c) Force is an action which creates effect.

Balanced and Unbalanced Forces

- a) Two equal force acting in opposite direction cancel the effect of each other. So the effect of force on body not appear . These are called balanced forces
- b) Two unequal forces acting in opposite direction may reduce the effect of one another, in such a way effect of resultant force appear in the direction of bigger force.

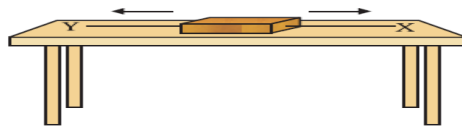
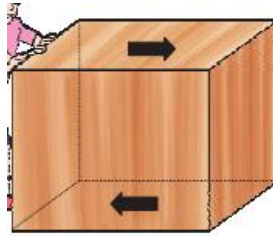


Figure shows a wooden block on a horizontal table. Two strings X and Y are tied to the two opposite faces of the block as shown. If we apply a force by pulling the string X, the block begins to move to the right. Similarly, if we pull the string Y, the block moves to the left. But, if the block is pulled from both the sides with equal forces, the block will not move. Such forces are called balanced forces and do not change the state of rest or of motion of an object. Now, let us consider a situation in which two opposite forces of different magnitudes pull the block. In this case, the block would begin to move in the direction of the greater force. Thus, the two forces are not balanced and the unbalanced force acts in the direction the block moves. This suggests that unbalanced force acting on an object brings it in motion.

- c) Forces always acting in pair. A single force acting on stationary body is always paired with an opposing force, that is called friction force.
- d) An object moves with a uniform velocity when the forces (pushing force and frictional force) acting on the object are balanced and there is no net external force on it.
- e) If an unbalanced force is applied on the object, there will be a change either in its speed or in the direction of its motion



As shown in figure if a boy pushes the box with a small force, the box does not move because of friction acting in a direction opposite to the push. This friction force arises between two surfaces in contact; in this case, between the bottom of the box and floor's rough surface. It balances the pushing force and therefore the box does not move.

If the boy push the box harder but the box still does not move. This is because the friction force still balances the pushing force.

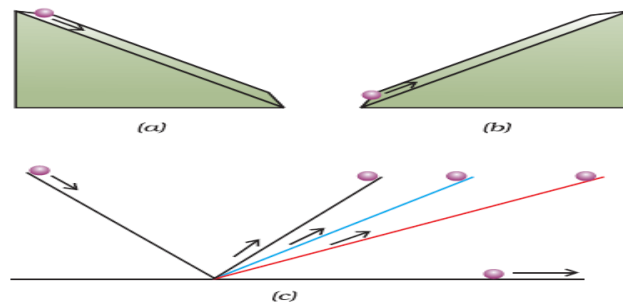
If the boy push the box harder still, the pushing force becomes bigger than the friction force. There is an unbalanced force. So the box starts moving.

- f) To accelerate the motion of an object, an unbalanced force is required. And the change in its speed (or in the direction of motion) would continue as long as this unbalanced force is applied. However, if this force is removed completely, the object would continue to move with the velocity it has acquired till then.

Galileo Galilei Hypothesis

By observing the motion of objects on an inclined plane Galileo deduced that objects move with a constant speed when no force acts on them

- a) When a marble rolls down an inclined plane, its velocity increases shown in figure (a)
- b) Its velocity decreases when it climbs up as shown in figure (b).
- c) As shown in figure (c), a marble resting on an ideal frictionless plane inclined on both sides. Galileo argued that when the marble is released from left, it would roll down the slope and go up on the opposite side to the same height from which it was released.



- d) If the inclinations of the planes on both sides are equal then the marble will climb the same distance that it covered while rolling down.
- e) If the angle of inclination of the right-side plane were gradually decreased, then the marble would travel further distances till it reaches the original height.
- f) If the right-side plane were ultimately made horizontal (that is, the slope is reduced to zero), the marble would continue to travel forever trying to reach the same height that it was released from.

Newton's First Laws of Motion

An object remains in a state of rest or of uniform motion in a straight line unless compelled to change that state by an applied force.

- In other words, all objects resist a change in their state of motion.
- In a qualitative way, the tendency of undisturbed objects to stay at rest or to keep moving with the same velocity is called inertia.
- This is why, the first law of motion is also known as the law of inertia.
- Certain experiences that we come across while travelling in a motorcar can be explained on the basis of the law of inertia. We tend to remain at rest with respect to the seat until the driver applies a braking force to stop the motorcar. With the application of brakes, the car slows down but our body tends to continue in the same state of motion because of its inertia.

Inertia and Mass

There is a resistance offered by an object to change its state of motion. If it is at rest it tends to remain at rest; if it is moving it tends to keep moving. This property of an object is called its inertia.

Heavier or more massive objects offer larger inertia. Quantitatively, the inertia of an object is measured by its mass. We may thus relate inertia and mass as follows:

Inertia is the natural tendency of an object to resist a change in its state of motion or of rest. The mass of an object is a measure of its inertia.

Second Law of Motion

The first law of motion indicates that when an unbalanced external force acts on an object, its velocity changes, that is, the object gets acceleration.

We would now like to study how the acceleration of an object depends on the force applied to it and how we measure a force?

The impact produced by the objects depends on their mass and velocity.

If an object is to be accelerated, we know that a greater force is required to give a greater velocity. In other words, there appears to exist some quantity of importance that combines the object's mass and its velocity.

One such property called momentum was introduced by Newton. The momentum, $\mathbf{p}$ of an object is defined as the product of its mass, $\mathbf{m}$ and velocity, $\mathbf{v}$.

$$\text{That is, } \mathbf{p} = m\mathbf{v}$$

Momentum has both direction and magnitude. Its direction is the same as that of velocity, $\mathbf{v}$.

The SI unit of momentum is *kilogram-metre per second (kg m s^{-1})*.

Since the application of an unbalanced force brings a change in the velocity of the object, it is therefore clear that a force also produces a change of momentum.

The force necessary to change the momentum of an object depends on the time rate at which the momentum is changed.

The second law of motion states that the rate of change of momentum of an object is proportional to the applied unbalanced force in the direction of force.

$$\text{Force} \propto \text{The change in momentum with respect to time}$$

Mathematical Formulation of Second Law of Motion

Suppose an object of mass, $\mathbf{m}$ is moving along a straight line with an initial velocity, $\mathbf{u}$. It is uniformly accelerated to velocity, $\mathbf{v}$ in time, $\mathbf{t}$ by the application of a constant force, $\mathbf{F}$ throughout the time, $\mathbf{t}$.

Let The initial momentum of the object $\mathbf{p}_1 = m\mathbf{u}$

Final momentum of the object $\mathbf{p}_2 = m\mathbf{v}$

$$\text{The change in momentum} \propto \mathbf{p}_2 - \mathbf{p}_1$$

$$\propto mv - mu$$

$$\propto m \times (v - u).$$

$$\text{The rate of change of momentum } \propto m \times (v - u) / t$$

By Newton's second law of motion

$$\text{Force } \propto \text{The change in momentum with respect to time (Rate of change of momentum)}$$

So

$$\text{The applied force, } F \propto m \times (v - u) / t$$

$$F = k \ m \times (v - u) / t$$

The rate of change of velocity is defined as acceleration as

$$a = (v - u) / t$$

So

$$F = kma$$

The quantity, k is a constant of proportionality.

The unit of force is so chosen that the value of the constant, k becomes one. For this, one unit of force is defined as the amount that produces an acceleration of 1 m s^{-2} in an object of 1 kg mass. That is,

$$1 \text{ unit of force} = k \times (1 \text{ kg}) \times (1 \text{ m s}^{-2}).$$

$$\text{Thus, the value of } k \text{ becomes } 1$$

$$F = ma$$

The unit of force is kg m s^{-2} or Newton, which has the symbol **N**.

The second law of motion gives us a method to measure the force acting on an object as a product of its mass and acceleration.

Newton's First Law from Second Law

The first law of motion can be mathematically stated from the mathematical expression for the second law of motion.

$$F = ma$$

$$\text{or } F = m(v-u) / t$$

$$Ft = mv - mu$$

That is, when $F = 0$ then $v = u$ for whatever time, t is taken. This means that the object will continue moving with uniform velocity, u throughout the time, t . If u is zero then v will also be zero. That is, the object will remain at rest.

Newton's Third Law of Motion

The third law of motion states that when one object exerts a force on another object, the second object instantaneously exerts a force back on the first.

These two forces are always equal in magnitude but opposite in direction.

These forces act on different objects and never on the same object.

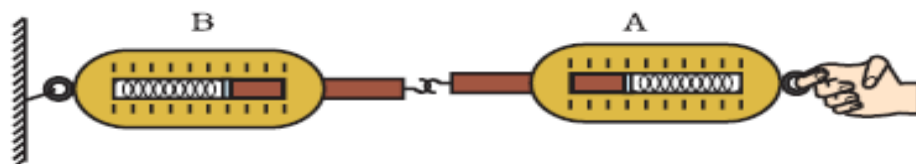
The two opposing forces are also known as action and reaction forces.

Every action there is an equal and opposite reaction. However, it must be remembered that the action and reaction always act on two different objects.

It is important to note that even though the action and reaction forces are always equal in magnitude; these forces may not produce accelerations of equal magnitudes. This is because each force acts on a different object that may have a different mass.

Example 1

Let us consider two spring balances connected together as shown in figure. The fixed end of balance B is attached with a rigid support, like a wall. When a force is applied through the free end of spring balance A, it is observed that both the spring balances show the same readings on their scales. It means that the force exerted by spring balance A on balance B is equal but opposite in direction to the force exerted by the balance B on balance A. The force which balance A exerts on balance B is called the action and the force of balance B on balance A is called the reaction.



Example 2

Suppose you are standing at rest and intend to start walking on a road. You must accelerate, and this requires a force in accordance with the second law of motion.

Which is this force? Is it the muscular effort you exert on the road? Is it in the direction we intend to move?

No, you push the road below backwards. The road exerts an equal and opposite reaction force on your feet to make you move forward.

Example 3

When a gun is fired, it exerts a forward force on the bullet. The bullet exerts an equal and opposite reaction force on the gun. This results in the recoil of the gun.

Since the gun has a much greater mass than the bullet, the acceleration of the gun is much less than the acceleration of the bullet.

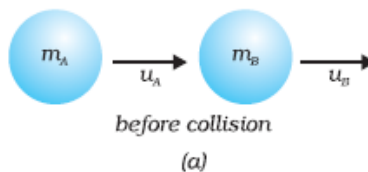
Example 4

When a sailor jumps out of a rowing boat. As the sailor jumps forward, the force on the boat moves it backwards.

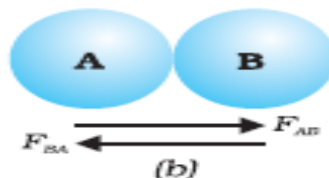
Conservation of Momentum

Suppose two objects (two balls *A* and *B*, say) of masses m_A and m_B are travelling in the same direction along a straight line at different velocities u_A and u_B , respectively.

There are no other external unbalanced forces acting on them.



Let $u_A > u_B$ and the two balls collide with each other. During collision which lasts for a time t , the ball **A** exerts a force F_{AB} on ball B and the ball **B** exerts a force F_{BA} on ball A.



Suppose v_A and v_B are the velocities of the two balls **A** and **B** after the collision, respectively

The the momenta (plural of momentum) of ball A and B Before the collision

For Ball A is $p_A = m_A * u_A$

For Ball B is $p_B = m_B * u_B$

The rate of change of its momentum of Ball A for F_{AB} action during the collision will be

$$= m_A (v_A - u_A) / t$$

The rate of change of its momentum of Ball B for F_{BA} action during the collision will be

$$= m_B (v_B - u_B) / t$$

According to the third law of motion, the force F_{AB} exerted by ball A on ball B (action) and the force F_{BA} exerted by the ball B on ball A (reaction) must be equal and opposite to each other.

Therefore,

$$F_{AB} = -F_{BA}$$

$$m_A (v_A - u_A) / t = -m_B (v_B - u_B) / t$$

$$m_A (v_A - u_A) = -m_B (v_B - u_B)$$

$$m_A v_A - m_A u_A = -m_B v_B + m_B u_B$$

$$m_A v_A + m_B v_B = m_A u_A + m_B u_B$$

The total momentum of the two balls A and B before the collision

$$= m_A u_A + m_B u_B$$

The total momentum of the two balls A and B before the collision

$$= m_A v_A + m_B v_B$$

We observe that the total momentum of the two balls remains unchanged or conserved provided no other external force acts.

As a result of this ideal collision experiment, we say that the sum of momenta of the two objects before collision is equal to the sum of momenta after the collision provided there is no external unbalanced force acting on them.

This is known as the law of conservation of momentum.

This statement can alternatively be given the total momentum of the two objects is unchanged or conserved by the collision.