

SUBJECT: MATHEMATICS
TOPIC: PROBABILITY
CLASS: X

Introduction :

Probabilities are based on the results of an actual experiment. For this reason, they are called experimental or empirical probabilities. In fact, experimental probabilities are based on the results of actual experiments and adequate recordings of the happening of the events. We may get different data giving different probability estimates.

*An event having only one outcome of the experiment is called an elementary event.

* The sum of the probabilities of all the elementary events of an experiment is **1**.

* The event $\bar{E}$, representing 'not E', is called the complement of the event E. We also say that E and $\bar{E}$ are complementary events.

*The probability of an event which is impossible to occur is 0. Such an event is called an impossible event.

*The probability of an event which is sure (or certain) to occur is 1. Such an event is called a sure event or a certain event.

Formulae:

i) $P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$

ii) $P(E) + P(\bar{E}) = 1$ or $P(E) = 1 - P(\bar{E})$.

Working Rule:

Step I: Write the total number of outcomes of the event.

Step II: Write the number of favourable outcomes.

Step III: $P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$

***Step IV:** For complementary events, $P(E) = 1 - P(\bar{E})$.

TYPE I

In this category of questions, you may find what you have done in your previous class (IX).

Q.A bag contains 9 black and 12 white balls. One ball is drawn at random. What is the probability that the ball drawn is black?

Solution: Total number of outcomes = $9 + 12 = 21$

Number of favourable outcomes = 9

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

$$\begin{aligned}\therefore P(\text{getting a black ball}) &= \frac{9}{21} \\ &= \frac{3}{7}\end{aligned}$$

TYPE II

Problems based on coins:

*Total outcomes of a single coin = 2

**Total outcomes of a Two coins = 4

Coin I	Coin II	Outcomes
H	H	(HH)
H	T	(HT)
T	H	(TH)
T	T	(TT)

*** Total outcomes of a Three coins = 8

Coin I	Coin II	Coin III	Outcomes
H	H	H	(HHH)
H	H	T	(HHT)
H	T	H	(HTH)
H	T	T	(HTT)
T	H	H	(THH)
T	H	T	(THT)
T	T	H	(TTH)
T	T	T	(TTT)

Q. Find the probability of getting a head when a coin is tossed once.

Solution: Total number of outcomes = 2

Number of favourable outcomes = 1

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

$$\therefore P(\text{getting a head}) = \frac{1}{2}$$

Q. Harpreet tosses two different coins simultaneously (say, one is of Re 1 and other of Rs 2). What is the probability that she gets at (i) at least one head, (ii) exactly one tail?

Solution: Total number of outcomes = 2

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

(i) Number of favourable outcomes = 3

$$\therefore P(\text{getting at least one head}) = \frac{3}{4}$$

(ii) Number of favourable outcomes = 2

$$\therefore P(\text{getting exactly one tail}) = \frac{2}{4} = \frac{1}{2}$$

Coin I	Coin II	Outcomes
H	H	(HH)
H	T	(HT)
T	H	(TH)
T	T	(TT)

Q. Saif tosses three coins simultaneously. What is the probability that he gets at (i) at least one head, (ii) exactly two tails, (iii) at least two heads and, (iv) at most two heads?

Solution: Total number of outcomes = 8

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

(i) Number of favourable outcomes = 7

$$\therefore P(\text{getting at least one head}) = \frac{7}{8}$$

(ii) Number of favourable outcomes = 3

$$\therefore P(\text{getting exactly two tails}) = \frac{3}{8}$$

(iii) Number of favourable outcomes = 3

$$\therefore P(\text{getting at least two heads}) = \frac{4}{8} = \frac{1}{2}$$

(iv) Number of favourable outcomes = 6

$$\therefore P(\text{getting at most two heads}) = \frac{6}{8} = \frac{3}{4}$$

Coin I	Coin II	Coin III	Outcomes
H	H	H	(HHH)
H	H	T	(HHT)
H	T	H	(HTH)
H	T	T	(HTT)
T	H	H	(THH)
T	H	T	(THT)
T	T	H	(TTH)
T	T	T	(TTT)

TYPE III

Problems based on dice:

*Total outcomes of a single dice = 6

**Total outcomes of a two dice = 36

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Duplets = {(1,1),(2,2),(3,3),(4,4),(5,5),(6,6)}

Q.When a dice is rolled, find the probability of getting an even prime number, prime numbers and factors of 6?

Solution: Total number of outcomes = 6

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

(i)Number of favourable outcomes = 1

$$\therefore P(\text{getting an even prime number}) = \frac{1}{6}$$

(ii)Number of favourable outcomes =

$$\therefore P(\text{getting a prime numbers}) = \frac{3}{6} = \frac{1}{2}$$

(iii)Number of favourable outcomes = {1,2,3,6}= 4

$$\therefore P(\text{getting factors of 6}) = \frac{4}{6} = \frac{2}{3}$$

Q.When a two dice are rolled simultaneously, find the probability of getting a duplet, sum as 10, sum as prime number.

Solution: Total number of outcomes = 36

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

(i)Number of favourable outcomes = 6

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

$$\therefore P(\text{getting a duplet}) = \frac{6}{36} = \frac{1}{6}$$

(ii) Number of favourable outcomes = $\{(4,6), (5,5), (6,4)\}$

$$\therefore P(\text{getting sum as 10}) = \frac{3}{36} = \frac{1}{12}$$

(iii) Number of favourable outcomes = $\{(1,1), (2,3), (3,2), (1,4), (4,1), (1,6), (6,1), (5,2), (2,5), (3,4), (4,3)\} = 11$

$$\therefore P(\text{getting sum as prime number}) = \frac{11}{36}$$

TYPE IV

Problems based on cards:

Playing cards consists of 52 cards which are divided into 4 suits of 13 cards each—

spades (♠), hearts (♥), diamonds (♦) and clubs (♣)

The cards in each suit are ace, king, queen, jack, 10, 9, 8, 7, 6, 5, 4, 3 and 2.

Kings, queens and jacks are called face cards. Thus, total face cards are 12.

Q. A card is drawn at random from a well-shuffled deck of playing cards. Find the probability of drawing a,

- i) a red card (ii) a face card (iii) a king (iv) a king of black

Solution: Total number of outcomes = 52

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

(i) Number of favourable outcomes = 26

$$\therefore P(\text{getting a red card}) = \frac{26}{52} = \frac{1}{2}$$

(ii) Number of favourable outcomes = 12

$$\therefore P(\text{getting a face card}) = \frac{12}{52} = \frac{3}{13}$$

(iii) Number of favourable outcomes = 4

$$\therefore P(\text{getting a king}) = \frac{4}{52} = \frac{1}{13}$$

TYPE IV

Problems based on complements:

Use the formula $P(E) = 1 - P(\bar{E})$.

Q. One card is drawn from a well-shuffled deck of 52 cards. Calculate the probability that the card will not be an ace.

Solution: Total number of outcomes = 52

Number of favourable outcomes = $52 - 4 = 48$

$$\therefore P(\text{getting a face card}) = \frac{48}{52} = \frac{12}{13}$$

Q. A bag contains 9 black and 12 white balls. One ball is drawn at random. What is the probability that the ball drawn is not black?

Solution: Total number of outcomes = $9 + 12 = 21$

Number of favourable outcomes = $21 - 9 = 12$

$$\therefore P(E) = \frac{\text{Number of favourable outcomes}}{\text{Number of all possible outcomes}}$$

$$\begin{aligned}\therefore P(\text{not getting a black ball}) &= \frac{12}{21} \\ &= \frac{4}{7}\end{aligned}$$

WORKSHEET: Questions based on above types.

1. A bag contains 9 black and 12 white balls. One ball is drawn at random. What is the probability that the ball drawn is black?
2. Find the probability that a number selected from the numbers 1 to 25 which is not a prime number when each of the given number is equally likely to be selected.
3. A bag contains 10 red, 5 blue and 7 green balls. A ball is drawn at random. Find the probability of this ball being not a blue ball.
4. Two dice are thrown at the same time and the product of numbers appearing on them is noted. Find the probability that the product is less than 9.
5. Cards, marked with numbers 5 to 50, are placed in a box and mixed thoroughly. A card is drawn from the box at random. Find the probability that the number on the taken out card is:
 - (i) a prime number less than 10.
 - (ii) a number which is a perfect square.
6. Two dice are thrown simultaneously. What is the probability that
 - (i) 5 will not come up on either of them?
 - (ii) 5 will come up on at least one?
 - (iii) 5 will come up at both dice?
7. From a well shuffled pack of playing cards, black jacks, black kings and black aces are removed. A card is then drawn from the pack. Find the probability of getting.
 - (i) a red card
 - (ii) not a diamond card.
8. A bag contains cards which are numbered from 2 to 90. A card is drawn at random from the bag. Find the probability that it bears.
 - (i) a two-digit number
 - (ii) a number which is a perfect square.

9. Cards numbered 1 to 30 are put in a bag. A card is drawn at random from this bag. Find the probability that the number on the drawn card is:

- (i) not divisible by 3.
- (ii) a prime number greater than 7.
- (iii) not a perfect square number.

10. Two different dice are tossed together. Find the probability:

- (i) That the numbers on either die is even.
 - (ii) That the sum of numbers appearing on the two dice is 5.
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1. Cards each marked with one of the numbers 6, 7, 8,, 15 and placed in a box and mixed thoroughly. One card is drawn at random from the box. What is the probability of getting a card with number less than 10?

2. A card is drawn at random from a well-shuffled deck of 52 cards. What is the probability of getting a black king?

3. What is the probability that two different friends have different birthdays? (ignoring leap year).

4. A box contains 3 blue, 2 white and 4 red marbles. If a marble is drawn at random from the box, what is the probability that it will not be a white marble?

5. From a well-shuffled pack of cards, a card is drawn at random. Find the probability of getting a black queen.

6. A bag contains 4 red and 6 black balls. A ball is taken out of the bag at random. Find the probability of getting a black ball.

7. A die is thrown once. Find the probability of getting a number less than 3.

8. A die is thrown once. Find the probability of getting a number greater than 5.

9. Cards bearing numbers 3 to 20 are placed in a bag and mixed thoroughly. A card is taken out from the bag at random. What is the probability that the number on the card taken out is an even number?

10. Two friends were born in the year 2000. What is the probability that they have the same birthday?

11. A box contains cards marked with numbers 5 to 20. A card is drawn from the bag at random. Find the probability of getting a number which is a perfect square.

12. Two dice are thrown at the same time. Find the probability of getting different numbers on the dice.

13. Cards with numbers 2 to 101 are placed in a box. A card is selected at random. Find the probability that the card has a square number.

14. In a game of chance there is spinning of an arrow which comes to rest pointing at one of the numbers 1, 2, 3, 4, 5, 6, 7, 8 and there are equally likely outcomes. What is the probability that it will point at

- (i) 7?
- (ii) an odd number?
- (iii) a number less than 9?

15. A game consists of tossing a one-rupee coin 3 times and noting its outcome each time. Hanif wins if all the tosses give the same result, i.e., three heads or three tails, and loses otherwise, calculate the probability that Hanif will lose the game.

16. Find the probability that a number selected at random from numbers 3, 4, 5,, 25 is prime.

17. The king, queen and jack of diamonds are removed from a pack of 52 cards and then the pack is well-shuffled. A card is drawn from the remaining cards. Find the probability of getting a card of

- (i) diamonds
- (ii) a Jack

- 18.** A bag contains 5 red, 4 blue and 3 green balls. A ball is taken out of the bag at random. Find the probability that the selected ball is
- (i) of red colour
 - (ii) not of green colour.
- 19.** A card is drawn at random from a well-shuffled deck of playing cards. Find the probability of drawing a
- (i) face card
 - (ii) card which is neither a king nor a red card.
- 20.** A bag contains tickets, numbered 11, 12, 13, ..., 30. A ticket is taken out from the bag at random. Find the probability that the number on the drawn ticket
- (i) is a multiple of 7
 - (ii) is greater than 15 and a multiple of 5.
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Undertaking

I, Jagdish Gannavarapu mathematics teacher in Jain Heritage A Cambridge School, Nagpur hereby declare that the above teaching material(format) is purely drafted by me under the guidance of my Principal Mr.Praveen Sharma.