

Master Lesson Plan

For

Playing with Numbers

Board	Standard	Subject	Chapter	Language	Reference Link	Creation date
CBSE	STD VI	Mathematics	Playing with Numbers	English	Playing with Numbers	2019-09-01 22:22:36

Disclaimer

- * Strictly not for Commercial use. *Provided on 'as is' basis with no warranties of any kind.
- * Content that falls in Public Domain or common Knowledge facts can be used freely.
- *Some of the contents are owned by the Third parties and are used in compliance with their licensing conditions. Any one infringing the Copyright of such Third parties will be doing so at their own risks and costs.
- * Content can be downloaded and used for Personal, educational and informational purposes only. Any attempt to remove, alter, circumvent or distort the data that is accessed Is Illegal and strictly prohibited.

Playing with Numbers

1. MS_Objectives_Playing with Numbers

OBJECTIVES

Notes to teacher:

This asset lays down the proposed plan for transacting this chapter. It states the asset objectives of the MLP. This asset is for teacher's reference and need not be taught to students.

Students will be able to

- Demonstrate factors and multiples of a given number
- Define and explain factors of a number
- Define and explain multiples of a number
- Appreciate Hardy-Ramanujam's number
- Define and explain even, odd, prime and composite and twin prime numbers
- Use Sieve of Eratosthenes to find prime numbers
- Identify a prime number by checking if it is divisible by all prime numbers less than its square root
- Define and explain perfect numbers
- Solve question related to types of numbers
- Explain the tests of divisibility of 2, 4, 3, 5, 7, 8, 9, 10 and 11 and explain the rules of divisibility
- Solve questions related to tests of divisibility
- Explain prime factorization, factor tree method and division method for prime factorization
- Explain highest common factor of any two numbers
- Explain least common multiple of any two numbers
- State the relation between HCF, LCM of any two numbers with their product
- Correlate LCM to situations in daily life
- Solve questions related to HCF and LCM
- Demonstrate factors, multiples, HCF and LCM
- Correlate factors, multiples, HCF and LCM to situations in daily life
- Practice love and kindness as highest common factors of their character

Time to teach	Asset Type	Theme	SubTheme
5 Minutes	Day-to-day Relevance	Factors & Multiples	Factors, Multiples, HCF,

2. SA_Activity Corner

Aim: To demonstrate factors and multiples of a number (6).

Resources Required: 50 gm biscuit packet 1(6 biscuits), paper, scale, pencil

Setting for the Activity: Indoor

Type of Activity: Demonstration

Procedure:

Instructions to teacher

1. Divide the whole class into groups of 6.
2. Select a group leader for each group.
3. In a previous class, ask students to bring a small biscuit packet (6 biscuits) each.

Proceed with the activity as follows

Group leader of each group will conduct the activity as follows:

1. Draw a tabular column, discuss with others in the group and fill up the table as follows (need not open all packets)

Number of packets	Number of biscuits
1	6
2	12
3	18
4	24
5	30
6	36

Note to teacher:

Teacher can draw this tabular column on the blackboard and mention that 6, 12, 18, 24, 30 and 36 are multiples of 6.

2. Group leader will give each one (including oneself) 1 biscuit from his/her packet. Group leader will make a tabular column and note down the entries as follows:

Number of biscuits	Number of children given	Number of biscuits given to one person	Number of biscuits remaining
6	6	1	0

2. Now another child in the group will select 2 children from the group and give them biscuits from his/her packet one after another alternatively till all biscuits are given. Group leader has to maintain the discipline in the group and make entries in the tabular column.

Number of biscuits	Number of children given	Number of biscuits given to one person	Number of biscuits remaining
6	6	1	0
6	2	3	0

3. Another child in the group will choose 3 of them in the group and give biscuits to them equally from his/her packet.

4. Another child in the group will choose 4 of them in the group and give biscuits from his/her packet equally.

5. Another child will choose 5 of them from the group and give biscuits from his/her packet equally.

6. The last person will choose any 1 person from the group and give the whole biscuit packet to him/her.

Group leader has to give a fair chance to everyone in the group and keep entries in the tabular column:

Number of biscuits	Number of children given	Number of biscuits given to one person	Number of biscuits remaining
6	6	1	0
6	2	3	0
6	3	2	0
6	4	1	2
6	5	1	1
6	1	6	0

Note to teacher:

Teacher can draw the above tabular column on the blackboard and highlight the numbers in the second column for which the entries in the last column is 0.

Number of biscuits	Number of children given	Number of biscuits given to one person	Number of biscuits remaining
6	⑥	1	0
6	②	3	0
6	③	2	0
6	4	1	2
6	5	1	1
6	①	6	0

Note to teacher:

Teacher can ask children to note down the highlighted numbers in this way

6 – 1, 2, 3, 6

And mention that 1, 2, 3 and 6 are factors of 6.

Observation:

1. Multiples of 6 are 6, 12, 18, 24, 30 and 36
2. Factors of 6 are 1, 2, 3 and 6
3. There are no biscuits remaining after equal distribution in the case of factors of 6.

Conclusion:

Factors and multiples of a number (6) can be demonstrated through an activity.

Note to teacher: Teacher can decide if he/she wants to conduct the activity with biscuits, coins, gems, etc. The number of items can also be decided by the teacher. Tabular columns and observations have to be modified accordingly.

Time to teach	Asset Type	Theme	SubTheme
20 Minutes	Suggested Activity	Factors & Multiples	Properties of Factors and Multiples

3. MS_Introduction to Factors

Example 1:

Sai's father bought a medium pizza  for New Year party. It has 6 slices. He told Sai to distribute it among his family members so that each one gets the **same number** of slices. Sai's family consist of 6 members i.e. father, mother, Sai's younger sister, grandfather and grandmother. Let us see the different ways he can do this

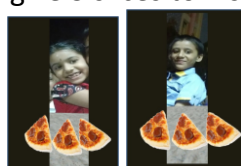
1. He can give each one slice and keep 1 for himself



$$1 \times 6 = 6$$

He can give 2 slices to his sister, father and keep 2 for himself: $2 \times 3 = 6$

He can give 3 slices to his sister and keep 3 for himself: $3 \times 2 = 6$



Can he give 4 slices to his sister and keep 2 for himself?

No, since they will not get the same number of slices.

Similarly giving 5 slices to his sister and keeping 1 for himself will also be an unequal distribution.

He can give all the six slices to his younger sister: $6 \times 1 = 6$

In all the cases 1,2,3 and 5, note that the numbers 1,2,3 and 6 divide 6 exactly giving 0 as the remainder

	Dividend				
Divisor	1	) 6 (6	Quotient	2	) 6 (3
		<u>-6</u>		<u>-6</u>	
		0		0	
	Remainder				

	3	) 6 (2		6	) 6 (1
		<u>-6</u>		<u>-6</u>	
		0		0	

EXAMPLE 2

Now let us say you have a magic show in your class and your class teacher wants to arrange chairs in rows such that each row has the same number of chairs. There are 18 students in the class. She can arrange the chairs in the following ways:

She can put all the chairs in one row : $1 \times 18 = 18$



She can put 2 rows with 9 chairs each : $2 \times 9 = 18$

She can put 3 rows with 6 chairs each : $3 \times 6 = 18$

She can put 6 rows with 3 chairs each : $6 \times 3 = 18$



1) She can put 9 rows with 2 chairs in each row : $9 \times 2 = 18$

2) She can put 18 rows with 1 chair each : $18 \times 1 = 18$

3) Can your Ma'am put 4 rows of 4 chairs each? : $(4 \times 4) + (1 \times 2) = 18$

No, since 2 chairs are left out and this is not an equal distribution.

In the above example we see that the numbers 1,2,3,6,9 and 18 divide the number 18 exactly without leaving any remainder. Other numbers like 4 or 5 don't divide 18 exactly.

1) 18 (18	2) 18 (9	3) 18 (6	6) 18 (3	9) 18 (2	18) 18 (1
<u>-18</u>	<u>-18</u>	<u>-18</u>	<u>-18</u>	<u>-18</u>	<u>-18</u>
0	0	0	0	0	0

Factors

Numbers which divide another number exactly without leaving any remainder are called the factors of that number.

In the above examples we notice that

1. 1 is a factor of 6 and 18.
2. 6 is a factor of 6
3. 18 is a factor of 18
4. 6 has 4 factors and 18 has 6 factors.

Facts about factors

- Every number has at least 2 factors, 1 and itself.
- The number of factors of any number are finite.
- Every factor is less than or equal to the number.

Image Source:

Chairs: <https://pixabay.com/en/chairs-furniture-seats-comfortable-575886/>

Pizza : <https://pixabay.com/photos/america-american-baked-bread-1238740/>

Pictures of people: Self Clicked by keshabam@gmail.com

Pizza: <https://pixabay.com/en/pizza-food-dinner-italian-2819821/>

Time to teach	Asset Type	Theme	SubTheme
10 Minutes	Main Script	Factors & Multiples	Properties of Factors and Multiples

4. MS_Multiples

MULTIPLES

Example 1

Prashant is helping his mother with cooking. She gives him 10 green peas to shell.

Each one has 6 seeds. How many seeds will he get?

He will get $10 \times 6 = 60$ seeds.

Example 2

Aarti wants to buy chocolates to give her friends in her school on her birthday. She buys 3 packets of chocolates. Each packet has 15 chocolates. Among how many of her friends can she distribute them?

She has $3 \times 15 = 45$ chocolates. She can distribute them among 45 of her friends.

Multiples of a number are got by multiplying it by any natural number.

In the above example 60 is a multiple of 10. 45 is a multiple of 15.

Let us write the first five multiples of 5, 7 and 9 till 5.

$5 \times 1 = 5$	$7 \times 1 = 7$	$9 \times 1 = 9$
$5 \times 2 = 10$	$7 \times 2 = 14$	$9 \times 2 = 18$
$5 \times 3 = 15$	$7 \times 3 = 21$	$9 \times 3 = 27$
$5 \times 4 = 20$	$7 \times 4 = 28$	$9 \times 4 = 36$
$5 \times 5 = 25$	$7 \times 5 = 35$	$9 \times 5 = 45$

Thus, the multiples of 5 are 5, 10, 15, 20, 25...

Multiples of 7 are 7, 14, 21, 28, 35...

Multiples of 9 are 9, 18, 27, 36, 45...

There are infinite multiples of a number.

A number is called an **even number** when it is a multiple of 2. Otherwise, it is called an **odd number**.

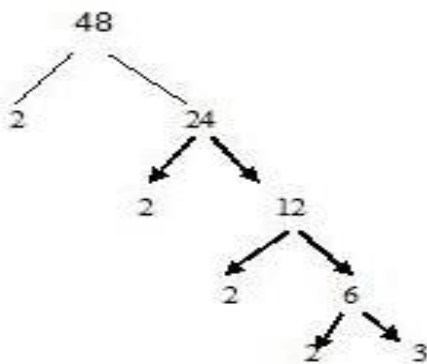
For example, 42 and 98 are even numbers, whereas 33 and 85 are odd numbers.

Factor tree

What is a family tree?

It tells us about all the members in your family.

Similarly, we could draw a factor tree showing all the factors of a number. This is useful for finding the factors of big numbers.



The factors of 48 are 2, 24, 12, 6 and 3.

$$48 = 1 \times 48$$

$$48 = 2 \times 24$$

$$48 = 4 \times 12$$

$$48 = 8 \times 6 \quad \text{or}$$

$$48 = 16 \times 3$$

Thus, 48 is a multiple of 1, 2, 3, 4, 6, 8, 24 and 48.

When a number is multiplied by itself a given number of times, it can be expressed in the **exponential form**.

$$2 \times 2 = 2^2$$

$$2 \times 2 \times 2 = 2^3$$

$$2 \times 2 \times 2 \times 2 = 2^4 \text{ etc.}$$

In all the above, 2 is called the base and the superscript is called the exponent. The notation, for example, 2^2 , is called the power.

Any number 'm' multiplied 'n' number of times, that is $m \times m \times m \times m \times m \dots \times m$, n times can be expressed as m^n . In this, 'm' is the base and 'n' is the exponent and m^n is the power.

For example:

$5 \times 5 = 5^2$ is read as 5 raised to 2. In this, the base is 5 and exponent is 2. This is also known as square of 5.

$7 \times 7 \times 7 = 7^3$ is read as 7 raised to 3. Here the base is 7 and exponent 3. This is also called the cube of 7.

Note that any number raised to 1 is itself. For example, 7^1 is 7.

Note that 2^3 is not the same as 2×3 .

Facts about multiples

1. Every number and its exponent is a multiple of itself.
2. Every multiple of a number is greater than or equal to the number.
3. A number is a multiple of each of its factors.
4. There are infinite multiples of a number.

Time to teach	Asset Type	Theme	SubTheme
10 Minutes	Main Script	Factors & Multiples	Properties of Factors and Multiples

5. IA_Hardy Ramanujan's number

Hardy Ramanujan's number

Do you know about the great Indian Mathematician genius Srinivasa Ramanujan (22 Dec 1887 - April 1920)?

In 2012, December 22 was declared to be National Mathematics Day marking his 125th birth anniversary.

He loved to play with numbers.

The number 1729 is known as the Hardy Ramanujan's number. G.H.Hardy was an English mathematician. Ramanujan and Hardy worked together in the field of research in Cambridge University. Once when Ramanujan was sick and admitted to a hospital in London, Hardy went to see him in a taxi number 1729. He thought that it was not a special number and also could be a bad omen. When he told this to Ramanujan, he immediately said that it is a special number as it is the smallest number that can be expressed as the sum of two cubes in two different ways.

$$1729 = 1^3 + 12^3 = 9^3 + 10^3$$

Time to teach	Asset Type	Theme	SubTheme
5 Minutes	Interesting Asides	Factors & Multiples	Properties of Factors and Multiples

6. MS_Types of numbers

Even and Odd numbers

Numbers which have 2,4,6,8 or 0 in their unit's place are known as **even** numbers. They are multiples of 2. For instance 45672382 is an even number because it has 2 in its unit's place. If you

have an even number you can get the next even number by adding 2 to it. Consecutive even numbers differ by 2. For example, 28 and 30 differ by 2.

What about the numbers which don't end with 2,4,6,8 or 0? They are the **odd** numbers. They are between 2 consecutive even numbers. For example, 29 is an odd number between 28 and 30. Consecutive odd numbers also differ by 2. 29 and 31 are consecutive odd numbers and differ by 2.

There are infinite even and odd numbers.

Prime numbers

Let us write down the factors of numbers from 1 to 10

- 1 - 1
- 2 - 1, 2
- 3 - 1, 3
- 4 - 1, 2, 4
- 5 - 1, 5
- 6 - 1, 2, 3, 6
- 7 - 1, 7
- 8 - 1, 2, 4, 8
- 9 - 1, 3, 9
- 10 - 1, 2, 5, 10

We see that the number 1 has only one factor, numbers like **2, 3, 5 and 7** have two factors (1 and themselves) whereas other numbers 4,6,8,9,10 have more than two factors.

All the numbers which have only two factors (1 and themselves) are known as prime numbers.

All the numbers which have more than two factors are called composite number.

Out of numbers from 1 to 10,

- 2, 3, 5 and 7 are prime numbers.
- 4, 6, 8, 9 and 10 are composite numbers.

Some more examples of prime numbers are 13, 17, 23, 31, 97 etc.

Twin prime numbers

If the difference between two prime numbers is 2, they are called twin prime numbers.

For example, 29 and 31 are twin prime numbers since they differ by 2.

17 and 19 are another example for twin prime numbers.

Facts about prime and composite numbers

- 1 is the only number that has only one factor that is itself. 1 is neither prime nor composite.
- There are 25 prime numbers between 1 and 100.
- 2 is the smallest prime number.
- 2 is the only even number which is prime. All the other prime numbers are odd.
- 4 is the smallest composite number.
- Any multiple of a prime number is composite.
- There is no number which has no factor (Since 1 is a factor of any number).
- Every even number greater than 4 can be expressed as the sum of two odd prime numbers.

For example, $6 = 3+3$ $24 = 11+13$ $98 = 67+31$

- There are infinite prime numbers. This was stated by the Greek Mathematician Euclid as early as 3rd century. The largest known prime number has 23,249,425 digits.

Time to teach	Asset Type	Theme	SubTheme
10 Minutes	Main Script	Factors & Multiples	Speciality of 2(even and odd prime numbers)

7. SA_Sieve of Eratosthenes

Sieve of Eratosthenes method-Prime and composite numbers

Aim: To make the students find out the prime numbers and the composite numbers from 1 to 100.

Materials required: Paper, scale, colour pencils

Procedure:

Teacher can give an introduction to the activity as follows:

Eratosthenes (276 BC- 194 BC) was a Greek Mathematician. He devised a simple method to separate prime numbers from the rest using his sieve.

Suppose we have to find all the prime numbers from 1 to 100. In this method, the idea is to find all the numbers in the table that are multiples of a prime and therefore composite and discard them as primes. In this process, the left out numbers are prime.

We know that any multiple of a prime is a composite number. 2 is the smallest prime number. Since all the other even numbers are multiples of 2, they are composite.

Teacher can ask students to proceed as follows:

1. Draw a table of 10 rows and 10 columns like in the figure below and write down all the numbers from 1 to 100.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

- Strike off 1 since 1 is neither prime nor composite. Circle 2 and strike off all the multiples of 2.
- Circle the next prime number 3 and strike off all of its multiples.
- Now let us do this same method for the next 2 prime numbers 5 and 7. Circle 5 and 7 and strike off their multiples. All the rest of the numbers which are not struck off are prime numbers. Circle them.
- Now write down all the prime numbers between 1 and 100. They are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89 and 97.

The rest are all composite numbers.

Conclusion: We conclude that 1 is neither a prime number nor a composite number. We had found all the prime numbers and composite numbers from 1 to 100.

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Suggested Activity	Factors & Multiples	Sieve of Eratosthenes

8. IQ Think and answer

Think and answer

In the Sieve of Eratosthenes method of finding prime numbers between 1 and N, we find all the multiples of prime numbers 2, 3, 5, 7, ..., P and discard them as composites. We take the remaining numbers as primes.

- What is P?
- What is P when N is 200 or 1000?
- How can one use P to check if a number is prime or not?

Solution

1) If P is a prime number such that $P > \sqrt{N}$, then any number $K < N$, cannot have a prime divisor $> P$.

i.e., if $K < N$, then all its prime divisors P satisfy $P < \sqrt{N}$.

Therefore, the highest prime P , such that $P < \sqrt{N}$ is chosen.

i.e. if a number $K < N$ and it has no prime divisor in the list $2, 3, 5, 7, \dots, P$, it cannot have a prime divisor greater than P . Therefore, we can assume that K is prime.

2) When $P = 13$, $P^2 = 169 < 200$.

For the next prime 17 , $17^2 = 289 > 200$.

Thus, if any number K , in the list between 1 to 200 , is not a multiple of primes $2, 3, 5, 7, 11, 13$, then it is prime.

When $N=1000$

$31^2 = 961 < 1000$. The next prime is 37 , $37^2 > 1000$.

If a number K , between 1 and 1000 , has no prime divisor between $2, 3, 5, 7, \dots, 31$, then it is prime.

3) For example, take the number 313 .

$17^2 = 289 < 313$, the next prime number is 19 . $19^2 > 313$.

Let us therefore divide 313 by $2, 3, 5, 7, 11, 13$ and 17

2) 313 (156	3) 313 (104	5) 313 (62	7) 313 (44	11) 313 (28	13) 313 (24	17) 313 (18
<u>-312</u>	<u>-312</u>	<u>-310</u>	<u>-308</u>	<u>-308</u>	<u>-312</u>	<u>-306</u>
1	1	3	5	5	1	7

Since $2, 3, 5, 7, 11, 13$ and 17 do not divide 313 exactly, 313 is a prime number.

Time to teach	Asset Type	Theme	SubTheme
8 Minutes	Inquisitive Questions	Factors & Multiples	Prime Numbers

9. MS_Perfect numbers

Perfect numbers

Let us take the number 6 . The proper divisors of 6 are $1, 2$ and 3 .

$$1+2+3=6$$

Now let us take the number 28 . The proper divisors of 28 are $1, 2, 4, 7$ and 14 .

$$1+2+4+7+14=28.$$

A perfect number is a positive number that is equal to the sum of its proper divisors.

Alternate definition

A number is called a perfect number if all its factors add up to twice of itself.

Note: In the above examples that sum of all the factors of 6 is $1+2+3+6=12=2 \times 6$

Sum of all the factors of 28 = $1+2+4+7+14+28=56$

$$56 = 2 \times 28$$

The perfect numbers known till now are

6, 28, 496, 8128, 33550336, 8589869056, 137438691328,

2305843008139952128

Time to teach	Asset Type	Theme	SubTheme
3 Minutes	Main Script	Factors & Multiples	Perfect number

10. QA_Exercise-Types of Numbers

Assessment questions

- 13 and 31 are two prime numbers having the same digits. Find other three pairs of 2 digit prime numbers that have the same digits.
- List all the 2 digit prime numbers whose digits are also prime numbers.
- Express the following as the sum of twin prime numbers
 - a) 24
 - b) 144
- Check if the following numbers are prime numbers or not
 - a) 131
 - b) 203
 - c) 157
- Find the set of any 7 consecutive composite numbers between 1 and 100.
- State true or false
 - a) 1 is the smallest prime number.
 - b) The sum of three odd numbers is even.
 - c) All prime numbers except 2 are odd.
 - d) 28 is a perfect number.
 - e) There are infinite prime numbers.

Answers

1. (17,71), (37,73) and (79,97)
2. 23, 37, 53, 73
3. a) 13 and 11 b) 73 and 71
4. a) 131

Since $11^2 = 121 < 131$ and $13^2 = 169 > 131$

Check whether the prime numbers 2, 3, 5, 7, 11

divide 131.

None of them divide 131. Hence 131 is prime.

b) 203

Since $13^2 = 169 < 203$. The next prime is 17 and $17^2 = 289 > 203$

Check whether the primes 2, 3, 5, 7, 11 and 13 divide

203.

Since 203 is divisible by 7, it is not a prime number.

c) 157

Since $11^2 = 121 < 157$ and $13^2 = 169 > 157$

Check if 157 is divisible by 2, 3, 5, 7, 11.

None of them divide 157. Hence 157 is prime.

5. 90, 91, 92, 93, 94, 95, 96

6. a) false. 1 is neither prime nor composite

b) false. Sum of three odd numbers is odd.

c) true

d) true

e) true

Interesting fact : Make a three digit number with 1, 1 and 3. All the possible combinations are 113, 131 and 311. All these are prime numbers.

This is an example for permutable primes.

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Assessments	Factors & Multiples	Prime Numbers

11. MS_Tests of divisibility of numbers

Tests of Divisibility for numbers

Let us say you appear for Maths Olympiad exam and you get a question to check if 54, 32, 669 is divisible by 11 or not?

You have to give the answer in 30 seconds. What will you do? Of course, you can do the normal division. But this will take more time.

Now we will learn shortcuts to answer these kinds of questions. If you know the divisibility test for 11, you can answer this easily.

Let us learn the divisibility test for some numbers.

Divisibility test for 2

If the digit in unit's place of a number is even then the number is divisible by 2.

For example, 118 is divisible by 2 since the digit in the unit's place is 8 which is even.

Divisibility test for 5

If the digit in the unit's place of a number is either 0 or 5 then it is divisible by 5.

For example, 110 and 115 are divisible by 5 since the digit in the unit's place is 0 and 5 respectively.

Divisibility test for 10

If the digit in the unit's place of a number is 0, it is divisible by 10.

For example, 230 is divisible by 10 since the digit in the unit's place is 0.

Divisibility test for 3

If the sum of all the digits of a number is a multiple of 3, then the number is divisible by 3.

For example, in 108, sum of the digits = $1 + 0 + 8 = 9$.

9 is a multiple of 3.

Therefore, 108 is a multiple of 3.

Divisibility test for 9

If the sum of the digits of any number is a multiple of 9, then the number is divisible by 9.

For example, in 729, sum of the digits is $7+2+9=18$.

Since 18 is a multiple of 9, 729 is divisible by 9.

Divisibility test for 7

Subtract twice the digit in the unit's place of the number from the rest of the number till you get a single digit.

If it is either 0 or 7, then the number is divisible by 7.

For example, let us take the number 1281 and see if it is divisible by 7. We have to follow the following steps for divisibility test for 7.

1. Take the digit in the unit's place and multiply by 2. Here 1 is there in the unit's place we get $1 \times 2 = 2$
2. Subtract twice the digit in unit's place from rest of the number. Here, $128 - 2 = 126$
3. Repeat the above steps till you get a single digit. If you get either 0 or 7, then the number is divisible by 7.
4. Now note that in 126, 6 is in the unit's place. $2 \times 6 = 12$.

Subtracting 12 from the rest of the number that is 12, we get $12 - 12 = 0$. Hence, 1281 is divisible by 7.

Divisibility test for 4

For numbers with three or more digits, if the number formed by the last two digits of the number is a multiple of 4, then it is divisible by 4.

For example, let us take the numbers 1416, 232. Note that the last two digits in these numbers (16, 32) are multiples of 4. Hence, 1416 and 232 are divisible by 4.

For two digit numbers, you have to divide in the normal way.

Divisibility test for 8

In a number of four or more digits, if the number formed by the last three digits is divisible by 8, then the number is divisible by 8.

For example, let us take the number 1248. Since 248 is divisible by 8, 1248 is divisible by 8.

For three digit and two digit numbers you have to divide in the usual way.

Divisibility test for 11

If the difference between the sum of all the digits in the odd places and the sum of all the digits in the even places of a number is either 0 or a multiple of 11, then the number is divisible by 11.

Let us go back to the question if 54, 32, 669 is divisible by 11 or not.

1. Find the sum of all the digits in the odd places (from right). In the above number they are 9, 6, 3

and 5. $9+6+3+5=23$

2. Find the sum of all the digits in the even places (from right). In the above example they are 6, 2

and 4. $6+2+4=12$

3. Find the difference between the two sums got above. In the above example it is

$23-12=11$

If the number got in the 3rd step is either 0 or a multiple of 11, then the number is divisible by 11. In the above example, since we got 11, the number 54,32,669 is divisible by 11.

Let us summarize all the divisibility rules learnt above:

No.	Divisibility rule
2	The digit in the unit's place is either 0,2,4,6 or 8
3	Sum of all the digits of the number is a multiple of 3
4	Number formed by the last two digits of a number is a multiple of 4
5	The digit in the unit's place is either 0 or 5
7	Difference of twice the digit in the unit's place from the rest of the number repeated till a single digit is got is either 0 or 7
8	Number formed by the last three digits of a number is a multiple of 8
9	Sum of all the digits of the number is a multiple of 9.
11	The difference between the sum of the digits in odd places and sum of the digits in even places is either 0 or 11.

Rules of divisibility

1. If a number is divisible by another number, then the first number is divisible by each of the factors of the second number.

For example, 42 is divisible by 21. Is it divisible by the factors of 21? The factors of 21 are 1, 3 and 7 apart from itself.

$42 \div 3 = 14$, $42 \div 7 = 6$. $42 \div 1 = 42$.

Hence, we see that 42 is divisible by 21, it is also divisible by each of the factors of 21.

Let us take 24 is divisible by 12.

1, 2, 3, 4 and 6 are the other factors of 12.

Is 24 divisible by all the factors of 12?

$24 \div 1 = 24$, $24 \div 2 = 12$

$24 \div 3 = 8$

$24 \div 4 = 6$

$$24 \div 6 = 4$$

Again, we see that since 24 is divisible by 12, 24 is also divisible by each of the factors of 12.

2. If two numbers are divisible by a number, then their sum is also divisible by that number.

We know that both 20 and 25 are divisible by 5.

Is their sum $20+25=45$ also divisible by 5?

Yes, $45 \div 5 = 9$.

Let us take the number 54 and 63. Both are divisible by 9.

Is their sum $54+63=117$ also divisible by 9?

Yes, $117 \div 9 = 13$.

3. If two numbers are divisible by a number, then their difference is also divisible by that number.

For example, both 56 and 35 are divisible by 7. Their

difference $56-35=21$

Is 21 divisible by 7?

Yes, $21 \div 7 = 3$.

Both 99 and 33 are divisible by 11.

Is their difference $99-33=66$ also divisible by 11?

Yes, $66 \div 11 = 6$.

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Main Script	Factors & Multiples	Divisibility tests and rules for numbers, Divisibility tests and rules for numbers

12. QA_Exercise-Tests of Divisibility

I. Choose the right answer

1. 12458 is divisible by which of the following numbers

- a) 2 b) 3 c) 8 d) 4

2. 34850 is divisible by which of the following numbers

- a) 2 b) 10 c) 5 d) all of these

3. The number 72981 is divisible by

- a) 3 b) 6 c) 9 d) both 3 and 9

I. The smallest digit in the blank space of the number 73_2 to be divisible by 3 is

- a) 0 b) 1 c) 2 d) 3

I. The number 75 is divisible by

- a) 3 b) 5 c) 15 d) all of these

II. State true or false

1. If a number is divisible by 35, it is also divisible by 5 and 7.
2. Since the numbers 49 and 56 are divisible by 7, 105 is also divisible by 7.
3. The number 345728 is divisible by 8.
4. The number 8172 is divisible by both 9 and 6.
5. 145541 is divisible by 11.
6. If 240 and 216 are both divisible by 6, their difference is also divisible by 6.
7. If a number is divisible by 8, it is also divisible by 4.
8. If a number is divisible by 4, it is also divisible by 8.

Answer

I.

1. a
2. d
3. d
4. a
5. d

II.

1. True
2. True
3. True
4. True
5. True
6. True
7. True
8. False

Time to teach	Asset Type	Theme	SubTheme
---------------	------------	-------	----------

10 Minutes	Assessments	Factors & Multiples	Divisibility tests and rules for numbers,
------------	-------------	---------------------	---

13. MS_Prime factorization

Prime factorization

When we write $12 \times 3 = 36$ this is called multiplication.

When we write the other way around $36 = 12 \times 3$, we are writing 36 as a product of its factors, this is called **factorization**. We have written 36 as the product of a composite number and a prime number.

36 can also be written as $36 = 2 \times 2 \times 3 \times 3$. Now, we have written 36 as a product of only prime numbers.

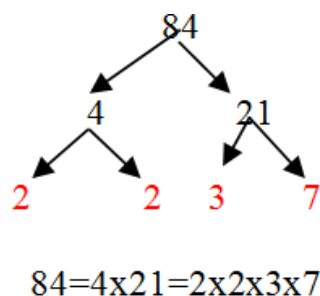
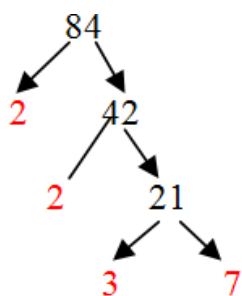
Let us take $42 = 6 \times 7$. We have written 42 as a product of a composite number and a prime number.

The other way to factorize is $42 = 2 \times 3 \times 7$. In this we have expressed 42 as a product of only prime numbers.

Expressing a composite number as a product of prime numbers is called prime factorization.

Factor tree method for Prime Factorization

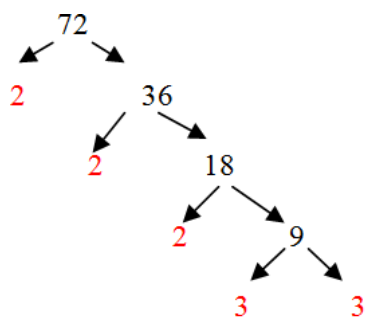
We have seen about the factor tree in a previous section. If we split the factors of a number till we get only prime numbers, we can get the prime factorization of the given number. Let us draw the factor tree for 84.



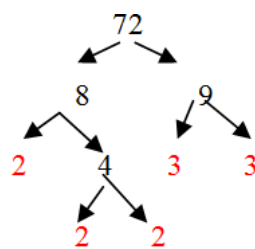
$$84 = 2 \times 42 = 2 \times 2 \times 21 = 2 \times 2 \times 3 \times 7$$

$$84 = 4 \times 21 = 2 \times 2 \times 3 \times 7$$

We see that in both ways 84 can be written as a unique product of primes $= 2 \times 2 \times 3 \times 7$. Let us take one more number 72 and draw the factor tree.



$$72 = 2 \times 36 = 2 \times 2 \times 2 \times 3 \times 3$$



$$72 = 8 \times 9 = 2 \times 4 \times 9 = 2 \times 2 \times 2 \times 3 \times 3$$

In this case also we see that 72 can be written as a unique product of primes = $2 \times 2 \times 2 \times 3 \times 3$.

Every composite number can be expressed as a product of primes in an unique way.

Try with two more composite numbers of your choice.

Division method for prime factorization

In this method we have to repeatedly divide the given number by least prime number it is divisible by, till we get a 1.

Let us take the number 112. What is the least prime number? Is 112 divisible by 2? Yes. Let us divide 112 by 2.

2	112	
2	56	→ Since 56 is also divisible by 2, we divide 56 by 2
2	28	→ Since 28 is also divisible by 2, we divide it by 2
2	14	→ Again 14 is divisible by 2, we divide it by 2
7	7	→ we get 7 which is a prime number .So divide it by itself.
	1	→ Since we got a 1, we will stop it here.

The prime factorization of 112 is $2 \times 2 \times 2 \times 2 \times 7$.

Let us take another number 120. Start dividing with the least prime number, it is divisible by. In this case 120 is divisible by 2.

2	120	→	Divide 120 by 2
2	60	→	Divide 60 by 2
2	30	→	Divide 30 by 2
3	15	→	since 15 is not divisible by 2, let us divide it with the next prime number 3
5	5	→	since 5 is a prime number we divide it by itself
	1	→	since we got a 1, we will stop it here

The prime factorization of 120 is $2 \times 2 \times 2 \times 3 \times 5$.

Time to teach	Asset Type	Theme	SubTheme
10 Minutes	Main Script	Factors & Multiples	Prime factorization

14. MS_Highest Common Factor

Highest Common factor (H.C.F)

Let us write down all the common factors of 12 and 16.

12- 1, 2, 3, 4, 6, 12

16- 1, 2, 4, 8, 16

The common factors of 12 and 16 are 1, 2 and 4. What is the highest common factor of 12 and 16? It is 4.

H.C.F of two or more numbers is the greatest of all the common factors of the given numbers.

The highest common factor is also called G.C.D (greatest common divisor).

Coprime numbers

Any two numbers whose H.C.F. is 1 are called coprime numbers.

For example

- Coprime numbers can be composite like 8 and 9.

Factors of 8 are 1, 2, 4 and 8

Factors of 9 are

1, 3 and 9 H.C.F is 1.

- Coprime numbers can be both prime like 2 and 3

Factors of 2 are 1 and 2

Factors of 3 are 1

and 3 H.C.F. is 1.

In fact, all pairs of prime numbers are coprime.

- Coprime numbers can be one prime and one composite like 4 and 5.

Factors of 4 are 1, 2 and 4

Factors of 5 are 1

and 5

H.C.F. is 1.

Rule of divisibility

If a number is divisible by two or more coprime numbers, then it is divisible by their product also.

For example, 24 is divisible by 3 and 4. It is also divisible by their product 12.

Finding H.C.F. of two numbers by listing common factors

In this method, all the factors of the two numbers are written down. Then the common factors are noted. The highest of all the common factors is the H.C.F of the two numbers.

For example, let us find the H.C.F. of 48 and 54.

Write down all the factors of 48 and 54.

48-- 1, 2, 3, 4, 6, 8, 16, 24, 48

54-- 1, 2, 3, 6, 9, 18, 27, 54

The common factors of 48 and 54 are 1, 2, 3, 6.

The highest common factor of 48 and 54 is 6.

H.C.F. of 48 and 54 is 6.

Finding H.C.F of two numbers using prime factorization

In this method, divide the two numbers separately with the least prime number they are divisible by. Continue the division of the quotient till you get quotient 1.

Express the numbers as the product of primes.

Mark the common factors and multiply them to find the H.C.F. of the numbers.

For example - Let us find out the H.C.F of 36 and 68.

<u>2</u>	36	<u>2</u>	68
2	18	2	34
3	9	17	17
3	3		1
	1		

$$36 = 2 \times 2 \times 3 \times 3$$

$$68 = 2 \times 2 \times 17$$

The common prime factors in both the numbers are 2 and 2.

Find their product.

$$2 \times 2 = 4.$$

Hence, the H.C.F of 36 and 68 is 4.

Examples

Let us see some examples from daily life where finding H.C.F. of two numbers is useful.

Example 1 –

Let us say that there are 24 boys and 18 girls in your class. Your Madam wants to make the least number of groups of boys and girls so that each group has the same number of students for a dance program. How many students will be in each group? Finding the H.C.F of 24 and 18, she can get the right number. Factors of

24 -- 1, 2, 3, 4, 6, 8, 12, 24

18-- 1, 2, 3, 6, 9, 18

Note that the highest common factor of 24 and 18 is 6. Hence, there can be 3 groups of 6 each for girls and 4 groups of 6 each for boys.

Example 2-

Let us say that you are collecting groceries for donating to a flood relief camp. You have collected 75 kg of rice and 60 kg toor dal (Arhar/Tuvar, Split pigeon peas). To pack them equally, you have to find the maximum value of weight which a bag can hold to pack and send them in equal amounts. Find the H.C.F of 75 and 60.

Factors of

75 -- 1, 3, 5, 15, 75

60 -- 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60.

H.C.F is 15. Hence, you can make 5 bags of rice weighing 15 kg each and 4 bags of dal weighing 15 kg each and send them. You need to buy

5+4=9 gunny bags which can hold 15 kg.

Properties of H.C.F

H.C.F is always smaller than or equal to the smallest of the given numbers.

If one of the two numbers is the factor of the other, then it is the H.C.F of the two numbers.

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Main Script	Factors & Multiples	HCF

15. MS_Least Common Multiple

Least Common Multiple

Let us write down the multiples of 3 and 4

₃ 3, 6, 9, 12, 18, 21, 24, 27, 30...

₄ 4, 8, 12, 16, 20, 24, 28, 32, 36, 40...

What are the common multiples of 3 and 4? They are 12, 24, 36, ...

The **least common multiple** is 12. **It is the smallest of the common multiples of any two numbers.** Its short form is L.C.M.

Properties of L.C.M.

L.C.M of two or more numbers is always greater than or equal to the largest number.

If one of the two numbers is a multiple of the other, then it is the L.C.M. of the two numbers.

There are two methods to find the L.C.M. of given numbers

Prime factorization method to find the L.C.M.

Let us take the number 20 and 36. Let us prime factorize them by factor tree method.



Now express the numbers in the exponential form.

$$20 = 2^2 \times 5$$

$$36 = 2^2 \times 3^2$$

Now, take each number with the greatest exponent and multiply.

Here, the highest power of 2 is 2, 3 is 2 and 5 is 1.

Hence, the L.C.M. = $2 \times 3^2 \times 5^2 = 4 \times 9 \times 5 = 180$

Division method to find the L.C.M.

In this method, we divide all the numbers by the least prime number that will divide all of them exactly. Repeat the process till the remainders are no more divisible by the same number. Multiply the numbers in the column and along the row at the end. This is the L.C.M of the numbers.

Example-

Let us take the numbers 24, 36 and 88.

2	24, 36, 84	→ divide all the numbers by the least prime number 2
2	12, 18, 42	→ divide all the numbers by 2
3	6, 9, 21	→ since they are not divisible by 2, divide them by 3
	2, 3, 7	→ stop here since they are all prime numbers

Multiply all the numbers along the red arrows.

L.C.M of 24, 36 and 84 = $2 \times 2 \times 3 \times 2 \times 3 \times 7 = 504$.

Note that the L.C.M. of two co-prime numbers is their product.

For example, the L.C.M. of co-prime numbers 4 and 5 is $4 \times 5 = 20$.

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Main Script	Factors & Multiples	LCM, LCM

16. MS_Relation between H.C.F and L.C.M.

Relation between H.C.F and L.C.M of any given numbers

Example -

Let us take 36 and 56 and prime factorize them

2	36
2	18
3	9
3	3
	1

2	56
2	28
2	14
7	7
	1

$36 = 2 \times 2 \times 3 \times 3$

$$56 = 2 \times 2 \times 2 \times 7$$

H.C.F. of 36 and 56 is $= 2 \times 2 = 4$

Expressing 36 and 56 in the exponential form we get

$$36 = 2^2 \times 3^2$$

$$56 = 2^3 \times 7$$

To find the L.C.M, take the greatest exponents of 2, 3 and 7.

The greatest exponent of 2 is 3, 3 is 2, 7 is 1.

$$\text{L.C.M} = 2^3 \times 3^2 \times 7 = 8 \times 9 \times 7 = 504$$

$$\text{H.C.F} \times \text{L.C.M} = 4 \times 504 = 2016$$

$$\text{Product of the numbers} = 36 \times 56 = 2016$$

$$36 \times 56 = 4 \times 504$$

$$\text{H.C.F} \times \text{L.C.M} = \text{Product of the numbers}$$

The product of H.C.F and L.C.M of any two numbers is equal to the product of the numbers.

For any two numbers a and b

$$\text{H.C.F} (a, b) \times \text{L.C.M} (a, b) = a \times b$$

This rule is true only for two numbers.

We can find out H.C.F of two number if we know their product and L.C.M

Example 1 -

If the product of two numbers is 144 and their L.C.M is 48, find their H.C.F.

Since $\text{H.C.F} \times \text{L.C.M} = \text{Product of the two numbers}$

$$\text{H.C.F.} = \text{Product of two numbers} / \text{L.C.M}$$

$$= 144/48 = 3$$

Thus H.C.F = 3.

Example 2-

If the H.C.F of two numbers is 2 and their product is 120, find their L.C.M.

Since $\text{H.C.F.} \times \text{L.C.M} = \text{Product of the numbers}$

$$\text{L.C.M} = \text{Product of the numbers} / \text{H.C.F}$$

$$= 120/2$$

$$= 60$$

Thus, L.C.M = 60.

Example 3-

If the H.C.F of two numbers is 6, L.C.M is 84 and one of the numbers is 12, find the other number.

Let the unknown number be 'y'.

Since product of the numbers = H.C.F. x L.C.M.

$$12 \times y = 6 \times 84$$

$$y = (6 \times 84) / 12 = 42$$

Thus, the other number is 42.

Time to teach	Asset Type	Theme	SubTheme
10 Minutes	Main Script	Factors & Multiples	LCM

17. DD_L.C.M.

Examples from daily life for application of L.C.M.

Example 1

Nishchay has a baby sister Niyathi. Nishchay has his meals- milk or snacks in an interval of every 3 hours and Niyathi has in an interval of every 2 hours. All the other members in his family have in an interval of 4 hours. Given that all of them had their breakfast at 8 a.m., when will they both have their next meal together with the rest of the family?

Solution:

Find the L.C.M. of 2,3 and 4

Multiples of 2 – 2, 4, 6, 8, 10, 12, 14,...

Multiples of 3 - 3, 6, 9, 12, 15,...

Multiples of 4 - 4, 8, 12, 16,...

The least common multiple of 2,3 and 4 is 12.

They had breakfast with rest of the family at 8 a.m.

They will have their next meal with the rest of the family at 8 a.m. + 12 hours = 8 p.m.

Example 2

Ronit, Sanjay and Praveen took part in a running race. Ronit takes 6 minutes to run one lap, Sanjay takes 3 minutes to run one lap and Praveen takes 5 minutes to run one lap. If all of them start the race at the same time, how many minutes will it take for all of them to be at the starting point again?

Solution:

Let us write down the time taken by them to complete the laps.

Ronit - 6, 12, 18, 24, 30

Praveen - 5, 10, 15, 20, 25, 30

Sanjay - 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

We see that the least common multiple is 30 minutes. They will all be together at the starting point after 30 minutes.

Time to teach	Asset Type	Theme	SubTheme
5 Minutes	Day-to-day Relevance	Factors & Multiples	LCM

18. QA_Exercise-H.C.F. and L.C.M.

1. Find the H.C.F of the following numbers by prime factorization

a) 48 and 94

b) 124 and 36

2. Find the L.C.M of the following numbers by prime factorization method

a) 24 and 36

b) 21 and 54

3. Find the L.C.M of the following numbers by division method

a) 90, 54, 36

b) 27, 99, 45

4. In an anganwadi (rural child care center), 80 L of milk and 60L of soup is served for breakfast along with toast for 400 children.

What is the maximum capacity of a measuring cup which can measure milk and soup in equal amounts for one child.

How much milk and soup will each child get?

5. Three traffic lights in a city at different junctions change in 54 seconds, 72 seconds and 108 seconds. Provided they all changed at 6 a.m, when will they change next?

Answers

1. a)

$$48 = 2 \times 2 \times 2 \times 3$$

$$94 = 2 \times 47$$

$$\text{H.C.F.}(48, 94) = 2$$

b)

$$124 = 2 \times 2 \times 31$$

$$36 = 2 \times 2 \times 3 \times 3$$

$$\text{H.C.F.}(124, 36) = 2 \times 2 = 4$$

2. a)

$$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$$

$$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$$

$$\text{L.C.M}(24, 36) = 2^3 \times 3^2 = 8 \times 9 = 72$$

b)

$$21 = 3^1 \times 7^1$$

$$54 = 2 \times 3 \times 3 \times 3 = 2^1 \times 3^3$$

$$\text{L.C.M.}(21, 54) = 2^1 \times 3^3 \times 7^1 = 378.$$

$$\begin{array}{l} 3. a) \quad \begin{array}{|l} 2 \quad 90, 54, 36 \\ \hline 3 \quad 45, 27, 18 \\ \hline 3 \quad 15, 9, 6 \\ \hline \end{array} \end{array}$$

$$\begin{array}{c} 5, 3, 2 \\ \longrightarrow \end{array}$$

$$\text{L.C.M.}(90, 54, 36) = 2 \times 3 \times 3 \times 5 \times 3 \times 2 = 540$$

$$\begin{array}{l} b) \quad \begin{array}{|l} 3 \quad 27, 99, 45 \\ \hline 3 \quad 9, 33, 15 \\ \hline \end{array} \end{array}$$

$$\begin{array}{c} 3, 11, 5 \\ \longrightarrow \end{array}$$

$$\text{L.C.M.}(27, 99, 45) = 3 \times 3 \times 3 \times 11 \times 5 = 1485$$

4. 80 L of milk and 60 L of soup is distributed among 400 children.

1 child gets $\frac{80,000}{400}$ ml of milk and $\frac{60,000}{400}$ ml of soup since 1 L = 1000 ml

Each child gets 200 ml milk and 150 ml of soup.

To find the capacity of a container which can measure 200 ml and 150 ml exactly, find

H.C.F.(200, 150)

$$200 = 2 \times 2 \times 2 \times 5 \times 5$$

$$150 = 2 \times 3 \times 5 \times 5$$

$$\text{H.C.F.}(200, 150) = 2 \times 5 \times 5 = 50$$

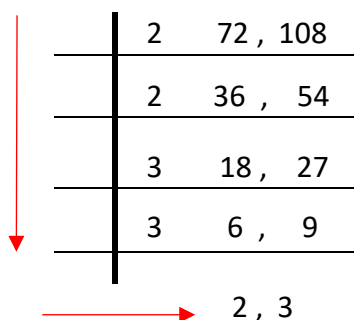
Hence, a 50 ml cup can measure the quantity of 200 ml of milk and 150 ml of soup exactly.

Each child will get 4 cups (200/50) of milk and 3 cups (150/50) of soup.

5. Since we have to find when the traffic lights change again together find the L.C.M. (54, 72, 108) seconds.

Since 108 is a multiple of 54, L.C.M. (108, 54) = 108

Let us find the L.C.M. of 72 and 108



L.C.M (54, 72, 108) = $2 \times 2 \times 3 \times 3 \times 2 \times 3 = 216$ seconds

(216)/60 minutes = 3.6 minutes = 3 minutes and 36 seconds

Since they all changed at 6:00:00 a.m, they will change again simultaneously at 6:03:36 a.m.

Time to teach	Asset Type	Theme	SubTheme
15 Minutes	Assessments	Factors & Multiples	LCM, HCF

19. SA_Activity in Factors and Multiples

Aim: To demonstrate factors, multiples, H.C.F. and L.C.M. through an activity.

Materials required – Buttons of two different colours, paper strips 15cmx5 cm, glue.

Procedure-

Each student will bring buttons of two different colours, paper strips and glue.

1. Factors –

Instructions to teacher: Teacher can demonstrate with 1 or 2 examples as follows:

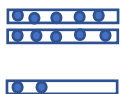
To find the factors of a given number, say 12, we have to take 12 buttons.

To check if 4 is a factor, place 4 buttons on a paper strip, 4 more on the second paper strip, repeat till all the 12 buttons are placed. If there are equal number of buttons on each strip and there are no more buttons remaining, then the number is a factor of the given number.



In this case, 4 is a factor of 12.

Let us see if 5 is a factor of 12 or not.



The number of buttons on each strip are not equal. There are 2 buttons (not 5) in the last strip, hence 5 is not a factor of 12.

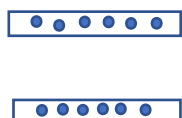
1. Choose any number. Do as shown above with all numbers less than or equal to the number chosen.
2. Note down all the factors of the number.

2. Multiples -

1. Choose any number, say 6.
2. Stick 6 buttons (the number chosen) on the paper strip.
3. Make 5 strips like this
4. Place the strips together and count the total number of buttons.

For example,

Take 2 strips, place them one below the other.



Count the number of buttons. There are 12. 12 is a multiple of 6.

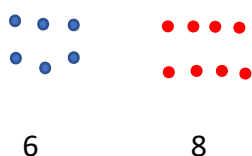
To find the next multiple, place 3 strips with 6 buttons each one below the other. Count the number of buttons. There are 18. 18 is a multiple of 6.

Similarly, place 4 strips and 5 strips of 6 buttons each one below the other and get the next two multiples of 6.

3.H.C.F. of two numbers

Let children take any two numbers of their choice, say 6 and 8.

Children have to take buttons of different colours and arrange them in rows such that there are equal number of buttons in each row for each number. You have to arrange the buttons in such a way that there are equal number of rows for both the numbers.



6

8

Now, count the number of rows. That is the H.C.F. of the two numbers.

H.C.F. of 6 and 8 is 2.

If you arrange in another way like:



3 rows with two buttons for 6

4 rows with 2 buttons for 8

You won't get the same number of rows.

Look for an arrangement with equal number of rows to find H.C.F.

4. To find L.C.M. of two numbers

Children again select any two numbers of their choice. Say 6 and 8.

Take some paper strips and stick 6 buttons of one colour.

Take some more paper strips and stick 8 buttons of another colour.

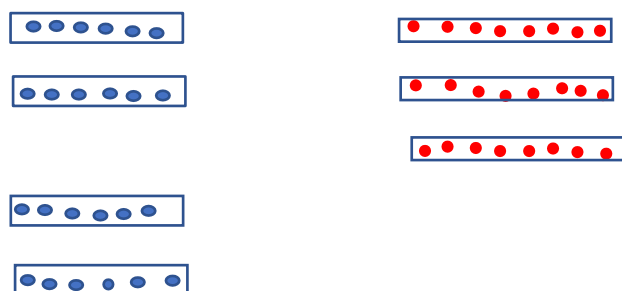
Let them arrange strips with 6 buttons one below the other and strips with 8 buttons one below the other in two rows, three rows and so on.

Count the total number of buttons of same colour.

If they are equal then the total number of buttons of same colour is the L.C.M. of the two numbers.

By placing two strips of 6 buttons and two strips of 8 buttons, we get 12 for 2 strips with 6 buttons and 16 for two strips with 8 buttons.

Since we did not get equal number of buttons, place four strips of 6 buttons and three strips of 8 buttons in three rows. We get 24 buttons for each colour.



Since we got equal number of buttons in this case, total number of buttons in one colour is the L.C.M. L.C.M. of 6 and 8 is 24.

Time to teach	Asset Type	Theme	SubTheme
20 Minutes	Suggested Activity	Factors & Multiples	LCM, HCF, LCM, HCF

20. DD_Factors and Multiples in daily life

Applications of H.C.F. and L.C.M. in everyday life

We use the knowledge of factors, multiples, H.C.F and L.C.M every day. Some of them are given below:

1. We go to market to buy 500 grams of apples, 500 is a factor of 1000 grams or 1 kg.

We pay money in rupees which is a multiple of standard rupee notes. If 1 kg of apples cost 200 rupees, 500 grams will cost 100 rupees. We can pay this by either giving ten 10 rupee notes, five 20 rupee notes or two 50 rupee notes. 100 is a multiple of 10, 20, 50.

2. Primary school children in your school have a period for 30 minutes and secondary school children have their period for 40 minutes. The snack break, lunch break etc. is calculated by L.C.M. of 30 and 40 minutes. So, all the children have their breaks at the same time.

3. When you have to stack the textbooks and notebooks in your rack such that each stack has the same thickness you have to calculate the H.C.F. of the thickness of textbook and notebook and put the number of books accordingly.

4. When your school starts after summer holidays, you cover all your textbooks and notebooks with brown paper. To know how much paper you have to buy, calculate the L.C.M. of the length of one textbook and notebook and multiply it with the number of books.

5. If three friends are going to school by different modes of transport like by bus, bike and cycle, to know when they meet you have to calculate the L.C.M of the time taken by each of them.

Now that you have learnt this chapter, relate what you see everyday to the terms used in this chapter. You will notice many more such applications.

Time to teach	Asset Type	Theme	SubTheme
5 Minutes	Day-to-day Relevance	Factors & Multiples	LCM, HCF, LCM, HCF

21. VC_Highest Common Factor

Highest common factor

In this chapter we have learned about the HCF (Highest common factor). The Highest Common Factor (HCF) of two or more given numbers is the highest (or

greatest) of their common factors.

For E.g. , the common factors of 12 and 16 are 1, 2 and 4 and the highest of these common factors is 4.

In this world, every group or team which works for a common objective need to have a highest common factor (HCF).

For example, a Football team's objective is to score goals against the opponent team and win a match with their opposite team. Hence the HCF is their desire to win the match and bring laurels to their team.

HCF in the teachers of a school as a group is to impart quality education to all the students in their school.

Generally, people in this world have some common physical & mental factors. But if Love and kindness is the highest common factor (HCF); that will make every one's life more comfortable and pleasant.

Discuss in the class:

1. Children! Can people have Highest Common factor?
2. Look for highest common factors (qualities) in your friends / everyone in the school.

Ans: Helpfulness, friendliness, sharing & caring, etc

3. What is the highest common factor of people in the world?

Ans: Honesty, compassion, tolerance, unity, love, kindness, etc

4. Rekha and Ramesh have both brown eyes, black hair; both are funny and both are kind. These are all common factors but what is the **highest** of these common factors?

Ans: Kindness

Time to teach	Value Type	Value Sub Type	Value Attribute
3 Minutes	Love		Group Activities

22. MS_Summary-Playing with Numbers

- Numbers which divide another number exactly without leaving any remainder are called the factors of that number.
- Multiples of a number are got by multiplying it by any natural number.
- Hardy-Ramanujan's number, 1729, is a special number.

- A number is called an even number when it is a multiple of 2. Otherwise, it is called an odd number.
- All the numbers which have only two factors (1 and themselves) are known as prime numbers.
- All the numbers which have more than two factors are called composite number.
- If the difference between two prime numbers is 2, they are called twin prime numbers.
- Prime numbers can be found out using the sieve of Eratosthenes.
- A number is prime if it is not exactly divisible by all the prime numbers which are less than its square root.
- A perfect number is a positive number that is equal to the sum of its proper divisors.
- Questions related to types of numbers can be solved.
- Factors of a number can be easily found using divisibility tests.
- Questions related to divisibility rules can be solved.
- Expressing a composite number as a product of prime numbers is called prime factorization.
- Every composite number can be expressed as a product of primes in an unique way.
- H.C.F of two or more numbers is the greatest of all the common factors
of the given numbers.
- Any two numbers whose H.C.F. is 1 are called coprime numbers.
- The least common multiple is the smallest of the common multiples of any two numbers.
- The product of H.C.F and L.C.M of any two numbers is equal to the product of the numbers.
- LCM can be used in daily life situations.
- Questions related to HCF and LCM can be solved.
- Factors, multiples, HCF and LCM can be demonstrated through an activity.
- Factors and multiples can be useful in daily life.
- Love and kindness can be practiced as highest common factor of a person's qualities.

Time to teach	Asset Type	Theme	SubTheme
5 Minutes	Main Script	Factors & Multiples	Factors, Multiples, HCF, LCM